Magneto double diffusive convection in Soret induced nanofluid layer

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ABSTRACT

The present study aims at investigating the combined effect of a uniform vertical magnetic field and Soret effect on the onset of double diffusive convection in a nanofluid layer. The linear stability analysis is based on normal mode technique. Galerkin method has been applied to find the critical Rayleigh number and the corresponding wave number in terms of various parameters numerically. The effects of Soret parameter, magnetic field, Lewis number, Modified diffusivity ratio, Concentration Rayleigh-Darcy number, Solutal Rayleigh number on the stability of the system has been investigated.

Keywords - Nanofluid, Magnetic field, Double diffusion, Soret parameter.

Nomenclature

c	Specific heat of nanofluid
g	Gravitational Acceleration
$t*$	Time
t	Dimensionless Time
$T*$	Nanofluid Temperature
(x^*, y^*, z^*)	Space Coordinates
(X, Y, Z)	Dimensionless Space Coordinates
S	Solute concentration
$S*$	Dimensionless Solute Concentration
μ	Viscosity
$(\rho c)_M$	Medium's effective heat capacity
$(\rho c)_F$	Fluid's effective heat capacity
$(\rho c)_P$	Effective heat capacity of nanoparticles
B_d	Brownian diffusion coefficient
B_t	Thermophoretic diffusion coefficient
S_d	Diffusion Coefficient
S_{ct}	Soret coefficient of salt
β_t	Thermal Volumetric Coefficient
β_c	Solutal Volumetric Coefficient
k_m	Effective thermal conductivity
α	Wave number
α_m	Thermal diffusivity of the porous medium
λ	Relaxation time

σ	Heat capacity ratio	$\in$	Porosity
ω	Frequency of oscillation	μ_e	Magnetic permeability
ψ^*	Volume fraction of Nanoparticle	σ'	Electrical conductivity of nanofluid
ψ_0^*	Reference value of nanoparticle volume fraction	K	Permeability

I. INTRODUCTION

Heat transfer mechanism has been improved by replacing micro sized particles with nano sized particles in conventional fluids. In 1992, Choi [1] observed that heat transfer is very excellent only when pumping power is increased and an expensive cryogenic system is maintained. The term nanofluid was first coined by Choi, he described the future and hope of the application of nanotechnology. The nanoparticles differ from conventional particles (milli-meter or micro scale) in sense that they stay in suspension in the fluid without sedimentation. Kumar, Prasad and Banerjee [2] established the utility of a particular nanofluid for its heat transfer application. Wong and Leon [3] focussed on giving the broad range of present and future applications of nanofluids.

The problem of thermal convection for a Newtonian fluid layer was discussed by Chandrasekhar [4] taking varying assumptions of hydro-dynamics and hydro-magnetism. In a horizontal porous layer of nanofluid, convection was studied by Nield and Kuznetsov [5], incorporating Brownian Motion and Thermophoresis. He observed that the critical Rayleigh number decreases or increases by a significant amount when the basic nanoparticle concentration is increased at the top or bottom.

Study of convective instability of the nanofluids has many uses in astrophysics and geophysics etc. Buongiorno [6] proposed the model for nanofluid convection. Later, Nield and Kuznetsov [7] revisited this problem by taking different types of non-dimensional variables. It was observed that nanofluids are more unstable than the pure fluids. Maxwell [8] gave mathematical model for non -Newtonian fluids exhibiting the elastic and viscous behaviour simultaneously. Khuzhayorov and Auriault [9] introduced law for linear flow of variety of viscoelastic fluids flow. Problem for Maxwell nanofluid taking into account thermophoresis and Brownian diffusion was studied by Jaimala, Singh and Tyagi [10].

The study of magnetic field effects on the onset of convection has important applications in physics and engineering. In metal casting and in cooling systems of electronic devices, magnetic field effects are of great importance. The nanofluid can be taken as a working medium in order to get effective heat performance of such devices. Rayleigh Benard Magneto-convection arises due to combined effect of buoyancy force and magnetic field induced Lorentz force. A non-dimensional parameter called Chandrasekhar number gets introduced due to Lorentz force. Heris, Salehi and Noie [11] observed the increase in thermal efficiency of a two-phase closed thermosyphon while experimental study in presence of magnetic field. The combined effect of a vertical magnetic field and the boundaries on the onset of convection in an electrically nanofluid layer heated from below was investigated by Yadav, Agrawal and Bhargava [12]. Effect of magnetic field considering internal heating after filling the space between plates with nanofluid was also studied by Yadav, Changhoon, Jinho and Hyung [13].

