Some Fixed Point Results on OWC Mappings for IFMS (Intuitionistic Fuzzy Metric Space).

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Abstract

OWC(occasionally weakly compatible mapping), Implicit relations, Complete intuitionistic fuzzy metric spaces, and a common fixed point.

Introduction

In 1965, Zadeh invented fuzzy set theory [17].Numerous writers have presented and analyzed diverse fuzzy metric space concepts in various ways [9], [4], and [5], also proved fixed point theorems with new findings in fuzzy metric spaces [6]. Park [12] introduced the idea of an intuitionistic fuzzy metric space and Alaca et al. [2] and Mohamad [10] have examined the fixed point outcomes in these spaces. The idea of OWC maps was first suggested by Al-Thagafi and N. Shahzad[3].

The aim of this paper is to show fixed point theorems for intuitionistic fuzzy metric spaces as an application of OWC mappings.

Preliminaries

Definition 1.1[2] If X is an arbitrary set, $\circ$ is a continuous t-norm[14], $\triangle$ is continuous t-conorm[14] and two fuzzy sets Φ, Ψ are defined on $X^2 \times (0, \infty)$ that holds:

(I-1) $\Phi(\nu, \omega, \tau) + \Psi(\nu, \omega, \tau) \leq 1$,
(I-2) $\Phi(\nu, \omega, 0) = 0$,
(I-3) $\Phi(\nu, \omega, \tau) = 1$ if and only if $\nu = \omega$,
(I-4) $\Phi(\nu, \omega, \tau) = \Phi(\omega, \nu, \tau)$,
(I-5) $\Phi(\nu, \omega, \tau) \circ \Phi(\omega, \lambda, s) \leq \Phi(\nu, \lambda, \tau + s)$,
(I-6) $\Phi(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ is left continuous,
(I-7) $\lim_{\tau\rightarrow\infty} \Phi(\nu, \omega, \tau) = 1$,
(I-8) $\Psi(\nu, \omega, 0) = 1$,
(I-9) $\Psi(\nu, \omega, \tau) = 0$ if and only if $\nu = \omega$,
(I-10) $\Psi(\nu, \omega, \tau) = \Psi(\omega, \nu, \tau)$,
(I-11) $\Psi(\nu, \omega, \tau) \triangle \Psi(\omega, \lambda, s) \leq \Psi(\nu, \lambda, \tau + s)$,
(I-12) $\Psi(\nu, \omega, .) : (0, \infty) \rightarrow (0,1]$ is right continuous,
(I-13) $\lim_{\tau\rightarrow\infty} \Psi(\nu, \omega, \tau) = 0$,

For all $\nu, \omega, \lambda \in X$ and $s, \tau > 0$, then (Φ, Ψ) is an intuitionistic fuzzy metric on X, where functions $\Phi(\nu, \omega, \tau)$ and $N(\nu, \omega, \tau)$ represent the degree of nearness and non-nearness of ν and ω with respect to τ, respectively. We say $(X, \Phi, \Psi, \circ, \triangle)$ is an intuitionistic fuzzy metric space or IFMS.

Remark 1.2 Every fuzzy metric space $(X, \Phi, \circ)$ is an IFMS of the form $(X, \Phi, 1\text{-}\Phi, \circ, \triangle)$ such that t-norm $\circ$ and t-conorm $\triangle$ are associated i.e. $\nu \triangle \omega = 1\text{-} ((1\text{-}\nu) \circ (1\text{-}\omega))$ for all $\nu, \omega \in X$.

Alaca et. Al.[2] also defined convergence of a sequence and Cauchy sequence and completeness of an intuitionistic fuzzy metric space.Park [13] has been given one important result about Cauchy sequence with certain condition as a lemma and other result has been very important for our research area as follows:

Lemma 1.3[13] For an IFMS $(X, \Phi, \Psi, \circ, \triangle)$, if $\Phi(\nu, \omega, \kappa\tau) \geq \Phi(\nu, \omega, \tau)$ and $\Psi(\nu, \omega, \kappa\tau) \leq \Psi(\nu, \omega, \tau)$ are true $\forall$ ν, $\omega \in X$, $\tau > 0$ and $\kappa \in (0, 1)$, then $\nu = \omega$.

Jungck [7, 8] defined compatible, weakly compatible mapping and coincidence points for two self mappings of an IFMS. We use the definition of OWC given by C. T. Aage and J. N. Salunke [1]. They give following lemma which is useful for our research as follows:

Lemma 1.4[1] For an IFMS $(X, \Phi, \Psi, \circ, \triangle)$, two owc self mappings F and G have unique point of coincidence such that $\mu = A\nu = S\nu$, then F and G have unique common fixed point μ.

