Algebraic operations on Pentapartitioned Neutrosophic Vague Sets

Radhika V R [1], Mohana K [2]

[1] Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India

e-mail: radhikaramakrishnan2805@gmail.com

[2] Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India

e-mail: riyaraju1116@gmail.com

*Corresponding author : Radhika V R; radhikaramakrishnan2805@gmail.com

---

Abstract

In this paper, we present some algebraic operations for Pentapartitioned Neutrosophic Vague Set as an extension of single valued Neutrosophic vague sets. Specifically, we introduce addition, multiplication, scalar multiplication and power for the Pentapartitioned Neutrosophic Vague Sets also the compact spaces with suitable examples and a theorem with proofs.

Keywords: *Neutrosophic Set , Vague Set, Neutrosophic Vague Set, Pentapartitioned Neutrosophic Vague Set, Compact space.*

1. *Introduction*:

In order to solve uncertain**ty** in decision making problems, some theories like *fuzzy set theory* [10] by Lofti A. *Zadeh* (1965), *Intuitionistic fuzzy set theory* [10] by Atanassov (1986), vague set theory [2] by Gau & Beuhrer (1993) were developed.
In fuzzy set theory, the uncertainty of a set is defined by membership function that fix values called membership degrees to the universal set elements within the range $[0, 1]$. As an extension, in *Intuitionistic fuzzy set theory*, non – membership degree is also taken into account. Later, to solve the problems involving incomplete, *indeterminate* and *inconsistent information, Smarandache* developed a new theory called *Neutrosophic set theory* [7] as it includes new membership called *indeterminacy – membership*. In this paper, We introduced a new concept called *Pentapartitioned Neutrosophic Vague Set* (*PNVS*) by which includes four membership

functions namely truth membership, contradiction membership, ignorance membership, unknown membership and false membership functions.

In 2010 Wang et al., [9] developed $single\ valued\ \ neutrosophic\ set$ ($SVNS$) and he defined some basic operations like subset, equality, complement, union and intersection on $SVNS$. Meanwhile Liu and Wang [2] introduced relations such as $addition, multiplication, scalar\ multiplication\ and\ power\ between\ two\ SVNSs$.

Many extensions of $Neutrosophic\ set$ were developed and areas like used in decision making, aggregation operators, image processing etc.,[5].

Following these developments, we present some algebraic operations for $Pentapartitioned\ Neutrosophic\ Vague\ Set$ and defined some algebraic operations on $PNVSs$ with examples.

2. Preliminaries

Definition 2.1 [7]: Let U be a universe. A $\boldsymbol{Neutrosophic\ set}$ A on X can be defined as follows:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in X\}$$

where T, I, F: $X \to [0,1]$ and $0 \le T_A(x)+I_A(x)+F_A(x) \le 3$.

Here $T_A(x)$ $is\ the\ degree\ of\ membership$, $I(x)$ $is\ the\ degree\ of\ indeterminacy$ and $F(x)$ $is\ the\ degree\ of\ non-membership.$

$For\ application\ in\ real\ scientific\ and\ engineering\ areas, Wang\ et\ al., proposed\ the\ concept\ of\ a\ single\ valued\ neutrosophic\ set\ as\ follows$:

$Operations\ between\ two\ SVN-numbers\ are\ defined\ by$ [6]. $It\ is\ recalled\ as\ follows$:

Let $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ be two $SVNS$, then the operations are defined as follows:

a) $x \oplus y = (T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2)$

b) $x \otimes y = (T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2)$

c) $\lambda x = \left(\left(1-(1-T_1)^\lambda\right), {I_1}^\lambda, {F_1}^\lambda\right)$

d) $x^\lambda = \left({T_1}^\lambda, \left(1-(1-I_1)^\lambda\right), \left(1-(1-F_1)^\lambda\right)\right)$

Definition 2.2 [8]: A $\boldsymbol{Neutrosophic\ vague\ set}$ ANV (NVS) on X written as

$A = \{\langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle; x \in X\}$, , $whose\ truth\ membership, indeterminacy\ membership\ and\ false\ membership\ functions\ is\ defined\ as$:

$$\hat{T}_A(x) = [T^-, T^+], \quad \hat{I}_A(x)[I^-, I^+], \quad \hat{F}_A(x) = [F^-, F^+],$$

Where, (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ and (3) $0 \leq T^- + I^- + F^- \leq 2^+$ when X is continuous, a NVS A can be written as

$$A = \int \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle /x, x \in X$$

$When\ X\ is\ discrete, a\ NVS\ A\ can\ be\ written\ as$

$$A = \sum_{i=1}^{n} \langle x; \hat{T}_A(x); \hat{I}_A(x); \hat{F}_A(x)\rangle / x_i, \ x_i \in X$$

Definition 2.3 [6]: A $Pentapartitioned\ Neutrosophic\ Vague\ Set\ A_{PNV}$ on the universe of discourse written as

$$A_{PNV} = \left\{\langle x; \ \hat{T}_{A_{PNV}}(x); \hat{C}_{A_{PNV}}(x); \ \hat{G}_{A_{PNV}}(x); \hat{U}_{A_{PNV}}(x); \hat{F}_{A_{PNV}}(x)\rangle; x \in X\right\}$$

$whose\ truth\ membership, contradiction\ membership, ignorance\ membership, unknown$

$membership, and\ false\ membership\ function\ is\ defined\ as$:

$$\hat{T}_{A_{PNV}}(x) = [T^-, T^+], \quad \hat{C}_{A_{PNV}}(x) = [C^-, C^+], \quad \hat{G}_{A_{PNV}}(x) = [G^-, G^+],$$

$$\hat{U}_{A_{PNV}}(x) = [U^-, U^+], \quad \hat{F}_{A_{PNV}}(x) = [F^-, F^+],$$

Where (1) $T^+ = 1 - F^-$ (2) $F^+ = 1 - T^-$ (3) $C^+ = 1 - U^-$ (4) $U^+ = 1 - C^-$

(5) $0 \leq T^- + C^- + G^- + U^- + F^- \leq 4^+$

Definition 2.4 [6]: Let A_{PNV} and B_{PNV} be two of the universe U. If $\forall x \in U$, $\hat{T}_{A_{PNV}}(x) \leq \hat{T}_{B_{PNV}}(x)$; $\hat{C}_{A_{PNV}}(x) \leq \hat{C}_{B_{PNV}}(x)$;

$\hat{G}_{A_{PNV}}(x) \geq \hat{G}_{B_{PNV}}(x)$; $\hat{U}_{A_{PNV}}(x) \geq \hat{U}_{B_{PNV}}(x)$; $\hat{F}_{A_{PNV}}(x) \geq \hat{F}_{B_{PNV}}(x)$ then the $PNVS$ A_{PNV} is included by B_{PNV}, denoted by $A_{PNV} \subseteq B_{PNV}$.