Due to vast applications double diffusion has become appreciably important now days. The difference of temperature affects the buoyancy force in double diffusive convection, but also the difference of concentration of the fluid affects the buoyancy force in double diffusion. A detailed study on double diffusive convection has been discussed in the books by Bejan and Nield [14], Ingham and Pop [15] and Vafai [16]. Horton and Rogers [17] and Lapwood [18] studied fluid instability in a horizontal layer. Further work was made by Haajizadeh, Ozgue and Tien [19], Gaikwad, Malashetty and Prasad [20], Malashetty and Swamy [21].

In fluid flow problems, Soret effect is phenomenon of generation of the concentration flux by temperature gradient. Soret effect induced in concentration on the convective instability of a regular Newtonian fluid saturated in a porous medium has been investigated by many researchers. Wang and Tan [22] investigated the Soret-driven convective instability in Benard cells in a non-Newtonian fluid. The role of Soret effect induced by the temperature gradient was explored by Singh, Bishnoi and Tyagi [23]. Bahlowl, Boutana and Vasseur [24] and Mansour, Amahmid, Hasnaoui and Bourich [25] worked on Soret effect in different forms of fluid layer. Postelnicu [26] and Rajput and Shareef [27] also studied the Soret effect incorporating magnetic field. The phenomenon in a horizontal porous layer taking the base fluid of the nanofluid as binary fluid was studied by Agarwal, Sacheti, Chandran and Bhadauria [28]. These nanofluids proved importance in electroplating and as a transfer medium in medical treatment by Buongiorno [29].

The literature survey indicates that no study has investigated the effect of magnetic field on double diffusive convection in a nanofluid layer with Soret factor. The present study examines the effect of vertical magnetic field on Soret induced double diffusive convection in a nanofluid layer.

II. MATHEMATICAL STATEMENT

We consider a layer of nanofluid confined between two infinite horizontal surfaces separated by a distance a, with z-axis vertically upward. Lower surface is maintained at higher temperature T_l^* and upper surface is maintained at temperature T_r^*. A uniform vertical magnetic field $M^* = (0, 0, M_0^*)$ is applied (See Fig. 1)

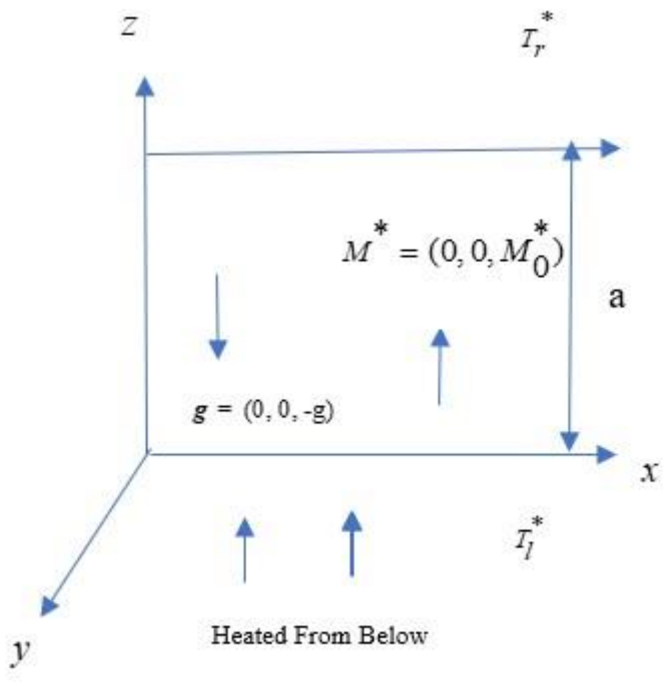


Fig. 1: Physical configuration of the problem

The governing equations for conservation of mass, momentum, energy and concentration of salt and nanoparticles, following Buongiorno [29], Nield and Kuznetsov [7] using modified Darcy Maxwell model [30] are as follows:

$$\nabla^* . \boldsymbol{q}_d^* = 0 \quad , \tag{1}$$

$$\frac{\mu}{K}\boldsymbol{q}_d^* = (1+\lambda^*\frac{\partial}{\partial t^*})[-\nabla^* p^* + (\psi^*\rho_P + (1-\psi^*)\left\{\rho\left(1-\beta_t(T^*-T_r^*)-\beta_c(S^*-S_r^*)\right)\right\})\boldsymbol{g} + \frac{\mu_e}{4\pi}(\nabla^*\times\boldsymbol{H}^*)\times\boldsymbol{H}^*], \quad (2)$$

$$(\rho c)_M \frac{\partial T^*}{\partial t^*} + (\rho c)_F \boldsymbol{V_D^*}.\nabla^* T^* = k_m \nabla^{*2} T^* + \in (\rho c)_P \left[B_d \nabla^*\psi^*.\nabla^* T^* + (\frac{B_t}{T_c^*})\nabla^* T^*.\nabla^* T^*\right] \quad (3)$$

$$\frac{\partial S^*}{\partial t^*} + \frac{1}{\in}\boldsymbol{V_D^*}.\nabla^* S^* = S_d \nabla^{*2} S^* + S_{ct}\nabla^{*2} T^*, \quad (4)$$

$$\frac{\partial \psi^*}{\partial t^*} + \frac{1}{\in}\boldsymbol{V_D^*}.\nabla^* \psi^* = B_d \nabla^{*2}\psi^* + \frac{B_t}{T_c^*}\nabla^{*2} T^*, \quad (5)$$