Main Results

Following theorem is given by [12] for fuzzy metric space:

Theorem. For a complete fuzzy metric space $(X, \Phi, \circ)$ four self mappings F, G, U and V having OWC in pairs {F, U} and {G, V} satisfying following condition such that

$$\Phi(Fx, Gy, qt) \geq \alpha_1 \Phi(Ux, Vy, t) + \alpha_2 \Phi(Fx, Vy, t) + \alpha_3 \Phi(Gy, Ux, t)$$

For $q \in (0, 1)$ and for all $x, y \in X$, where $\alpha_1, \alpha_2, \alpha_3 > 0$, $\alpha_1 + \alpha_2 + \alpha_3 > 1$, then there exist a unique point $u \in X$ such that $Fu = Uu = u$ and a unique point $v \in X$ such that $Gv = Vv = v$. Moreover, $v = u$, so that there is a unique common fixed point of F, G, U and V.

Here we generalized this theorem for IFMS as follows:

Theorem 2.1 For complete IFMS $(X, \Phi, \Psi, \circ, \triangle)$ let the pairs {A, S} and {B, T} are OWC self mappings on X, for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$\Phi(A\nu, B\omega, \kappa\tau) \geq a_1 \Phi(S\nu, T\omega, \tau) + a_2 \Phi(A\nu, T\omega, \tau) + a_3 \Phi(B\omega, S\nu, \tau) \quad \ldots(i)$$

and

$$\Psi(A\nu, B\omega, \kappa\tau) \leq b_1 \Psi(S\nu, T\omega, \tau) + b_2 \Psi(A\nu, T\omega, \tau) + b_3 \Psi(B\omega, S\nu, \tau), \quad \ldots(ii)$$

Inequalities (i) and (ii) are true for all $\nu, \omega \in X$, where $a_1, a_2, a_3, b_1, b_2, b_3 > 0$, $a_1 + a_2 + a_3 > 1$ and $b_1 + b_2 + b_3 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, is a unique common fixed point of all four self mappings on X.

Proof: It is given that the pairs {A, S} and {B, T} be OWC, so there are points $\nu, \omega \in X$ such that $A\nu = S\nu$ and $B\omega = T\omega$. We now show that, $A\nu = B\omega$. If not, by inequality (i)

$$\begin{aligned}\Phi(A\nu, B\omega, \kappa\tau) &\geq a_1 \Phi(S\nu, T\omega, \tau) + a_2 \Phi(A\nu, T\omega, \tau) + a_3 \Phi(B\omega, S\nu, \tau)\\ &= a_1\Phi(A\nu, B\omega, \tau) + a_2 \Phi(A\nu, B\omega, \tau) + a_3\Phi(B\omega, A\nu, \tau)\\ &= (a_1 + a_2 + a_3) \Phi(A\nu, B\omega, \tau)\end{aligned}$$

this gives contradiction since $a_1 + a_2 + a_3 > 1$.

Similarly, by inequality (ii)

$$\begin{aligned}\Psi(A\nu, B\omega, \kappa\tau) &\leq b_1 \Psi(S\nu, T\omega, \tau) + b_2 \Psi(A\nu, T\omega, \tau) + b_3 \Psi(B\omega, S\nu, \tau)\\ &= b_1 \Psi(A\nu, B\omega, \tau) + b_2\Psi(A\nu, B\omega, \tau) + b_3\Psi(B\omega, A\nu, \tau)\\ &= (b_1 + b_2 + b_3) \Psi(A\nu, B\omega, \tau)\end{aligned}$$

We get contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.3 $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Let us assume that there is another point λ such that $A\lambda = S\lambda$, then by (i) and (ii), we have $A\lambda = S\lambda = B\omega = T\omega$. This gives, $A\nu = A\lambda$ and $\mu = A\nu = S\nu$,we have conclude that μ is the unique point of coincidence of A and S. By Lemma 1.4 μ is the only common fixed point of A and S, i. e. $\mu = A\mu = S\mu$. Similarly there is a unique point $\lambda \in X$ such that $\lambda = B\lambda = T\lambda$.

Let us assume that $\mu \neq \lambda$. We have,

$$\begin{aligned}\Phi(\mu, \lambda, \kappa\tau) &= \Phi(A\mu, B\lambda, \kappa\tau)\\ &\geq a_1\Phi(S\mu, T\lambda, \tau) + a_2 \Phi(A\mu, T\lambda, \tau) + a_3\Phi(B\lambda, S\mu, \tau)\\ &= a_1\Phi(\mu, \lambda, \tau) + a_2\Phi(\mu, \lambda, \tau) + a_3\Phi(\lambda, \mu, \tau)\\ &= (a_1 + a_2 + a_3) \Phi(\mu, \lambda, \tau)\end{aligned}$$

this gives contradiction since $(a_1 + a_2 + a_3) > 1$.