Definition 2.5 [6]: The complement of $\boldsymbol{PNVS}\ A_{PNV}$ is denoted by A_{PNV}^{C} and is defined by $\hat{T}_{A_{PNV}}^{C}(x) = [1 - T^{+}, 1 - T^{-}]$, $\hat{C}_{A_{PNV}}^{C}(x) = [1 - C^{+}, 1 - C^{-}]$, $\hat{G}_{A_{PNV}}^{C}(x) = [1 - G^{+}, 1 - G^{-}]$, $\hat{U}_{A_{PNV}}^{C}(x) = [1 - U^{+}, 1 - U^{-}]$, $\hat{F}_{A_{PNV}}^{C}(x) = [1 - F^{+}, 1 - F^{-}]$.

Definition 2.6 [6]: Let A_{PNV} be a $PNVS$ *on the universe of discourse* U where $\forall x \in U$, $\hat{T}_{A_{PNV}}(x) = [1,1]$, $\hat{C}_{A_{PNV}}(x) = [1,1]$,

$\hat{G}_{A_{PNV}}(x) = [0,0]$, $\hat{U}_{A_{PNV}}(x) = [0,0]$, $\hat{F}_{A_{PNV}}(x) = [0,0]$. Then A_{PNV} is *called unit* $PNVS(1_{PNV}$ in *short*).

Definition 2.7 [6]: Let A_{PNV} be a $PNVS$ *on the universe of discourse* U where $\forall x \in U$,

$\hat{T}_{A_{PNV}}(x) = [0,0]$, $\hat{C}_{A_{PNV}}(x) = [0,0]$, $\hat{G}_{A_{PNV}}(x) = [1,1]$,
$\hat{U}_{A_{PNV}}(x) = [1,1]$, $\hat{F}_{A_{PNV}}(x) = [1,1]$

Then A_{PNV} *is called zero PNVS* (0_{PNV} *in short*).

Definition 2.8 [6]: Let A_{PNV} and B_{PNV} be two $PNVS$ of the universe U. If $\forall\ x \in U$,

$\hat{T}_{A_{PNV}}(x) = \hat{T}_{B_{PNV}}(x)$; $\hat{C}_{A_{PNV}}(x) = \hat{C}_{B_{PNV}}(x)$; $\hat{G}_{A_{PNV}}(x) = \hat{G}_{B_{PNV}}(x)$; $\hat{U}_{A_{PNV}}(x) = \hat{U}_{B_{PNV}}(x)$; $\hat{F}_{A_{PNV}}(x) = \hat{F}_{B_{PNV}}(x)$ then the $PNVS$ A_{PNV} and B_{PNV} are equal.

Definition 2.9 [6]: *The union of* $PNVSs$ two A_{PNV} and B_{PNV} is a $PNVS$ K_{PNV} *written as* $K_{PNV} = A_{PNV} \cup B_{PNV}$ whose *truth − membership, contradiction − membership, ignorance membership, unknown membership and false membership functions are related to those of* A_{PNV} *and* B_{PNV} *by*

$$\hat{T}_{K_{PNV}}(x) = \left[max\left(\hat{T}_{A_{PNV}}^{-}, \hat{T}_{B_{PNV}}^{-}\right), max\left(\hat{T}_{A_{PNV}}^{+}, \hat{T}_{B_{PNV}}^{+}\right)\right]$$

$$\hat{C}_{K_{PNV}}(x) = \left[max\left(\hat{C}_{A_{PNV}}^{-}, \hat{C}_{B_{PNV}}^{-}\right), max\left(\hat{C}_{A_{PNV}}^{+}, \hat{C}_{B_{PNV}}^{+}\right)\right]$$

$$\hat{G}_{K_{PNV}}(x) = \left[min\left(\hat{G}_{A_{PNV}}^{-}, \hat{G}_{B_{PNV}}^{-}\right), min\left(\hat{G}_{A_{PNV}}^{+}, \hat{G}_{B_{PNV}}^{+}\right)\right]$$

$$\hat{U}_{K_{PNV}}(x) = \left[min\left(\hat{U}_{A_{PNV}}^{-}, \hat{U}_{B_{PNV}}^{-}\right), min\left(\hat{U}_{A_{PNV}}^{+}, U_{B_{PNV}}^{+}\right)\right]$$

$$\hat{F}_{K_{PNV}}(x) = \left[min\left(\hat{F}_{A_{PNV}}^{-}, \hat{F}_{B_{PNV}}^{-}\right), min\left(\hat{F}_{A_{PNV}}^{+}, F_{B_{PNV}}^{+}\right)\right]$$

Definition 2.10 [6]: *The intersection* of two $PNVSs$ A_{PNV} and B_{PNV} is a $PNVS\ H_{PNV}$, written as $H_{PNV} = A_{PNV} \cap B_{PNV}$ whose *truth − membership, contradiction −*