Maxwell equations in modified form [4] are

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{\in}\left(\boldsymbol{V_D^*}.\nabla^*\right)\right)\boldsymbol{M^*} = \left(\boldsymbol{M^*}.\nabla^*\right)\frac{1}{\in}\boldsymbol{V_D^*} + \eta\nabla^{*2}\boldsymbol{M^*} \quad (6)$$

$$\nabla^*\cdot\boldsymbol{M^*} = 0, \quad \eta = \frac{1}{4\pi\mu_e\sigma'}, \quad (7)$$

where $\boldsymbol{q}_d^* = (u_{1d}^*, u_{2d}^*, u_{3d}^*)$.

For constant temperature and salt concentrations at the boundaries and zero nanoparticle flux (Flow conditions [31]), the boundary conditions are taken as

$$\boldsymbol{q}_d^* = 0, \ T = T_l^*, \ S^* = S_l^*, \ B_d\frac{\partial\psi^*}{\partial z^*} + \frac{B_t}{T_c^*}\frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = 0 \quad (8)$$

$$\boldsymbol{q}_d^* = 0, \ T = T_r^*, \ S^* = S_r^*, \ B_d\frac{\partial\phi^*}{\partial z^*} + \frac{B_t}{T_c^*}\frac{\partial T^*}{\partial z^*} = 0 \text{ at } z^* = a \quad (9)$$

We now non-dimensionalize the physical quantities in the following way:

$$(X,Y,Z) = \frac{(x^*,y^*,z^*)}{a}, \ t = \frac{t^*\alpha_m}{\sigma a^2}, \ (u_{1d},u_{2d},u_{3d}) = \frac{(u_{1d}^*,u_{2d}^*,u_{3d}^*)a}{\alpha_m}, \ p = \frac{p^*K}{\mu\alpha_m}, \ \psi = \frac{\psi^*-\psi_0^*}{\psi_0^*},$$

$$S = \frac{S^*-S_r^*}{S_l^*-S_r^*},$$

$$T = \frac{T^*-T_r^*}{T_l^*-T_r^*}, \ \lambda = \frac{\lambda^*\alpha_m}{a^2} \text{ and } (M_X, M_Y, M_Z) = \frac{(M_X^*, M_Y^*, M_Z^*)}{M_0^*},$$

On replacing $\boldsymbol{q}_d$ by $\boldsymbol{q}$,

$$\nabla.\boldsymbol{q} = 0, \quad (10)$$

$$\boldsymbol{q} = (1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t})[\{-\nabla p - R_m\hat{\boldsymbol{e}}_Z - R_n\phi\hat{\boldsymbol{e}}_Z + R_a T\hat{\boldsymbol{e}}_Z + \frac{R_s}{Ln}S\hat{\boldsymbol{e}}_Z\} + \frac{P_1}{P_{1m}}QD_a\left(\nabla\times M\right)\times M], \quad (11)$$

$$\frac{\partial T}{\partial t} + \left(\boldsymbol{q}.\nabla\right)T = \nabla^2 T + \frac{N_b}{Le}\nabla\psi.\nabla T + \frac{N_a N_b}{Le}\nabla T.\nabla T, \quad (12)$$

$$\frac{1}{\sigma}\frac{\partial S}{\partial t} + \frac{1}{\in}\boldsymbol{q}.\nabla S = \frac{1}{Ln}\nabla^2 S + N_{ct}\nabla^2 T \quad , \quad (13)$$

$$\frac{1}{\sigma}\frac{\partial\psi}{\partial t} + \frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\psi = \frac{1}{Le}\nabla^2\psi + \frac{N_a}{Le}\nabla^2 T, \quad (14)$$

$$\frac{1}{\sigma}\frac{\partial\boldsymbol{M}}{\partial t} + \frac{1}{\in}\left(\boldsymbol{q}.\nabla\right)\boldsymbol{M} = \frac{1}{\in}(\boldsymbol{M}.\nabla)\boldsymbol{q} + \frac{P_1}{P_{1m}}\nabla^2\boldsymbol{M} \quad , \quad (15)$$

