Rabinowitsch fluid flow with viscosity variarion between wide parallel rectangular-plate with squeeze-film characteristics

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Abstract

This study investigated the behavior of a squeeze film non-Newtonian fluid (NNF) flow with viscosity variation between wide parallel rectangular plates. Using the governing equation, boundary condition, and relationship between viscosity and film thickness, the non-linear modified Reynolds equation (NLMRE) is obtained. This equation has been solved by using the small perturbation technique and the Gauss quadrature formula. Numerous physical parameters including film pressure (FP), load carrying capacity (LCC), and squeeze response time (SRT) is calculated to find the characteristic of the wide parallel rectangular plate. According to the results, NNF with variable viscosity reduces the LCC, FP, and SRT of parallel plates. The influence of viscosity variation is one of the most important fluid properties that help in the design of bearing lubricants for engineering and industrial applications.

Keywords: squeeze film; viscosity variation; parallel rectangular-plate; Rabinowitsch fluid model

1. Introduction

To enhance the effectiveness of bearings, squeeze film devices between parallel plates have grown commonplace throughout time in a variety of engineering research and business fields. Gears, turbomachinery, disc clutches, viscous dampers, viscous-lock systems, and the mechanics of synovial joints in humans and other animals are a few examples of uses for the squeezing film structure.

The compression of Newtonian fluid-lubricated film bearings has been the subject of numerous studies. For squeeze film bearings, a wide range of film shapes have been examined by Pinkus and Sternlicht [1]. The squeeze film lubrication between two indefinitely long parallel plates was discussed by Cameron [2]. Hsu et al. [3] examined the squeezing film characteristics of conical bearings using the Rabinowitsch fluid model. By Naduvinamani et al. [4], the squeeze film lubrication between circular stepped plates is investigated.

Modified lubricants are used in commerce with a variety of additives to satisfy the diverse needs of machine systems. When additives are applied, these types of complicated fluids produce a NNF flow characteristic, which Spike [5] experimentally demonstrated. For NNF squeeze-film, researchers have developed a variety of squeeze film bearing models during the last few decades. For the two-dimensional Reynolds type equation for wide parallel rectangular plates, Lin et al. [6] employed the hydromagnetic fluid. The behavior of ferrofluid lubricant on rotating spheres and uneven plates was studied by Shah and Patel [7]. Wada and Hayashi [8, 9] carried out theoretical and experimental journal bearing research to investigate the properties of the Rabinowitsch fluid model (RFM). They discovered that the RFM is useful for the modern engineering variety of bearing models.

Lin et al. [10] used the RFM to study the impact of NNF on large parallel rectangular squeeze-film plates, while Huang and Tian [11] looked into the stable operation of a hydrostatic thrust bearing. The dilatant property increases the values of the bearing characteristics, while the effects of the pseudoplastic lubricant lead to a reduction in the bearing essence, according to their comparison with Newtonian fluid.

When analysing how the lubricant interacts with surfaces, it is better to take this into account because viscosity often varies with temperature and pressure. This covers both the difference in lubricant viscosity along the fluid layer and throughout it. A approach for examining the effects of viscosity fluctuation was put forth by Tipei [12], who considered a viscosity-film thickness relationship. A one-dimensional rough slider bearing surface served as the working example by Sinha et al. [13] as they investigated the effects of viscosity change brought on by the layer of micropolar fluid on rough slider bearing. The NLMRE for porous wall bearings was recently developed by Rao and Rahul [14] while taking viscosity fluctuation into account, and they discovered that porous material improves the bearings' performance.

The current model accounts for viscosity variation and squeeze film lubrication between wide parallel plates. It is resolved using the small perturbation approach and the five-point Gauss quadrature formula. Several physical parameters, including FP, LCC, and SRT, are plotted against film thickness, a non-linear parameter, and a viscous parameter to examine the features of the wide parallel plate. The most recent discoveries could be used in engineering research and business to develop bearing models that are more effective and efficient. The current problem is discussed in more detail in the section that follows.

2. Mathematical formulation

Shear stress (τ_{xy}) and strain rate $(\partial u/\partial z)$ are assumed to have a nonlinear (cubic) relationship that is

$$\tau_{xz} + k\,\tau_{xz}^3 = \mu\,\frac{\partial u}{\partial z}, \tag{1}$$

where $\hat{\mu}$ stand for the initial viscosity of the Newtonian lubricant. Three different types of fluids can be distinguished because of the non-linear component k, which described the properties of RFM:

- The lubricant is referred to as pseudo-plastic if $k > 0$,

- The lubricant is referred to as Newtonian if $k = 0$,
- The lubricant is referred to as dilatant if $k < 0$.