$membership, ignorance\ membership, unknown\ membership\ and\ false -$
$membership\ functions\ are\ related\ to\ those\ of\ A_{PNV}\ and\ B_{PNV}\ by$

$$\hat{T}_{H_{PNV}}(x) = \left[min\left(\hat{T}^{-}_{A_{PNV}}, \hat{T}^{-}_{B_{PNV}}\right), min\left(\hat{T}^{+}_{A_{PNV}}, \hat{T}^{+}_{B_{PNV}}\right)\right]$$

$$\hat{C}_{H_{PNV}}(x) = \left[min\left(\hat{C}^{-}_{A_{PNV}}, \hat{C}^{-}_{B_{PNV}}\right), min\left(\hat{C}^{+}_{A_{PNV}}, \hat{C}^{+}_{B_{PNV}}\right)\right]$$

$$\hat{G}_{H_{PNV}}(x) = \left[max\left(\hat{G}^{-}_{A_{PNV}}, \hat{G}^{-}_{B_{PNV}}\right), max\left(\hat{G}^{+}_{A_{PNV}}, \hat{G}^{+}_{B_{PNV}}\right)\right]$$

$$\hat{U}_{H_{PNV}}(x) = \left[max\left(\hat{U}^{-}_{A_{PNV}}, \hat{U}^{-}_{B_{PNV}}\right), max\left(\hat{U}^{+}_{A_{PNV}}, U^{+}_{B_{PNV}}\right)\right]$$

$$\hat{F}_{H_{PNV}}(x) = \left[max\left(\hat{F}^{-}_{A_{PNV}}, \hat{F}^{-}_{B_{PNV}}\right), max\left(\hat{F}^{+}_{A_{PNV}}, F^{+}_{B_{PNV}}\right)\right]$$

Definition 2.11 [6]: Let $\left\{A_{i_{PNV}} : i \in J\right\}$ be an arbitrary family of $PNVSs$. Then

$$\bigcup A_{i_{PNV}} = \left\{ \left\langle \begin{array}{c} x; \left(\max_{i\in J}\left(\hat{T}^{-}_{A_{i_{PNV}}}\right), \max_{i\in J}\left(\hat{T}^{+}_{A_{i_{PNV}}}\right)\right), \left(\max_{i\in J}\left(\hat{C}^{-}_{A_{i_{PNV}}}\right), \max_{i\in J}\left(\hat{C}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i\in J}\left(\hat{G}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{G}^{+}_{A_{PNV}}\right)\right) \\ \left(\min_{i\in J}\left(\hat{U}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{U}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i\in J}\left(\hat{F}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{F}^{+}_{A_{i_{PNV}}}\right)\right) \end{array} \right\rangle ; x \in X \right\}$$

$$\bigcap A_{i_{PNV}} = \left\{ \left\langle \begin{array}{c} x; \left(\max_{i\in J}\left(\hat{T}^{-}_{A_{i_{PNV}}}\right), \max_{i\in J}\left(\hat{T}^{+}_{A_{i_{PNV}}}\right)\right), \left(\max_{i\in J}\left(\hat{C}^{-}_{A_{i_{PNV}}}\right), \max_{i\in J}\left(\hat{C}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i\in J}\left(\hat{G}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{G}^{+}_{A_{i_{PNV}}}\right)\right) \\ \left(\min_{i\in J}\left(\hat{U}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{U}^{+}_{A_{i_{PNV}}}\right)\right), \left(\min_{i\in J}\left(\hat{F}^{-}_{A_{i_{PNV}}}\right), \min_{i\in J}\left(\hat{F}^{+}_{A_{i_{PNV}}}\right)\right) \end{array} \right\rangle ; x \in X \right\}$$

3. *Operations on Pentapartitioned Neutrosophic Vague Sets*:

This section introduces various algebraic operations on PNVS based on SVNS operations

$The\ following\ are\ the\ definitions\ of\ addition, multiplication, scalar\ multiplication, and$

$power\ operations$:

Definition 3.1:

Let

$A_{PNV} =$

$$\left\{\langle\begin{matrix} x;\ \left[\hat{T}^{-}{}_{A_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{C}^{-}{}_{A_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{G}^{-}{}_{A_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\right]; \\ \left[\hat{U}^{-}{}_{A_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{F}^{-}{}_{A_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\right] \end{matrix}\rangle ; x \in X\right\}$$

$B_{PNV} =$

$$\left\{\langle\begin{matrix} x;\ \left[\hat{T}^{-}{}_{B_{PNV}}(x), \hat{T}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{C}^{-}{}_{B_{PNV}}(x), \hat{C}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{G}^{-}{}_{B_{PNV}}(x), \hat{G}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{U}^{-}{}_{B_{PNV}}(x), \hat{U}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{F}^{-}{}_{B_{PNV}}(x), \hat{F}^{+}{}_{B_{PNV}}(x)\right] \end{matrix}\rangle ; x \in X\right\}$$

be two $PNVSS$. Then the addition of A_{PNV} and B_{PNV} denoted as $(A \oplus B)_{PNV}$ defined by

$$(A \oplus B)_{PNV} = \left\{\langle\begin{matrix} x; \left[\hat{T}^{-}{}_{A_{PNV}}(x) + \hat{T}^{-}{}_{B_{PNV}}(x) - \hat{T}^{-}{}_{A_{PNV}}(x)\hat{T}^{-}{}_{B_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x) + \hat{T}^{+}{}_{B_{PNV}}(x) - \hat{T}^{+}{}_{A_{PNV}}(x)\hat{T}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{C}^{-}{}_{A_{PNV}}(x) + \hat{C}^{-}{}_{B_{PNV}}(x) - \hat{C}^{-}{}_{A_{PNV}}(x)\hat{C}^{-}{}_{B_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x) + \hat{C}^{+}{}_{B_{PNV}}(x) - \hat{C}^{+}{}_{A_{PNV}}(x)\hat{C}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{G}^{-}{}_{A_{PNV}}(x)\hat{G}^{-}{}_{B_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\hat{G}^{+}{}_{B_{PNV}}(x)\right]; \left[\hat{U}^{-}{}_{A_{PNV}}(x)\hat{U}^{-}{}_{B_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\hat{U}^{+}{}_{B_{PNV}}(x)\right]; \\ \left[\hat{F}^{-}{}_{A_{PNV}}(x)\hat{F}^{-}{}_{B_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\hat{F}^{+}{}_{B_{PNV}}(x)\right] \end{matrix}\rangle\right\}$$

Example 3.2:

Let $U = \{m, n\}\ be\ a\ set\ of\ universe\ and\ let$

$$A_{PNV} = \left\{\begin{matrix} \dfrac{m}{\{[0.3,0.55],[0.45,0.5],[0.2,0.75],[0.55,0.78],[0.65,0.7]\}}, \\ \dfrac{n}{\{[0.4,0.85],[0.35,0.6],[0.45,0.8],[0.15,0.76],[0.37,0.9]\}} \end{matrix}\right\},$$

$$B_{PNV} = \left\{\begin{matrix} \dfrac{m}{\{[0.43,0.65], [0.56,0.7], [0.37,0.68], [0.59,0.8], [0.4,0.65]\}}, \\ \dfrac{n}{\{[0.32,0.65], [0.5,0.86], [0.25,0.5], [0.4,0.65], [0.6,0.95]\}} \end{matrix}\right\}$$

Then $(A \oplus B)_{PNV}$

$$= \left\{ \frac{m}{\{[0.601,0.8425],[0.758,0.85],[0.074,0.51],[0.3245,0.624],[0.26,0.455]\}}, \frac{n}{\{[0.592,0.9475],[0.675,0.944],[0.1125,0.4],[0.06,0.494],[0.18,0.855]\}} \right\}$$

which is again a $PNVS$.