Here $R_a(=\frac{\rho g\beta Kd\left(T_l^* - T_r^*\right)}{\mu\alpha_m})$, $R_n(=\frac{\left(\rho_P - \rho\right)\psi_0^* gKd}{\mu\alpha_m})$, $R_m(=\frac{\rho_P\psi_0^* + \rho(1-\psi_0^*)gKd}{\mu\alpha_m})$, $R_s(=\frac{\rho\beta_c gdK(S_l^* - S_r^*)}{\mu S_d})$ are thermal, concentration, basic density and solutal Rayleigh Drcy number respectively, $P_1(=\frac{\mu}{\rho\alpha_m})$ and $P_{1m}(=\frac{\mu}{\rho\eta})$ are Prandtl numbers, $Q(=\frac{\mu_e M_0^{*2} d^2}{4\pi\mu\eta})$ is Magnetic Chandrasekhar number, $D_a(=\frac{K}{a^2})$ is Darcy number, $N_{ct}(=\frac{S_{ct}(T_l^* - T_r^*)}{\alpha_m(S_l^* - S_r^*)})$ is Soret parameter, $N_a(=\frac{B_t(T_l^* - T_r^*)}{B_d T_r^* Q_0^*})$ and $N_b(=\frac{\left(\rho c\right)_P \in Q_0^*}{\left(\rho c\right)_F})$ are modified diffusivity ratio and modified particle density increment respectively, $Le(=\frac{\alpha_m}{B_d})$ and $Ln = \frac{\alpha_m}{S_d}$ are Lewis numbers for nanofluid and salt respectively.

The boundary conditions are

$$\boldsymbol{q} = 0, \ T=1, \ S=1, \quad \frac{\partial\psi}{\partial Z} + N_a\frac{\partial T}{\partial Z} = 0 \ \text{at } Z = 0 \quad (16)$$

$$\boldsymbol{q} = 0, \ T=0, \ S = 0, \quad \frac{\partial\psi}{\partial Z} + N_a\frac{\partial T}{\partial Z} = 0 \ \text{at } Z = 1 \quad (17)$$

A. Basic State

The basic time independent flow state is given by

$$\boldsymbol{q} = 0, p = p_{bs}(Z), \psi = \psi_{bs}\left(Z\right), T = T_{bs}\left(Z\right), S = S_{bs}(Z), \boldsymbol{M} = \hat{\boldsymbol{e}}_Z \quad (18)$$

where the suffix ‘bs’ refers to the basic flow.

Following Chandrasekhar [4], the basic volume fraction and temperature equations are given as

$$\frac{d^2\psi_{bs}}{dZ^2}+N_a\frac{d^2T_{bs}}{dZ^2}=0\ , \tag{19}$$

$$\frac{d^2T_{bs}}{dZ^2}+\frac{N_b}{Le}\frac{d\psi_{bs}}{dZ}\frac{dT_{bs}}{dZ}+\frac{N_aN_b}{Le}\left(\frac{dT_{bs}}{dZ}\right)^2=0\ , \tag{20}$$

$$\frac{1}{Ln}\frac{d^2S_{bs}}{dZ^2}+N_{ct}\frac{d^2T_{bs}}{dZ^2}=0 \tag{21}$$

where,

$$\boldsymbol{q}=0\ ,T_{bs}\left(Z\right)=1\,,S_{bs}(Z)=1,\ \ \frac{d\psi_{bs}}{dZ}+N_a\frac{dT_{bs}}{dZ}=0 \quad \text{at} \quad Z=0 \tag{22}$$

$$\boldsymbol{q}=0\,,T_{bs}\left(Z\right)=0\,,\ S_{bs}(Z)=0\ ,\ \frac{d\psi_{bs}}{dZ}+N_a\frac{dT_{bs}}{dZ}=0 \quad \text{at} \quad Z=1\ . \tag{23}$$

On solving, we get

$$T_{bs}=1-Z\ \ ,\ \psi_{bs}=\psi_0+N_aZ\,,\ \text{and}\ S_{bs}=1-Z$$

B. Perturbed State

On the basic state, we superimpose perturbations in the form

Let $\boldsymbol{q}=\boldsymbol{q'}$, $p=p_{bs}+p'$, $T=T_{bs}+T'$, $S=S_{bs}+S'$, $\phi=\phi_{bs}+\phi'$, and $\boldsymbol{M}=\hat{\boldsymbol{e}}_Z+\boldsymbol{M'}$,

where the primes denote infinitesimal small quantities. Ignoring the products of primed quantities and their derivatives, following linearised form of equations is obtained:

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t}-\frac{P_1}{P_{1m}}\nabla^2\right)\left[\nabla^2u_3'-\left(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left(R_a\nabla_H^2T'-R_n\nabla_H^2\phi'+\frac{R_s}{Ln}\nabla_H^2S'\right)\right]=(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t})Q\frac{P_r}{P_{rM}}\frac{D_a}{\in}\nabla^2\frac{\partial^2u_3'}{\partial Z^2}, \tag{24}$$

$$\frac{\partial T'}{\partial t}-u_3'=\nabla^2T'-\frac{N_aN_b}{Le}\frac{\partial T'}{\partial Z}-\frac{N_b}{Le}\frac{\partial\phi'}{\partial Z}, \tag{25}$$

$$\frac{1}{\sigma}\frac{\partial S'}{\partial t}-\frac{u_3'}{\in}=\frac{1}{Ln}\nabla^2S'+N_{ct}\nabla^2T'\ , \tag{26}$$

$$\frac{1}{\sigma}\frac{\partial\psi'}{\partial t}+\frac{1}{\in}(\boldsymbol{V'}.\nabla)\psi'+\frac{1}{\in}N_au_3'=\frac{1}{Le}\nabla^2\psi'+\frac{N_a}{Le}\nabla^2T'\ \ , \tag{27}$$

$$u_3'=0,T'=0,S'=0,\frac{\partial\psi'}{\partial Z}+N_a\frac{\partial T'}{\partial Z}=0\ \text{ at } Z=0 \text{ and } Z=1 \tag{28}$$

III. LINEAR STABILITY ANALYSIS

Following the linear stability theory by Chandrasekhar [4], the perturbations are taken of the form

$$\left(\psi', T', u_3', S'\right) = \left[\Phi(Z), \Theta(Z), \Omega(Z), \Psi(Z)\right] e^{st+iLX+iMY} \quad , \tag{29}$$

where L and M are dimensionless wave numbers in X and Y directions respectively.

On substituting the above values, we get

$$\begin{aligned}
&\left[\frac{s}{\sigma}\left(D^2-\alpha^2\right)-\frac{P_1}{P_{1m}}\left(D^2-\alpha^2\right)^2-(1+\frac{\lambda s}{\sigma})Q\frac{P_1}{P_{1m}}\frac{D_a}{\in}D^2\left(D^2-\alpha^2\right)\right]\Omega \\
&-R_a\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_1}{P_{1m}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Theta+R_n\alpha^2\left(1+\frac{\lambda s}{\sigma}\right)\left[\frac{P_1}{P_{1m}}\left(D^2-\alpha^2\right)-\frac{s}{\sigma}\right]\Phi \\
&+\frac{R_s}{Ln}\alpha^2(1+\frac{\lambda s}{\sigma})[\frac{s}{\sigma}-\frac{P_1}{P_{1m}}(D^2-\alpha^2)]\Psi=0,
\end{aligned} \tag{30}$$

$$\Omega+\left(D^2-\alpha^2-s-\frac{N_aN_b}{Le}D\right)\Theta-\frac{N_b}{Le}D\Phi=0, \tag{31}$$

$$\frac{\Omega}{\in}+N_{ct}(D^2-\alpha^2)\Theta+[\frac{1}{Ln}(D^2-\alpha^2)-\frac{s}{\sigma}]\Psi=0 \quad , \tag{32}$$

$$\frac{N_a}{\in}\Omega-\frac{N_a}{Le}(D^2-\alpha^2)\Theta-\left[\frac{1}{Le}(D^2-\alpha^2)-\frac{s}{\sigma}\right]\Phi=0 \quad , \tag{33}$$

$\Omega = 0 = \Theta$, $\Psi = 0$, $D\Phi + N_a D\Theta = 0$ at Z= 0 and Z = 1. (34)

Employing Galerkin method to solve equations (30)-(33) together with the boundary condition (34) and taking first estimation as N=1, we have

$$\Omega = A_1 \sin \pi Z \ , \ \Theta = B_1 \sin \pi Z \ , \ \Phi = -N_a C_1 \sin \pi Z \ , \Psi = D_1 \sin \pi Z \ .$$