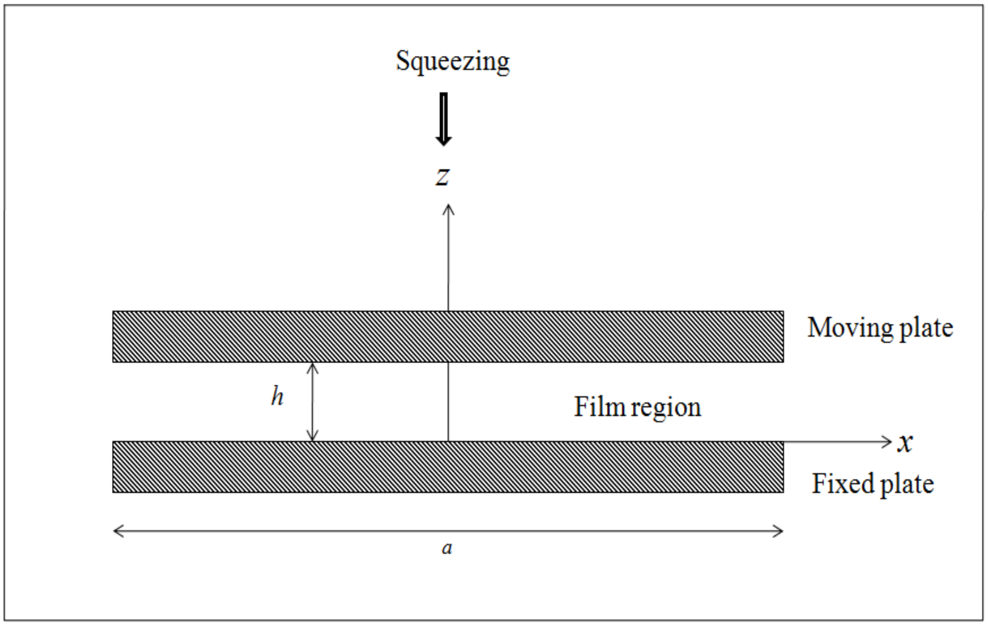

Fig. 1 Schematic of squeeze film configuration of parallel rectangular-plate lubricated with a RFM.

The schematic of a wide parallel plate with a squeeze film configuration is shown in Figure 1 and is lubricated with NNF. With a squeeze velocity of $(-dh/dt)$, the D width top plate moves in the $x-$direction. The hydrodynamic lubrication theory is assumed to be applicable. The film region is denoted by h and is located between the moving and fixed plates.There is no inertia or body forces in the assumed NNF. In Cartesian coordinates, the equations of incompressible continuity and momentum are written as (see Wada and Hayashi [8] and Lin et al. [10]):

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \tag{2}$$

$$\frac{\partial p}{\partial x} - \frac{\partial \tau_{xz}}{\partial z} = 0, \tag{3}$$

$$\frac{\partial p}{\partial z} = 0. \tag{4}$$

The following are the expressions for each component of velocity boundary conditions:

$$\text{(a)} \quad z = 0 : u = 0, \;\; w = 0, \tag{5}$$

$$\text{(b)} \quad z = h : u = 0, \; w = dh/dt. \tag{6}$$

where u and w are the velocity components in x and z directions.

3. Solution

The following method is used to determine the solution in order to determine the velocity in the fluid film in the positive $x-$direction. As a result of integrating the momentum equation (3),

$$\tau_{xz} = z\frac{\partial p}{\partial x} + c_1, \tag{7}$$

The velocity u will take the form after integrating, applying the boundary conditions, and including equation (7) into equation (1) as follows:

$$u = \frac{1}{2\mu}\left[\left(z^2 - zh\right)\frac{\partial p}{\partial x} + k\left(\frac{\partial p}{\partial x}\right)^3\left(\frac{z^4}{2} - h\,z^3 + \frac{3\,z^2h^2}{4} - \frac{z\,h^3}{4}\right)\right], \tag{8}$$

The continuity equation, velocity, and boundary condition have been applied to derive the NLMRE for the bearings characteristics, which is given by Eq. (9),