Definition 3.3:

Let

$A_{PNV} =$

$$\left\{ \left\langle \begin{matrix} x; \ [\hat{T}^-{}_{A_{PNV}}(x), \hat{T}^+{}_{A_{PNV}}(x)]; [\hat{C}^-{}_{A_{PNV}}(x), \hat{C}^+{}_{A_{PNV}}(x)]; [\hat{G}^-{}_{A_{PNV}}(x), \hat{G}^+{}_{A_{PNV}}(x)]; \\ [\hat{U}^-{}_{A_{PNV}}(x), \hat{U}^+{}_{A_{PNV}}(x)]; [\hat{F}^-{}_{A_{PNV}}(x), \hat{F}^+{}_{A_{PNV}}(x)] \end{matrix} \right\rangle ; x \in X \right\}$$

$B_{PNV} =$

$$\left\{ \left\langle \begin{matrix} x; \ [\hat{T}^-{}_{B_{PNV}}(x), \hat{T}^+{}_{B_{PNV}}(x)]; [\hat{C}^-{}_{B_{PNV}}(x), \hat{C}^+{}_{B_{PNV}}(x)]; [\hat{G}^-{}_{B_{PNV}}(x), \hat{G}^+{}_{B_{PNV}}(x)]; \\ [\hat{U}^-{}_{B_{PNV}}(x), \hat{U}^+{}_{B_{PNV}}(x)]; [\hat{F}^-{}_{B_{PNV}}(x), \hat{F}^+{}_{B_{PNV}}(x)] \end{matrix} \right\rangle ; x \in X \right\}$$

$be\ two\ PNVSS. Then\ the\ addition\ of$ A_{PNV} and B_{PNV} denoted as $(A \otimes B)_{PNV}$ $defined\ by$

$$(A \otimes B)_{PNV} = \left\{ \left\langle \begin{matrix} x; [\hat{T}^-{}_{A_{PNV}}(x)\hat{T}^-{}_{B_{PNV}}(x), \hat{T}^+{}_{A_{PNV}}(x)\hat{T}^+{}_{B_{PNV}}(x)]; [\hat{C}^-{}_{A_{PNV}}(x)\hat{C}^-{}_{B_{PNV}}(x), \hat{C}^+{}_{A_{PNV}}(x)\hat{C}^+{}_{B_{PNV}}(x)]; \\ [\hat{G}^-{}_{A_{PNV}}(x) + \hat{G}^-{}_{B_{PNV}}(x) - \hat{G}^-{}_{A_{PNV}}(x)\hat{G}^-{}_{B_{PNV}}(x), \hat{G}^+{}_{A_{PNV}}(x) + \hat{G}^+{}_{B_{PNV}}(x) - \hat{G}^+{}_{A_{PNV}}(x)\hat{G}^+{}_{B_{PNV}}(x)]; \\ [\hat{U}^-{}_{A_{PNV}}(x) + \hat{U}^-{}_{B_{PNV}}(x) - \hat{U}^-{}_{A_{PNV}}(x)\hat{U}^-{}_{A_{PNV}}(x), \hat{U}^+{}_{A_{PNV}}(x) + \hat{U}^+{}_{B_{PNV}}(x) - \hat{U}^+{}_{A_{PNV}}(x)\hat{U}^-{}_{B_{PNV}}(x)]; \\ [\hat{F}^-{}_{A_{PNV}}(x) + \hat{F}^-{}_{B_{PNV}}(x) - \hat{F}^-{}_{A_{PNV}}(x)\hat{F}^-{}_{A_{PNV}}(x), \hat{F}^+{}_{A_{PNV}}(x) + \hat{F}^+{}_{B_{PNV}}(x) - \hat{F}^+{}_{A_{PNV}}(x)\hat{F}^-{}_{B_{PNV}}(x)] \end{matrix} \right\rangle \right\}$$

Example 3.4:

Let $U = \{m, n\}\ be\ a\ set\ of\ universe\ and\ let$

$$A_{PNV} = \left\{ \frac{m}{\{[0.25,0.43],[0.36,0.48],[0.35,0.62],[0.47,0.6],[0.45,0.55]\}}, \frac{n}{\{[0.53,0.55],[0.4,0.85],[0.26,0.64],[0.5,0.75],[0.45,0.76]\}} \right\},$$

$$B_{PNV} = \left\{ \begin{array}{c} \frac{m}{\{[0.35,0.59],[0.43,0.55],[0.15,0.4],[0.29,0.65],[0.52,0.71]\}}, \\ \frac{n}{\{[0.45,0.8],[0.37,0.9],[0.25,0.5],[0.25,0.4],[0.47,0.5]\}} \end{array} \right\}$$

Then $(A \otimes B)_{PNV}$

$$= \left\{ \begin{array}{c} \frac{m}{\{[0.0875,0.2537],[0.1548,0.264],[0.4475,0.772],[0.6237,0.86],[0.736,0.8695]\}}, \\ \frac{n}{\{[0.2385,0.44],[0.148,0.765],[0.445,0.82],[0.625,0.85],[0.7085,0.88]\}} \end{array} \right\}$$

Definition 3.5:

Let

$A_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x;\ \left[\hat{T}^{-}{}_{A_{PNV}}(x), \hat{T}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{C}^{-}{}_{A_{PNV}}(x), \hat{C}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{G}^{-}{}_{A_{PNV}}(x), \hat{G}^{+}{}_{A_{PNV}}(x)\right]; \\ \left[\hat{U}^{-}{}_{A_{PNV}}(x), \hat{U}^{+}{}_{A_{PNV}}(x)\right]; \left[\hat{F}^{-}{}_{A_{PNV}}(x), \hat{F}^{+}{}_{A_{PNV}}(x)\right] \end{array} \right\rangle ; x \in X \right\}$$

be a $PNVS. Then\ the\ scalar\ (\lambda)\ multiplication\ of\ A_{PNV}\ denoted\ as\ \lambda A_{PNV} defined\ by$