Substituting these expressions in equations (30)-(33), we get

$$\begin{pmatrix}
\frac{s}{\sigma}\chi^2+\frac{P_1}{P_{1m}}\chi^4 & -R_a\alpha^2(1+\frac{\lambda s}{\sigma}) & -R_nN_a\alpha^2(1+\frac{\lambda s}{\sigma} & -\frac{R_s}{Ln}\alpha^2(1+\frac{\lambda s}{\sigma}) \\
+(1+\frac{\lambda s}{\sigma})\frac{QP_1}{P_{1m}}\frac{D_a}{\in}\chi^2\pi^2 & (\frac{s}{\sigma}+\frac{P_1}{P_{1m}}\chi^2) &)(\frac{s}{\sigma}+\frac{P_1}{P_{1m}}\chi^2) & (\frac{s}{\sigma}+\frac{P_1}{P_{1m}}\chi^2) \\
1 & -(\chi^2+s) & 0 & 0 \\
\frac{1}{\in} & \frac{\chi^2}{Le} & -(\frac{\chi^2}{Le}+\frac{s}{\sigma}) & 0 \\
\frac{1}{\in} & -\chi^2N_{ct} & 0 & -(\frac{\chi^2}{Ln}+\frac{s}{\sigma})
\end{pmatrix}
\begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = 0 \tag{35}$$

where $\chi^2 = \pi^2 + \alpha^2$.

Taking the determinant of above matrix equation as zero, the following Rayleigh number is obtained

$$R_a = \frac{\sigma}{\in \alpha^2}\left[\frac{\begin{array}{l} R_s\alpha^2(\lambda s+\sigma)(\sigma A\chi^2+s)(\sigma\chi^2+sLe)(\chi^2+s)\{\chi^2(\in N_{ct}-1)-s\} \\ -R_n N_a\alpha^2(\lambda s+\sigma)(\chi^2\sigma+sLn)(A\chi^2\sigma+s)\{\chi^2(\in +Le)+sLe\} \\ +\in(\chi^2\sigma+sLn)(\chi^2\sigma+sLe)(\chi^2+s)\{A\sigma\chi^4+B\pi^2\chi^2(\sigma+\lambda s)+s\chi^2\} \end{array}}{(\sigma\chi^2+sLn)(\sigma\chi^2+sLe)(\lambda s+\sigma)(A\chi^2\sigma+s)}\right], \quad (36)$$

where, $A = \frac{P_1}{P_{1m}}$ and $B = Q\frac{P_1}{P_{1m}}\frac{D_a}{\in}$.

IV. RESULTS AND DISCUSSION

A Stationary Convection

Taking s=0 in equation (37),

$$R_a^{st} = \frac{\chi^4}{\alpha^2} - \left(1+\frac{Le}{\in}\right)R_n N_a + \frac{QD_a\pi^2\chi^2}{\in \alpha^2} - \frac{R_s}{\in}(1-\in N_{ct}) \ . \quad (37)$$

From equation (37), the minimum Rayleigh number is given as

$$R_a^{st} = \pi^2\left[1+\left(1+\frac{QD_a}{\in}\right)^{1/2}\right]^2 - R_n N_a\left(1+\frac{Le}{\in}\right) - \frac{R_s}{\in}(1-\in N_{ct}) \quad (38)$$

at critical wave number

$$\alpha_c = \pi\left(1+\frac{QD_a}{\in}\right)^{1/4} \ .$$

In absence of magnetic field, eq. (37) converts to

$$R_a^{st} = 4\pi^2 - R_n N_a\left(1+\frac{Le}{\in}\right) - \frac{R_s}{\in}(1-\in N_{ct})$$

which is same as obtained by Singh, Bishnoi and Tyagi [23]. For the stationary convection it is clear from eq. (38) that the critical Rayleigh number whose value is $4\pi^2$, for Benard convection in Horton-Roger problem for a regular fluid in absence of magnetic field (Horton and Rogers [17]) is reduced with the presence of nanoparticles provided density of nanoparticles is greater than the density of base fluid. The presence of salt further reduces it but inclusion of Soret effect and magnetic field delays the convection. The stationary convection curves for Rayleigh number R_a versus the wave number α are shown in Fig. 2(a)-(f) by assigning fixed values.

N_a = 4, D_a = 0.2, Le = 10, R_n = 4, $\in$ = 0.4, Q = 800, R_s =5, N_{ct} =0.1

with variations in one of these parameters.

Fig 2(a) shows the effect of Darcy number. The increase in Darcy Number increases the Rayleigh Number resulting in delay in convection. Fig 2(b) displays the effect of porosity parameter $\in$. Porosity has stabilizing as well as destabilizing effect in presence of Q. Initially there is decrease in Rayleigh no. with increase in porosity and after a certain wave no. behaviour gets reversed. Fig 2(c) illustrates the behaviour of Rayleigh Number for different values of Lewis number. There is decrease in Rayleigh number with Le.

The effect of R_n on Rayleigh Number is shown in Fig 2(d). Different Curves show that Rayleigh Number is decreased with increase in R_n.

The graphs for Rayleigh Number R_a against the wave number α for various values of N_a and fixed values of other parameters are in Fig 2(e). It is evident that N_a advances the onset of stationary convection.