Similarly,

$$\begin{aligned}\Psi(\mu, \lambda, \kappa\tau) &= \Psi(A\mu, B\lambda, \kappa\tau) \\ &\le b_1 \Psi(S\mu, T\lambda, \tau) + b_2\Psi(A\mu, T\lambda, \tau) + b_3 \Psi(B\lambda, S\mu, \tau) \\ &= b_1 \Psi(\mu, \lambda, \tau) + b_2\Psi(\mu, \lambda, \tau) + b_3 \Psi(\lambda, \mu, \tau) \\ &= (b_1 + b_2 + b_3)\Psi(\mu, \lambda, \tau)\end{aligned}$$

again contradiction, because $(b_1 + b_2 + b_3) < 1$. And by Lemma 1.3, $\lambda = \mu$. Also by Lemma 1.4, λ is the common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (i) and (ii) ♦

Theorem 2.2 Let the pairs (A, S) and (B, T) are OWC self mappings on complete IFMS $(X, \Phi, \Psi, ^{o}, \triangle)$ for any $\nu, \omega \in X$ and $\tau > 0$ with positive number $\kappa \in (0, 1)$ such that

$$\Phi(A\nu, B\omega, \kappa\tau) \ge a_1 \min\{\Phi(S\nu, T\omega, \tau), \Phi(S\nu, A\nu, \tau)\} + b_1\min\{\Phi(B\omega, T\omega, \tau), \Phi(A\nu, T\omega, \tau)\} + c_1\Phi(B\omega, S\nu, \tau) \quad \ldots(iii)$$

and

$$\Psi(A\nu, B\omega, \kappa\tau) \le a_2 \min\{\Psi(S\nu, T\omega, \tau), \Psi(S\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, T\omega, \tau), \Psi(A\nu, T\omega, \tau)\} + c_2 \Psi(B\omega, S\nu, \tau) \quad \ldots(iv)$$

Inequalities (iii) and (iv) are true for all $\nu, \omega \in X$, where $a_1, a_2, b_1, b_2, c_1, c_2 > 0$, $a_1 + b_1 + c_1 > 1$ & $a_2 + b_2 + c_2 < 1$ then there exist a unique point $\mu \in X$ such that $A\mu = S\mu = \mu$ and a unique point $\lambda \in X$ such that $B\lambda = T\lambda = \lambda$. Moreover, $\lambda = \mu$, is a unique common fixed point of all four self mappings on X.

Proof : - It is given that the pairs {A, S} and {B, T} are OWC, so there are points ν, ω in IFM Such that $A\nu = S\nu$ and $B\omega = T\omega$. We claim that, $A\nu = B\omega$. If not, by inequality (iii)

$$\begin{aligned}\Phi(A\nu, B\omega, \kappa\tau) &\ge a_1 \min\{\Phi(S\nu, T\omega, \tau), \Phi(S\nu, A\nu, \tau)\} + b_1 \min\{\Phi(B\omega, T\omega, \tau), \Phi(A\nu, T\omega, \tau)\} + c_1\Phi(B\omega, S\nu, \tau) \\ &= a_1 \min\{\Phi(A\nu, B\omega, \tau), \Phi(A\nu, A\nu, \tau)\} + b_1\min\{\Phi(B\omega, B\omega, \tau), \Phi(A\nu, B\omega, \tau)\} + c_1\Phi(B\omega, A\nu, \tau) \\ &= a_1 \min\{\Phi(A\nu, B\omega, \tau), 1\} + b_1 \min\{1, \Phi(A\nu, B\omega, \tau)\} + c_1\Phi(B\omega, A\nu, \tau) \\ &= a_1 \Phi(A\nu, B\omega, \tau) + b_1 \Phi(A\nu, B\omega, \tau) + c_1\Phi(A\nu, B\omega, \tau) \\ &= (a_1 + b_1 + c_1)\Phi(A\nu, B\omega, \tau)\end{aligned}$$