$\lambda A_{PNV} =$

$$\left\{ \left\langle \begin{array}{c} x; \left[1-\left(1-\hat{T}^{-}{}_{A_{PNV}}(x)\right)^{\lambda}, 1-\left(1-\hat{T}^{+}{}_{A_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[1-\left(1-\hat{C}^{-}{}_{A_{PNV}}(x)\right)^{\lambda}, 1-\left(1-\hat{C}^{+}{}_{A_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{G}^{-}{}_{A_{PNV}}(x)\right)^{\lambda}, \left(\hat{G}^{+}{}_{A_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{U}^{-}{}_{A_{PNV}}(x)\right)^{\lambda}, \left(\hat{U}^{+}{}_{A_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{F}^{-}{}_{A_{PNV}}(x)\right)^{\lambda}, \left(\hat{F}^{+}{}_{A_{PNV}}(x)\right)^{\lambda}\right] \end{array} \right\rangle ; x \in X \right\}$$

Example 3.6:

Let $U = \{m, n\}\ be\ a\ set\ of\ universe, let$

$$A_{PNV} = \left\{ \frac{m}{\{[0.3,0.5],[0.4,0.6],[0.2,0.7],[0.3,0.6],[0.5,0.8]\}}, \frac{n}{\{[0.1,0.6],[0.6,0.7],[0.3,0.4],[0.5,0.7],[0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 3$, then

λA_{PNV}

$$= \left\{ \frac{m}{\{[1-(1-0.3)^3, 1-(1-0.5)^3], [1-(1-0.4)^3, 1-(1-0.6)^3], [(0.2)^3, (0.7)^3], [(0.3)^3, (0.6)^3], [(0.5)^3, (0.8)^3]\}}, \frac{n}{\{[1-(1-0.1)^3, 1-(1-0.6)^3], [1-(1-0.6)^3, 1-(1-0.7)^3], [(0.3)^3, (0.4)^3], [(0.5)^3, (0.7)^3], [(0.6)^3, (0.9)^3]\}} \right\}$$

$$\lambda A_{PNV} = \left\{ \frac{m}{\{[0.657,0.875], [0.784,0.936], [0.008,0.343], [0.027,0.216], [0.125,0.512]\}}, \frac{n}{\{[0.271,0.936], [0.936,0.973], [0.027,0.064], [0.125,0.343], [0.216,0.729]\}} \right\}$$

Definition 3.7:

Let

$A_{PNV} =$

$$\left\{ \left\langle \begin{matrix} x;\ [\hat{T}^-{}_{A_{PNV}}(x), \hat{T}^+{}_{A_{PNV}}(x)]; [\hat{C}^-{}_{A_{PNV}}(x), \hat{C}^+{}_{A_{PNV}}(x)]; [\hat{G}^-{}_{A_{PNV}}(x), \hat{G}^+{}_{A_{PNV}}(x)]; \\ [\hat{U}^-{}_{A_{PNV}}(x), \hat{U}^+{}_{A_{PNV}}(x)]; [\hat{F}^-{}_{A_{PNV}}(x), \hat{F}^+{}_{A_{PNV}}(x)] \end{matrix} \right\rangle ; x \in X \right\}$$

be a $PNVS$.

Then, $PNVS$ power denoted by ${A_{PNV}}^{\lambda}$ is defined by

$${A_{PNV}}^{\lambda} = \left\{ \left\langle \begin{matrix} x; \left[\left(\hat{T}^-{}_{A_{PNV}}(x)\right)^{\lambda}, \left(\hat{T}^+{}_{A_{PNV}}(x)\right)^{\lambda} \right]; \\ \left[\left(\hat{C}^-{}_{A_{PNV}}(x)\right)^{\lambda}, \left(\hat{C}^+{}_{A_{PNV}}(x)\right)^{\lambda} \right]; \\ \left[1-\left(1-\hat{G}^-{}_{A_{PNV}}(x)\right)^{\lambda}, 1-\left(1-\hat{G}^+{}_{A_{PNV}}(x)\right)^{\lambda} \right]; \\ \left[1-\left(1-\hat{U}^-{}_{A_{PNV}}(x)\right)^{\lambda}, 1-\left(1-\hat{U}^+{}_{A_{PNV}}(x)\right)^{\lambda} \right]; \\ \left[1-\left(1-\hat{F}^-{}_{A_{PNV}}(x)\right)^{\lambda}, 1-\left(1-\hat{F}^+{}_{A_{PNV}}(x)\right)^{\lambda} \right] \end{matrix} \right\rangle ; x \in X \right\}$$

Example 3.8:

Let $U = \{m, n\}$ be a set of universe, let

A_{PNV}

$$= \left\{ \frac{m}{\{[0.3,0.5], [0.4,0.6], [0.2,0.7], [0.3,0.6], [0.5,0.8]\}}, \frac{n}{\{[0.1,0.6], [0.6,0.7], [0.3,0.4], [0.5,0.7], [0.6,0.9]\}} \right\}$$

be a $PNVS$ and if $\lambda = 2$, then

$$A_{PNV}{}^{\lambda} = \left\{ \frac{m}{\{[(0.3)^2,(0.5)^2],[(0.4)^2,(0.6)^2],[1-(1-0.2)^2,1-(1-0.7)^2]],[[1-(1-0.3)^2,1-(1-0.6)^2]],[[1-(1-0.5)^2,1-(1-0.8)^2]]\}}, \frac{n}{\{[(0.1)^2,(0.6)^2],[(0.6)^2,(0.7)^2],[[1-(1-0.3)^2,1-(1-0.4)^2]],[[1-(1-0.5)^2,1-(1-0.7)^2]],[[1-(1-0.6)^2,1-(1-0.9)^2]]\}} \right\}$$

$$A_{PNV}{}^{\lambda} = \left\{ \frac{m}{\{[0.09,0.25],[0.16,0.36],[0.36,0.91],[0.51,0.84],[0.75,0.96]\}}, \frac{n}{\{[0.01,0.36],[0.36,0.49],[0.51,0.64],[0.75,0.91],[0.84,0.99]\}} \right\}$$