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x} + \frac{3}{20}kh^5\left(\frac{\partial p}{\partial x}\right)^3\right] = 12\mu\frac{dh}{dt}. \tag{9}$$

and pressure boundary conditions is represented by Eq.(10):

$$\left.\begin{aligned} &p = 0 \text{ at } x = \pm\frac{a}{2}, \\ &\frac{\partial p}{\partial x} = 0 \text{ at } x = 0 \end{aligned}\right\} \tag{10}$$

When examining the bearing property with viscosity fluctuation along the squeeze-film, the following viscosity-film thickness relation is taken into account (see Sinha et al. [13]).

$$\frac{\mu}{\mu_0} = \left(\frac{h}{h_0}\right)^Q; \; 0 \le Q \le 1, \tag{11}$$

where μ_0, h_0 initial viscosity and thickness.

The NLMRE in terms of viscosity parameter is given by substituting Eq. (11) into Eq. (9).

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x} + \frac{3}{20}kh^5\left(\frac{\partial p}{\partial x}\right)^3\right] = 12\mu_0\left(\frac{h}{h_0}\right)^Q\frac{dh}{dt}. \tag{12}$$

The parameters and variables with no dimensions are presented:

$$\left.x^* = \frac{x}{a},\; h^* = \frac{h}{h_0},\; P^* = \frac{ph_0^3}{\mu_0 a^2\left(-dh/dt\right)},\; \beta = \frac{k\mu_0^2 a^2\left(-dh/dt\right)^2}{h_0^4}.\right\}, \tag{13}$$

The dimensionless NLMRE would be as follows when using the dimensionless variables and parameters from Eq. (13) in Eq.

$$\frac{\partial}{\partial x^*}\left[h^{*3}\frac{\partial P^*}{\partial x^*}+\frac{3}{20}\beta h^{*5}\left(\frac{\partial P^*}{\partial x^*}\right)^3\right]=-12\left(h^*\right)^Q, \tag{14}$$

where β is a dimensionless non-linear factor of RFM. NNF behavior in prospective of β, the NNF behaves dilatants fluids for negative value and pseduplastic fluids for positive value of β as comrade to Newtonian fluids. The following is the dimensionless form of the boundary condition on film pressure:

$$\left.\begin{aligned} &P^*=0 \text{ at } x^*=\pm\frac{1}{2},\\ &\frac{\partial P^*}{\partial x^*}=0 \text{ at } x^*=0.\end{aligned}\right\} \tag{15}$$

The non-dimensional NLMRE (14) is remarkably non-linear when porosity and viscosity fluctuation parameters are included.

It is noticed that, as the value of $\beta \to 0$, The NLMRE is reduced to a Newtonian lubricant case.

$$\frac{\partial}{\partial x^*}\left[h^{*3}\frac{\partial P^*}{\partial x^*}\right]=-12\left(h^*\right)^Q. \tag{16}$$

The film pressure is difficult to calculate from NLMRE. Apply a small perturbation method to obtaine the film pressure, β at low for parameter values, $-0.0001\le\beta\le 0.0001$, then

$$P^*=P_0^*+\beta P_1^*+O(\beta^2) \tag{17}$$

By substituting this expansion into the non-dimensional NLMRE (14) and assembling the order $O(\beta^0)$ and order $O(\beta^1)$, one can extract two coupled equations that are responsible for the zero order pressure and the first order pressure, respectively.

$$\frac{d}{dx^*}\left[h^{*3}\frac{dP_0^*}{dx^*}\right]=-12\left(h^*\right)^Q, \tag{18}$$

and the other

$$\frac{d}{dx^*}\left[h^{*3}\frac{dP_1^*}{dx^*}\right]=-\frac{3}{20}h^{*5}\frac{d}{dx^*}\left(\frac{dP_0^*}{dx^*}\right)^3. \tag{19}$$

The boundaries for FP are as follows:

$$\left.\begin{aligned} &P_0^*=0 \quad \text{at } x^*=\pm\frac{1}{2};\ \frac{\partial P_0^*}{\partial x}=0 \text{ at } x^*=0,\\ &P_1^*=0 \quad \text{at } x^*=\pm\frac{1}{2};\ \frac{\partial P_1^*}{\partial x}=0 \text{ at } x^*=0.\end{aligned}\right\} \tag{20}$$

One can obtain by resolving these two differential equations

$$P^*=\frac{3h^{*Q}\left(1-4x^{*2}\right)}{2h^{*3}}+\frac{81\beta}{20h^{*7}}h^{*3Q}\left(16x^{*4}-1\right). \tag{21}$$