Above inequality gives us contradiction because $a_1 + b_1 + c_1 > 1$

Similarly, by inequality (iv)

$$\begin{aligned}\Psi(A\nu, B\omega, \kappa\tau) &\le a_2 \min\{\Psi(S\nu, T\omega, \tau), \Psi(S\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, T\omega, \tau), \Psi(A\nu, T\omega, \tau)\} + c_2\Psi(B\omega, S\nu, \tau) \\ &= a_2 \min\{\Psi(A\nu, B\omega, \tau), \Psi(A\nu, A\nu, \tau)\} + b_2 \min\{\Psi(B\omega, B\omega, \tau), \Psi(A\nu, B\omega, \tau)\} + c_2\Psi(B\omega, A\nu, \tau) \\ &= a_2 \min\{\Psi(A\nu, B\omega, \tau), 1\} + b_2 \min\{1, \Psi(A\nu, B\omega, \tau)\} + c_2\Psi(B\omega, A\nu, \tau) \\ &= a_2 \Psi(A\nu, B\omega, \tau) + b_2 \Psi(A\nu, B\omega, \tau) + c_2 \Psi(A\nu, B\omega, \tau) \\ &= (a_2 + b_2 + c_2)\Psi(A\nu, B\omega, \tau)\end{aligned}$$

We get again contradiction, since $a_2 + b_2 + c_2 < 1$. And by Lemma 1.3, $A\nu = B\omega$, i.e. $A\nu = S\nu = B\omega = T\omega$. Suppose that there is another point λ such that $A\lambda = S\lambda$, then by (iii) and (iv), we have $A\lambda = S\lambda = B\omega = T\omega$. Hence we have, $A\nu = A\lambda$ and $\mu = A\nu = S\nu$ is the unique point of coincidence of A and S. By Lemma 1.4, μ is the only common fixed point of A and S, i. e. $\mu = A\mu = S\mu$. Similarly there is a unique point $\lambda \in X$ such that $\lambda = B\lambda = T\lambda$.

Let(Hyp.) $\mu \ne \lambda$. We have,

$$\begin{aligned}\Phi(\mu, \lambda, \kappa\tau) &= \Phi(A\mu, B\lambda, \kappa\tau) \\ &\ge a_1 \min\{\Phi(S\mu, T\lambda, \tau), \Phi(S\mu, A\mu, \tau)\} + b_1 \min\{\Phi(B\lambda, T\lambda, \tau), \Phi(A\mu, T\lambda, \tau)\} + c_1\Phi(B\lambda, S\mu, \tau) \\ &= a_1\min\{\Phi(\mu, \lambda, \tau), \Phi(\mu, \mu, \tau)\} + b_1 \min\{\Phi(\lambda, \lambda, \tau), \Phi(\mu, \lambda, \tau)\} + c_1\Phi(\lambda, \mu, \tau) \\ &= a1\min\{\Phi(\mu, \lambda, \tau), 1\} + b_1 \min\{1, \Phi(\mu, \lambda, \tau)\} + c_1 \Phi(\mu, \lambda, \tau) \\ &= a_1\Phi(\mu, \lambda, \tau) + b_1 \Phi(\mu, \lambda, \tau) + c_1\Phi(\mu, \lambda, \tau) \\ &= (a_1 + b_1 + c_1)\Phi(\mu, \lambda, \tau)\end{aligned}$$

We again get contradiction since $a_1 + b_1 + c_1 > 1$

Similarly,

$$\begin{aligned}\Psi(\mu, \lambda, \kappa\tau) &= \Psi(A\mu, B\lambda, \kappa\tau) \\ &\le a_2 \min\{\Psi(S\mu, T\lambda, \tau), \Psi(S\mu, A\mu, \tau)\} + b_2 \min\{\Psi(B\lambda, T\lambda, \tau), \Psi(A\mu, T\lambda, \tau)\} + c_2\Psi(B\lambda, S\mu, \tau) \\ &= a_2 \min\{\Psi(\mu, \lambda, \tau), \Psi(\mu, \mu, \tau)\} + b_2 \min\{\Psi(\lambda, \lambda, \tau), \Psi(\mu, \lambda, \tau)\} + c_2\Psi(\lambda, \mu, \tau) \\ &= a_2 \min\{\Psi(\mu, \lambda, \tau), 1\} + b_2 \min\{1, \Psi(\mu, \lambda, \tau)\} + c_2\Psi(\mu, \lambda, \tau) \\ &= a_2 \Psi(\mu, \lambda, \tau) + b_2 \Psi(\mu, \lambda, \tau) + c_2 \Psi(\mu, \lambda, \tau) \\ &= (a_2 + b_2 + c_2)\Psi(\mu, \lambda, \tau)\end{aligned}$$

Again gives contradiction, because $a_2 + b_2 + c_2 < 1$. And by Lemma 1.3 $\lambda = \mu$. Also by Lemma 1.4, λ is the common fixed point of A, B, S and T. The uniqueness of the fixed point holds from (iii) and (iv) ♦

Conclusion

We establish common fixed point solutions for occassionally weakly compatible in intuitive fuzzy metric spaces, which enhances and generalizes the work of several writers who have previously presented their findings in the fixed point theory of fuzzy metric spaces.

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