Theorem 3.9:

Let A_{PNV} and B_{PNV} be two $PNVS$ and $\lambda, \lambda_1, \lambda_2 > 0$. Then

i) $(A \oplus B)_{PNV} = (B \oplus A)_{PNV}$

ii) $(A \otimes B)_{PNV} = (B \otimes A)_{PNV}$

iii) $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$

iv) $\lambda_1 A_{PNV} \oplus \lambda_2 A_{PNV} = (\lambda_1 \oplus \lambda_2) A_{PNV}$

v) $A_{PNV}{}^{\lambda_1} \otimes A_{PNV}{}^{\lambda_2} = A_{PNV}{}^{\lambda_1+\lambda_2}$

vi) $A_{PNV}{}^{\lambda} \otimes B_{PNV}{}^{\lambda} = (A \otimes B)_{PNV}{}^{\lambda}$

Proof:

$The\ results\ (i), (ii), (iv)\ and\ (vi)\ are\ obvious, thus\ we\ prove\ the\ others.$

$\boldsymbol{Proof\ of\ (iii)}$

$By\ definition, we\ have$

$$(A \oplus B)_{PNV} = \left\{ \left\langle \begin{array}{c} x; \left[\hat{T}^-{}_{A_{PNV}}(x) + \hat{T}^-{}_{B_{PNV}}(x) - \hat{T}^-{}_{A_{PNV}}(x)\hat{T}^-{}_{B_{PNV}}(x), \hat{T}^+{}_{A_{PNV}}(x) + \hat{T}^+{}_{B_{PNV}}(x) - \hat{T}^+{}_{A_{PNV}}(x)\hat{T}^+{}_{B_{PNV}}(x)\right]; \\ \left[\hat{C}^-{}_{A_{PNV}}(x) + \hat{C}^-{}_{B_{PNV}}(x) - \hat{C}^-{}_{A_{PNV}}(x)\hat{C}^-{}_{B_{PNV}}(x), \hat{C}^+{}_{A_{PNV}}(x) + \hat{C}^+{}_{B_{PNV}}(x) - \hat{C}^+{}_{A_{PNV}}(x)\hat{C}^+{}_{B_{PNV}}(x)\right]; \\ \left[\hat{G}^-{}_{A_{PNV}}(x)\hat{G}^-{}_{B_{PNV}}(x), \hat{G}^+{}_{A_{PNV}}(x)\hat{G}^+{}_{B_{PNV}}(x)\right]; \left[\hat{U}^-{}_{A_{PNV}}(x)\hat{U}^-{}_{B_{PNV}}(x), \hat{U}^+{}_{A_{PNV}}(x)\hat{U}^+{}_{B_{PNV}}(x)\right]; \\ \left[\hat{F}^-{}_{A_{PNV}}(x)\hat{F}^-{}_{B_{PNV}}(x), \hat{F}^+{}_{A_{PNV}}(x)\hat{F}^+{}_{B_{PNV}}(x)\right] \end{array} \right\rangle \right\}$$

$\lambda(A \oplus B)_{PNV}$

$$= \left\{ \langle \begin{array}{c} x; \left[1-\left(1-\left(\hat{T}^{-}{}_{A_{PNV}}(x)+\hat{T}^{-}{}_{B_{PNV}}(x)-\hat{T}^{-}{}_{A_{PNV}}(x)\hat{T}^{-}{}_{B_{PNV}}(x)\right)\right)^{\lambda}, 1-\left(1-\left(\hat{T}^{+}{}_{A_{PNV}}(x)+\hat{T}^{+}{}_{B_{PNV}}(x)-\hat{T}^{+}{}_{A_{PNV}}(x)\hat{T}^{+}{}_{B_{PNV}}(x)\right)\right)^{\lambda}\right]; \\ \left[1-\left(1-\left(\hat{C}^{-}{}_{A_{PNV}}(x)+\hat{C}^{-}{}_{B_{PNV}}(x)-\hat{C}^{-}{}_{A_{PNV}}(x)\hat{C}^{-}{}_{B_{PNV}}(x)\right)\right)^{\lambda}, 1-\left(1-\left(\hat{C}^{+}{}_{A_{PNV}}(x)+\hat{C}^{+}{}_{B_{PNV}}(x)-\hat{C}^{+}{}_{A_{PNV}}(x)\hat{C}^{+}{}_{B_{PNV}}(x)\right)\right)^{\lambda}\right]; \\ \left[\left(\hat{G}^{-}_{A_{PNV}}(x)\hat{G}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{G}^{+}_{A_{PNV}}(x)\hat{G}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{U}^{-}_{A_{PNV}}(x)\hat{U}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{U}^{+}_{A_{PNV}}(x)\hat{U}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{F}^{-}_{A_{PNV}}(x)\hat{F}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{F}^{+}_{A_{PNV}}(x)\hat{F}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right] \end{array} \rangle ; x \in X \right\}$$

We solve truth membership functions

$$= 1-\left(1-\left(\hat{T}^{-}{}_{A_{PNV}}(x)+\hat{T}^{-}{}_{B_{PNV}}(x)-\hat{T}^{-}{}_{A_{PNV}}(x)\hat{T}^{-}{}_{B_{PNV}}(x)\right)\right)^{\lambda}$$

$$= 1-\left(1-\hat{T}^{-}_{A_{PNV}}(x)-\hat{T}^{-}_{B_{PNV}}(x)+\hat{T}^{-}_{A_{PNV}}(x)\hat{T}^{-}_{B_{PNV}}(x)\right)^{\lambda}$$

$$= 1-\left(\left(1-\hat{T}^{-}_{A_{PNV}}(x)\right)-\hat{T}^{-}_{B_{PNV}}(x)\left(1-\hat{T}^{-}_{A_{PNV}}(x)\right)\right)^{\lambda}$$

$$= 1-\left(\left(1-\hat{T}^{-}_{A_{PNV}}(x)\right)\left(1-\hat{T}^{-}_{B_{PNV}}(x)\right)\right)^{\lambda}$$