The LCC can be calculated by integrating the FP over the interval $\left[\frac{-a}{2},\frac{+a}{2}\right]$.

$$W = D\int_{\frac{-a}{2}}^{\frac{a}{2}} p\ dx, \tag{22}$$

One can derive the non-dimensional LCC following by performing above integration.

$$W^* = \frac{Wh_0^3}{\mu_0 a^3 D\left(-dh/dt\right)} = \frac{h^{*Q}}{h^{*3}} - \frac{81\beta}{25}\left(\frac{h^{*3Q}}{h^{*7}}\right). \tag{23}$$

The differential equation governing the variation of the fluid film height with the SRT must be obtained after substitution the value of t^* (dimensionless SRT) into Eq. (23).

$$\frac{dh^*}{dt^*} = -\left[\frac{h^{*Q}}{h^{*3}} - \frac{81\beta}{25}\left(\frac{h^{*3Q}}{h^{*7}}\right)\right] \tag{24}$$

where $t^* = \frac{Wh_0^2}{\mu_0 a^3 D} t$.

After separating the variables and integrating the equation, the SRT is derived by integrating the differential equation mentioned above with respect to h^* across the interval $\left[h^*, 1\right]$ obtained under the starting condition $h^*\left(t^*=0\right)=1$.

$$t^* = \int_{h^*}^{h^*=1} -\left[\frac{h^{*Q}}{h^{*3}} - \frac{81\beta}{25}\left(\frac{h^{*3Q}}{h^{*7}}\right)\right] dh^*, \tag{25}$$

Eq. (25) solved by using Gaussian Quadrature method to find the SRT.

4. Results and discussion

The RFM is used in this article to calculate the combined effects of NNF viscosity variation in squeeze film lubrication between wide parallel-plates. Furthermore, the results of Lin at el. [10] are retrieved as a limiting situation with a constant viscosity ($Q \to 0$) from the current problem. A small perturbation method is used to solve the NLMRE with viscosity parameters. Moreover, the Gaussian quadrature integral formula is utilized to compute the LCC and SRT of the plate, which was then resolved by Mathematica software. A non-linear dimensionless parameter $\beta\left(-1 \ll \beta \ll +1\right)$ distinguishes the NNF, while $\beta \to 0$ classifies a linear Reynolds equation. The change of the FP (P^*) with a coordinate axis (x^*) is depicted in Fig. 2 for various values of a β and Q with $h^* = 0.2$. The finding of Lin et al. [10] has been recovered as a limiting example of the iso-viscous $(Q=0.0)$ with varied values of β. In contrast to fluids with viscosity change $(Q=0.25$ and $Q=0.50)$, the FP is seen to be more prominent for fluids with constant viscosity $(Q=0.0)$. In addition, it explores how, in contrast to Newtonian fluid, the FP increases and decreases for dilatant fluid and pseudoplastic fluid. Fig. 3 compares the impact of viscosity change and NNF on the LCC of the wide parallel-plate. By graphing the variability of the LCC (W^*) with proportion to film thickness (h^*) for various values of the β and Q, this comparison is conducted in this figure. Dilatant, pseudo-plastic, and Newtonian fluids achieve maximum, least, and intermediate LCC for the isoviscous $(Q=0.0)$ condition, respectively.

Furthermore, they all deteriorate across the range. A similar pattern is exhibited when viscosity varies, with the exception of their magnitudes; in this case, the change in viscosity is to blame for a further drop in the LCC of the plates. For different values of Q and β, Figure4 illustrates the SRT (t^*) behavioral trend as a function of film thickness.

It has been shown that variances in outcomes lead to a reduction in the relationship between SRT and film height. The linear variation of LCC with non-linear parameter may be realized since the approach introduced a small disturbance to the NLMRE driving the FP, which similarly affected the integrated LCC.

5. Validation

The limiting $Q=0$ example provides as a representation of the Lin [8] case study. As observed in Fig. 2-4, it is observed that the performance of wide parallel plates with isoviscous case and RFM behavior with are strongly agree with Lin [8].