Fig 2(f) shows the variation of Rayleigh Number for different values of Q. It is clear from the figure that there is a significant increase in the value of critical Rayleigh Number with increase in Q. Thus, the magnetic field stabilises the nanofluid layer and the increase in magnetic field increases the stabilising effect.

Fig. 2(g) shows the effect of Soret parameter. Critical Rayleigh no. increases with increase in Soret parameter and hence responsible for promoting the stability of the flow.

It is clear from Fig. 2(h) on increasing the solutal Rayleigh number critical Rayleigh no. is decreased, thus resulting in an early convection.

B **Oscillatory Convection**

Taking $s = i\omega$ in equation (35), we get the following Rayleigh Number:

$$R_a^{osc} = \frac{\sigma}{\alpha^2 \in}\left[\frac{(X_1 + X_2 + X_3)U - (Y_1 + Y_2 + Y_3)V}{U^2 + \omega^2 V^2}\right] \qquad (40)$$

where $A = \frac{P_r}{P_{rM}}$, $B = Q\frac{P_r}{P_{rM}}\frac{D_a}{\in}$

$$X_1 = \begin{bmatrix} R_s\alpha^2[\sigma\chi^2(\in N_{ct} - 1)\{A\chi^2(\sigma^2\chi^2 - \lambda\omega^2 Le) - \omega^2(Le + \lambda\chi^2)\} \\ -\omega^2\{\lambda\omega^2 Le - \sigma^2 A\chi^2(Le + \lambda\chi^2) - \sigma^2\chi^2\}] \end{bmatrix},$$

$$X_2 = \begin{bmatrix} -R_n N_a\alpha^2[(\in + Le)\{A\chi^4\sigma(\sigma^2\chi^2 - \lambda\omega^2 Ln) - \omega^2\sigma\chi^2(\lambda\chi^2 + Ln)\} \\ -\omega^2\chi^2\sigma^2 Le(1 + \lambda A\chi^2) + \omega^2 LeLn(\lambda\omega^2 - \sigma^2 A\chi^2)] \end{bmatrix},$$

$$X_3 = \begin{bmatrix} \in[\sigma\chi^4(A\chi^2 + B\pi^2)\{\sigma^2\chi^4 - \omega^2 LnLe - \sigma\omega^2(Ln + Le)\} \\ -\omega^2\chi^2(1 + \lambda\pi^2 B)\{\sigma^2\chi^4 - \omega^2 LnLe + \sigma\chi^4(Ln + Le)\}] \end{bmatrix},$$

$$Y_1=\begin{bmatrix} R_s\alpha^2[(\in N_{ct}-1)\{A\chi^4\sigma^2\omega^2(\lambda\chi^2+Le)+\omega^2\chi^2(\sigma^2\chi^2-\lambda\omega^2 Le)\} \\ +\omega^2\sigma(\lambda\omega^2 A\chi^2 Le+\omega^2 Le-\sigma^2 A\chi^4+\lambda\omega^2\chi^2)] \end{bmatrix},$$

$$Y_2=\begin{bmatrix} -R_n N_a\alpha^2[\omega^2\chi^2(\in+Le)(A\chi^4\lambda\sigma^2+A\chi^2\sigma^2 Ln+\sigma^2\chi^2-\lambda\omega^2 Ln) \\ -\omega^2 Le\{\lambda\chi^2\omega^2\sigma(1+ALn)+\sigma\omega^2 Ln-A\sigma^3\chi^4\}] \end{bmatrix},$$

$$Y_3=\begin{bmatrix} \in[\sigma\chi^2\omega^2(A\chi^2+B\pi^2)\{\sigma^2\chi^4-\omega^2 LnLe+\sigma\chi^4(Ln+Le)\} \\ -\omega^2\chi^4(1+\lambda\pi^2 B)\{\sigma\omega^2(Ln+Le)-\chi^4\sigma^2+\omega^2 LnLe\}] \end{bmatrix},$$

$$U=\begin{bmatrix} \omega^2\{\lambda\omega^2 LnLe-\chi^2\sigma^2(\chi^2\lambda+Ln+Le)\}+ \\ A\chi^2\sigma^2\{\sigma^2\chi^4-\omega^2 LnLe-\lambda\omega^2\chi^2(Ln+Le)\} \end{bmatrix},$$

$$V=\begin{bmatrix} \sigma\omega^2\{\lambda\chi^2(Ln+Le)+LeLn\}-\chi^4\sigma^3 \\ -A\chi^2\sigma\{\sigma^2\chi^2(Ln+Le)+\lambda(\sigma^2\chi^4-\omega^2 LnLe)\} \end{bmatrix}$$