By similar calculation, we get

$\lambda(A \oplus B)_{PNV} =$

$$\left\{ \langle x; \begin{array}{c} \left[1-\left(\left(1-\hat{T}^{-}_{A_{PNV}}(x)\right)\left(1-\hat{T}^{-}_{B_{PNV}}(x)\right)\right)^{\lambda}, 1-\left(\left(1-\hat{T}^{+}_{A_{PNV}}(x)\right)\left(1-\hat{T}^{+}_{B_{PNV}}(x)\right)\right)^{\lambda}\right]; \\ \left[1-\left(\left(1-\hat{T}^{-}_{A_{PNV}}(x)\right)\left(1-\hat{T}^{-}_{B_{PNV}}(x)\right)\right)^{\lambda}, 1-\left(\left(1-\hat{T}^{+}_{A_{PNV}}(x)\right)\left(1-\hat{T}^{+}_{B_{PNV}}(x)\right)\right)^{\lambda}\right]; \\ \left[\left(\hat{G}^{-}_{A_{PNV}}(x)\hat{G}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{G}^{+}_{A_{PNV}}(x)\hat{G}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{U}^{-}_{A_{PNV}}(x)\hat{U}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{U}^{+}_{A_{PNV}}(x)\hat{U}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right]; \\ \left[\left(\hat{F}^{-}_{A_{PNV}}(x)\hat{F}^{-}_{B_{PNV}}(x)\right)^{\lambda}, \left(\hat{F}^{+}_{A_{PNV}}(x)\hat{F}^{+}_{B_{PNV}}(x)\right)^{\lambda}\right] \end{array} \rangle x \in X \right\} \rightarrow (1)$$

Now

$$\lambda A_{PNV} \oplus \lambda B_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \left[\begin{array}{c} \left(\left(1 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) + \left(1 - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right) - \left(1 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) \left(1 - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right) \right), \\ \left(\left(1 - \left(1 - \hat{T}^{+}_{A_{PNV}}(x) \right)^{\lambda} \right) + \left(1 - \left(1 - \hat{T}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right) - \left(1 - \left(1 - \hat{T}^{+}_{A_{PNV}}(x) \right)^{\lambda} \right) \left(1 - \left(1 - \hat{T}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right) \right) \end{array} \right]; \\ \left[\begin{array}{c} \left(\left(1 - \left(1 - \hat{C}^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) + \left(1 - \left(1 - \hat{C}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right) - \left(1 - \left(1 - C^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) \left(1 - \left(1 - \hat{C}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right) \right), \\ \left(\left(1 - \left(1 - \hat{C}^{+}_{A_{PNV}}(x) \right)^{\lambda} \right) + \left(1 - \left(1 - \hat{C}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right) - \left(1 - \left(1 - \hat{C}^{+}_{A_{PNV}}(x) \right)^{\lambda} \right) \left(1 - \left(1 - \hat{C}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right) \right) \end{array} \right]; \\ \left[\left(\hat{G}^{-}_{A_{PNV}}(x) \right)^{\lambda} \left(\hat{G}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{G}^{+}_{A_{PNV}}(x) \hat{G}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right]; \left[\left(\hat{U}^{-}_{A_{PNV}}(x) \right)^{\lambda} \left(\hat{U}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{U}^{+}_{A_{PNV}}(x) \hat{U}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right]; \\ \left[\left(\hat{F}^{-}_{A_{PNV}}(x) \hat{F}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{F}^{+}_{A_{PNV}}(x) \hat{F}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right] \end{array} \right\rangle ; x \in X \right\}$$

Consider the truth membership function

$$= \left(1 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) + \left(1 - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right) - \left(1 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \right) \left(1 - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right)$$

$$= 2 - \left[\left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} + \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right] - \left[1 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} + \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right]$$

$$= 2 - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} - \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} - 1 + \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} + \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} - \left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda}$$

$$= 1 - \left(\left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right)^{\lambda} \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right)^{\lambda} \right)$$

$$= 1 - \left[\left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right) \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right) \right]^{\lambda}$$

In the similar manner, we have

$$\lambda A_{PNV} \oplus \lambda B_{PNV} =$$

$$\left\{ \left\langle \begin{array}{c} x; \left[1 - \left(\left(1 - \hat{T}^{-}_{A_{PNV}}(x) \right) \left(1 - \hat{T}^{-}_{B_{PNV}}(x) \right) \right)^{\lambda}, 1 - \left(\left(1 - \hat{T}^{+}_{A_{PNV}}(x) \right) \left(1 - \hat{T}^{+}_{B_{PNV}}(x) \right) \right)^{\lambda} \right]; \\ \left[1 - \left(\left(1 - \hat{C}^{-}_{A_{PNV}}(x) \right) \left(1 - \hat{C}^{-}_{B_{PNV}}(x) \right) \right)^{\lambda}, 1 - \left(\left(1 - \hat{C}^{+}_{A_{PNV}}(x) \right) \left(1 - \hat{C}^{+}_{B_{PNV}}(x) \right) \right)^{\lambda} \right]; \\ \left[\left(\hat{G}^{-}_{A_{PNV}}(x) \hat{G}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{G}^{+}_{A_{PNV}}(x) \hat{G}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right]; \left[\left(\hat{U}^{-}_{A_{PNV}}(x) \hat{U}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{U}^{+}_{A_{PNV}}(x) \hat{U}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right] \\ \left[\left(\hat{F}^{-}_{A_{PNV}}(x) \hat{F}^{-}_{B_{PNV}}(x) \right)^{\lambda}, \left(\hat{F}^{+}_{A_{PNV}}(x) \hat{F}^{+}_{B_{PNV}}(x) \right)^{\lambda} \right] \end{array} \right\rangle ; x \in X \right\} \quad \rightarrow (2)$$

From (1) and (2), we have $\lambda(A \oplus B)_{PNV} = \lambda A_{PNV} \oplus \lambda B_{PNV}$

Proof of (v):

$The\ proof\ is\ similar\ as\ the\ proof\ of\ (\boldsymbol{iii})$.