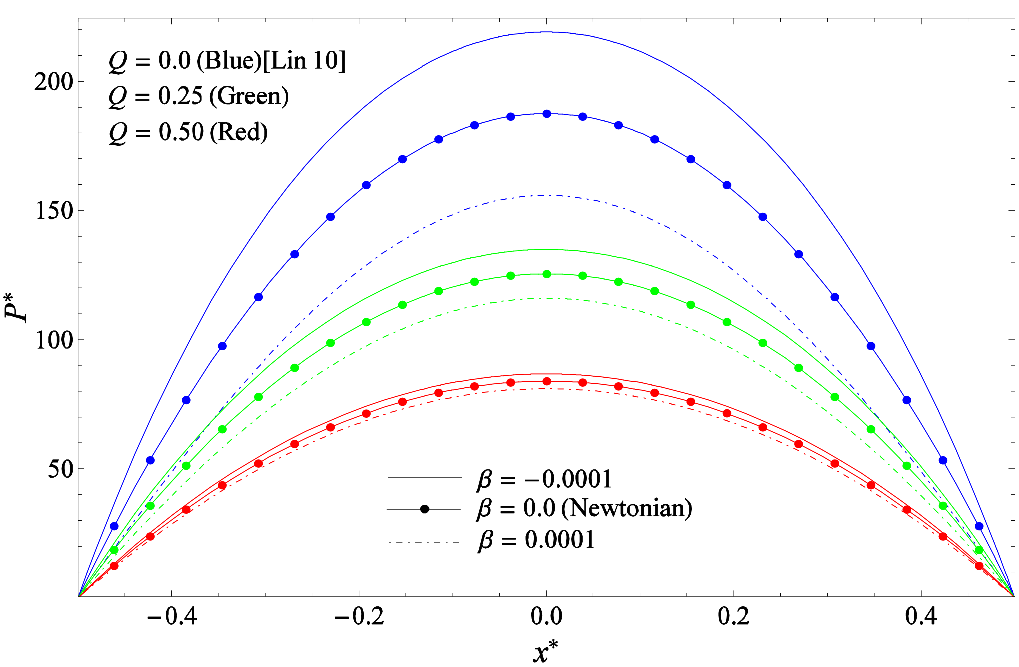

Fig. 2 Variation of the FP (P^*) vs coordinate axis (x^*) for different values of β, Q with $h^* = 0.2$
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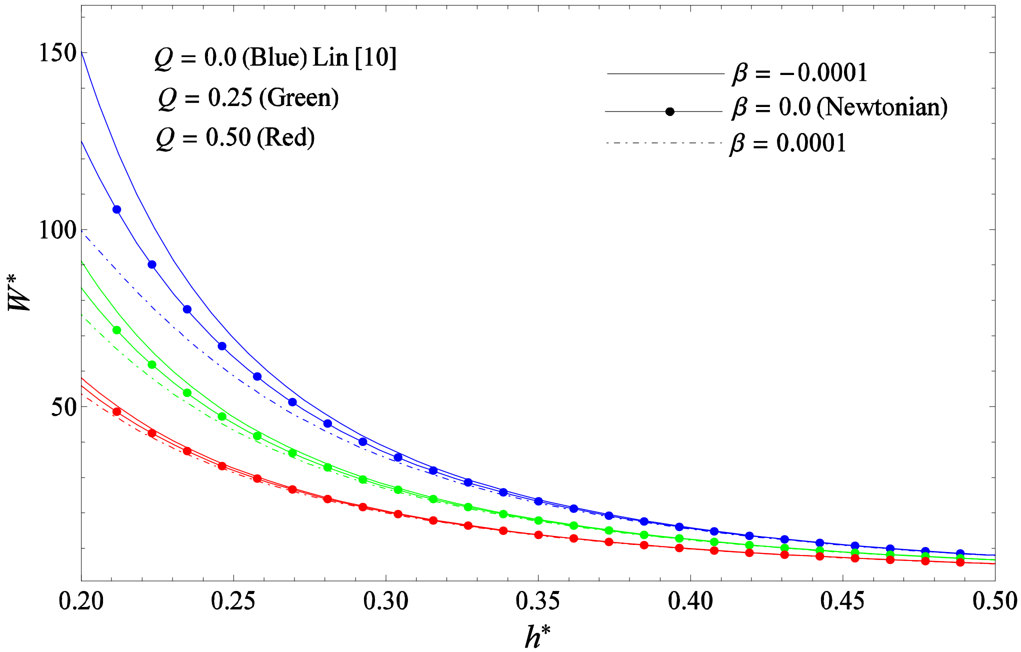

Fig. 3 Variation of the LCC (W^*) vs film thickness (h^*) for different values of β and Q .