4. $\boldsymbol{Pentapartitioned\ Neutrosophic\ Vague\ Compact\ Space}$

Definition 4.1:

Let (X_{PNV}, τ_{PNV}) be $PNVTS$.

i) If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A\rangle; i \in J\}$ of $PNVOS$ in X $satisfies\ the\ condition$ $\cup \{\langle x, T_A, C_A, G_A, U_A, F_A\rangle; i \in J\} = 1_{PNV}$, then it is called $Pentapartitioned\ Neutrosophic\ vague\ open\ cover\ of\ X$. A finite subfamily of $Pentapartitioned\ neutrosophic\ vague\ open\ cover$ $\{\langle x, T_A, C_A, G_A, U_A, F_A\rangle : i \in J\}$ of X, which is also a $pentapartitioned\ neutrosophic\ vague\ cover$ of X, is called $Pentapartitioned\ neutrosophic\ vague\ finite\ subcover$ of $\{\langle x, T_A, C_A, G_A, U_A, F_A\rangle : i \in J\}$.

ii) A family $\{\langle x, T_B, C_B, G_B, U_B, F_B\rangle; i \in J\}$ of $PNVCS$ in X $satisfies\ finite\ intersection\ property\ iff\ every\ finite\ subfamily$ $\{\langle x, T_B, C_B, G_B, U_B, F_B\rangle; i = 1, 2, \dots \dots n\}$ of the $family\ satisfies\ the\ condition$ $\bigcap_{i=1}^{n}\{\langle x, T_{B_i}, C_{B_i}, G_{B_i}, U_{B_i}, F_{B_i}\rangle\} \neq 0_{QNV}$.

Definition 4.2: A $PNVTS$ (X_{PNV}, τ_{PNV}) is $Pentapartitioned\ neutrosophic\ vague\ compact$ iff every $Pentapartitioned\ neutrosophic\ vague$ $open\ cover$ of X has a $Pentapartitioned\ neutrosophic\ vague\ finite\ subcover$.

Corollary 4.3: A $PNVTS$ (X_{PNV}, τ_{PNV}) is $Pentapartitioned\ neutrosophic\ vague\ compact$ iff every family $\{\langle x, T_B, C_B, G_B, U_B, F_B\rangle : i \in J\}$ of $PNVCSs$ in X having the FIP has nonempty intersection.

Corollary 4.4: Let (X_{PNV}, τ_{PNV}), (Y_{PNV}, σ_{PNV}) be $PNVTSs$ and $f: (X_{PNV}, \tau_{PNV}) \to (Y_{PNV}, \sigma_{PNV})$ a $Pentapartitioned\ neutrosophic\ vague\ continuous\ surjection$. If (X_{PNV}, τ_{PNV}) is $Pentapartitioned\ neutrosophic\ vague\ compact$, then so is (Y_{PNV}, σ_{PNV}).

Definition 4.5: Let (X_{PNV}, τ_{PNV}) be $PNVTS$ and A_{PNV} a $PNVS$ in X.

i) If a family $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$ of $PNVOS$ in $X\ satisfies\ the\ condition$ $A_{PNV} \subseteq \cup\{\langle\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\rangle\}$, then it is called $Pentapartitioned\ neutrosophic\ vague\ open$ cover of A_{PNV}. A finite subfamily of Pentapartitioned $neutrosophic\ vague\ open$ cover $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$ of A_{PNV}, which is also a pentapartitioned $neutrosophic\ vague\ cove$r of A_{PNV}, is called Pentapartitioned $neutrosophic\ vague\ finite\ subcover$ of $\{\langle x, T_A, C_A, G_A, U_A, F_A \rangle : i \in J\}$.

ii) A $PNVS$ in a $PNVTS$ (X_{PNV}, τ_{PNV}) is called Pentapartitioned n$eutrosophic$ $vague$ compact iff every Pentapartitioned $neutrosophic\ vague\ cover\ A_{PNV}$ of has finite subcover.

Corollary 4.6: Let (X_{PNV}, τ_{PNV}), (Y_{PNV}, σ_{PNV}) be $PNVTSs$ and $f: (X_{PNV}, \tau_{PNV}) \to (Y_{PNV}, \sigma_{PNV})$ a Pentapartitioned $neutrosophic\ vague$ continuous function. If A_{PNV} is Pentapartitioned $neutrosophic\ vague\ compact$ in (X_{PNV}, τ_{PNV}), then so if $f(A_{PNV})$ in (Y_{PNV}, σ_{PNV}).

Conclusion

In this paper, we defined some new operations on Pentapartitioned Neutrosophic Vague Set under neutrosophic environment. The basic algebraic operations on Pentapartitioned neutrosophic vague sets namely addition, multiplication, scalar multiplication and power also compact space along with illustrative examples were presented. Subsequently, the basic properties of these operations such as commutative law and relevant laws are mathematically proven. This new extension will broaden the fundamental knowledge of existing set theories and subsequently could be applied to real life experiments where truthness, indeterminacy and falsity could be dealth with.

References

[1]. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 1986; 20, pp. 87–96.

[2]. W.-L. Gau and D. J. Buehrer, “Vague sets,” *IEEE Trans. Syst. Man. Cybern.*, vol. 23, no. 2, pp. 610–614, 1993.

[3]. Hazwani Hashim, Lazim Abdullah and Ashraf Al-Quran, “Algebraic operations on new interval neutrosophic vague sets, ICoAIMS 2019 , IOP Publishing ,Journal of Physics: Conference Series **1366** (2019) 012062”.

[4]. P. Liu and Y. Wang, “Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean,” *Neural Comput. Appl.*, vol. 25, no. 7–8, pp. 2001–2010, Dec. 2014.

[5]. X. Peng and J. Dai, “A bibliometric analysis of neutrosophic set: two decades review from 1998 to 2017,” *Artificial Intelligence Review*, Springer Netherlands, pp. 1–57, 18-Aug-2018.

[6]. V. R. Radhika , K. Mohana. Pentapartitioned Neutrosophic Vague Sets and its Topological spaces(Communicated)

[7]. F. Smarandache, *Neutrosophic set, a generalisation of the intuitionistic fuzzy sets*, in “Inter. J. Pure Appl. Math.”, 24 (2005), 287-297.

[8] Shawkat Alkhazaleh. Neutrosophic vague set theory, Critical Review. 2015; 10, pp. 29–39.

[9]. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, “Single valued neutrosophic set,” in *Multispace and Multistructure*, vol. IV, 2010, pp. 410–413.

[10]. L. A. Zadeh, “Fuzzy sets,” *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.