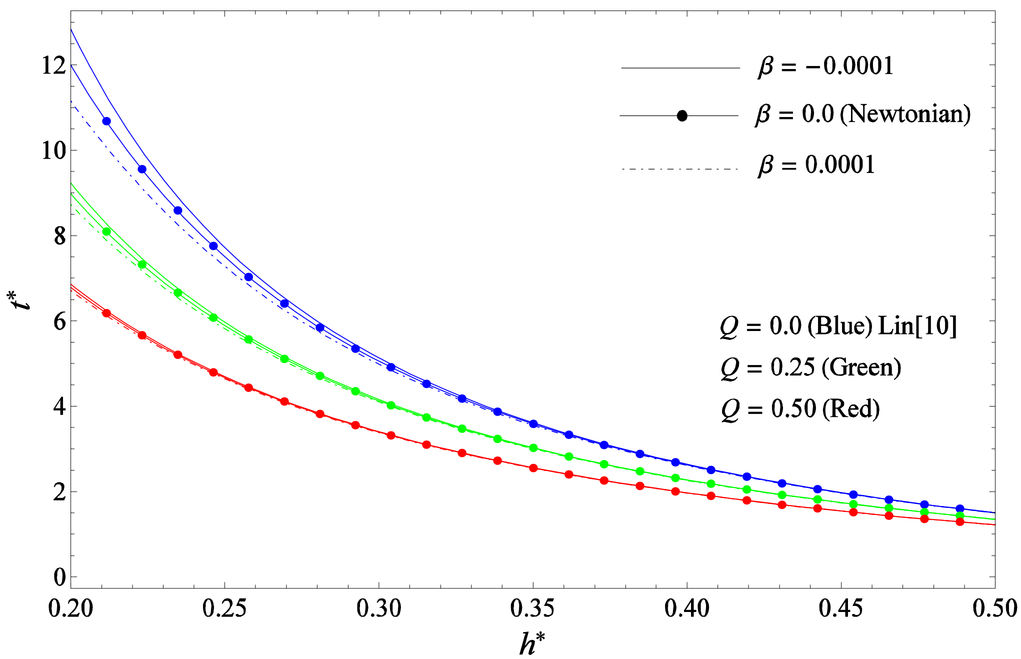

Fig. 4 Variation of SRT (t^*) vs film thickness (h^*) for different values of β and Q .

6. Conclusions

In the current study, squeeze film lubrication between a system of wide parallel rectangular plates is theoretically investigated together with NNF flow with viscosity change. Major conclusions include the following:

1. A reduction in squeeze FP, LCC, and SRT addressed with viscosity variation of NNF.
2. For viscosity variation of NNF flow between wide parallel rectangular plates, the effect of pseudo-plastic $(\beta = 0.0001)$ behaviour yields in decreasing these entities as compared to Newtonian fluid $(\beta = 0.0)$, where dilatants $(\beta = -0.0001)$ features magnify the physical entities $(P^*, W^*$ and $t^*)$.
3. The differences in viscosity do not appear to play a substantial influence in LCC and SRT for higher values of film thickness $h^* \geq 0.35$.

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List of symbol

D, a	Width and length of the plate
h_0, h_1	Inlet and outlet film thickness
h	Film thickness
h^*	Dimensionless film thickness defined in Eq. (23)
dh/dt	Squeeze velocity
x, z	Horizontal and vertical rectangular coordinates
x^*	Non-dimensional coordinate defined in Eq. (23)
u, w	Velocity components in x and z directions
k	Nonlinear factor accounting for non-Newtonian effects
Q	Viscosity variation factor
$\overline{p}$	Pressure in the porous region
p	Film pressure
P^*	Dimensionless film pressure defined in Eq. (23)
W	Load carrying capacity
W^*	Dimensionless load-carrying capacity defined in Eq. (33)
t	Squeeze response time
t^*	Dimensionless squeeze response time defined in Eq. (34)

Greek symbols

μ_0, μ_1	Inlet and outlet viscosity coefficient
μ	Viscosity of the Newtonian fluid
τ_{xz}	Shear stress
β	Dimensionless non-linear factor defined in Eq. (23)

Abbreviation

NLMRE	Non-linear modified Reynolds equation
NNF	Non-Newtonian fluids
FP	Film pressure
LCC	Load carrying capacity
SRT	Squeeze response time