The frequency of oscillation is given by

$$(X_1'L_1-X'L')\omega^6+(X_1'M_1+Z_1'L_1-Y'L'-X'N')\omega^4+(Y_1'L_1+Z_1'M_1-X'M'-Y'N')\omega^2+(Y_1'M_1-Y'M')=0. \quad (41)$$

Here

$$X_1'=\left[-R_n N_a\alpha^2\lambda LeLn+\chi^2\in LeLn(1+\lambda\pi^2 B)-\lambda R_s\alpha^2 Le\right],$$

$$Y_1'=\left[\sigma^3 A\chi^6 R_s\alpha^2(\in N_{ct}-1)+\in\sigma^3\chi^8(A\chi^2+B\pi^2)-R_n N_a\alpha^2 A\chi^6\sigma^3(\in+Le)\right],$$

$$Z_1'=\begin{bmatrix} R_n N_a\alpha^2[\sigma\chi^2\{\lambda\chi^2(\in+Le)+\sigma Le+\in Ln+LeLn\}+A\chi^4\lambda\sigma Ln(\in+Le)+\lambda A\chi^4\sigma^2 Le \\ +\sigma^2 A\chi^2 LeLn]+R_s\alpha^2[\chi^2\sigma(Le-\in N_{ct}Le+\sigma-\lambda\chi^2\in N_{ct}+\lambda\chi^2)+\sigma^2 A\chi^2 Le \\ -\lambda A\chi^4\sigma Le(\in N_{ct}-1)+\lambda\sigma^2 A\chi^4-\in\chi^4\sigma[\chi^2(1+\lambda\pi^2 B)(Le+\sigma+Ln) \\ +(A\chi^2+B\pi^2)\{\sigma(Ln+Le)+LnLe\} \end{bmatrix},$$

$$L_1=\sigma\lambda\chi^2(Ln+Le)+\sigma LeLn+A\sigma\chi^2\lambda LnLe,$$

$$M_1=-A\sigma\chi^2\{\sigma^2\chi^2(Ln+Le)+\lambda\sigma^2\chi^4\}-\chi^4\sigma^3,$$

$$X'=\begin{bmatrix} -R_n N_a\alpha^2[\lambda\chi^2\{\sigma Le+Ln(\in+Le)\}+\sigma LeLn(1+\lambda A\chi^2)] \\ +\in[\chi^4(1+\lambda\pi^2 B)\{\sigma(Ln+Le)+LeLn\}+\sigma\chi^2 LnLe(A\chi^2+B\pi^2)] \\ -R_s\alpha^2[\chi^2\lambda\sigma+Le\{\sigma+\chi^2\lambda(1-\in N_{ct})\}+\lambda\sigma A\chi^2 Le] \end{bmatrix},$$

$$Y'=\begin{bmatrix} R_n N_a\alpha^2[\chi^4\sigma^2(\in+Le)+A\sigma^2\chi^4\{(\in+Le)(Ln+\lambda\chi^2)+\sigma Le\}] \\ -\in\sigma^2\chi^6\{\chi^2(1+\lambda\pi^2 B)+(A\chi^2+B\pi^2)(Ln+Le+\sigma)\} \\ +R_s\alpha^2\chi^4\sigma^2[(1-\in N_{ct})\{1+A(Le+\lambda\chi^2)+\sigma A\}] \end{bmatrix},$$

$$L'=\lambda LnLe,$$

$$M' = A\chi^6\sigma^4 ,$$

and $N' = -\chi^2\sigma^2\{\chi^2\lambda + Ln + Le + ALnLe + A\chi^2\lambda(Ln + Le)\}$.

For oscillatory convection , R_a vs α curves are shown in figure 3 (a)-(k) for fixed value of $R_n, Le, Q, D_a, \varepsilon, N_a, \sigma, \lambda$, Ln , R_s and N_{ct} with variations in one of these parameters. No critical Rayleigh number is obtained for oscillatory convection.

Fig 3(a), (d), (e), (f), (j) and (k) depict the effects of D_a , N_a , R_n , Q , R_s and λ . In each of these graphs, Rayleigh no. increases with increase in each of the respective parameters.

Fig 3(b), (g), (h) and (i) show the effects of ε , Ln , σ and N_{ct} . In each of these graphs, Rayleigh no. decreases with increase in each of these respective parameters. Fig. 3(c) shows the dual effect of Lewis number on oscillatory Rayleigh number. Initially Rayleigh number decreases with increase in Lewis no. but after a certain wave no. the effect gets reversed.

The comparison of stability curves for both linear stationary convection and the linear oscillatory convection are drawn in figure 4(a)-(h). For different values of parameters, a comparison between two convections is shown.

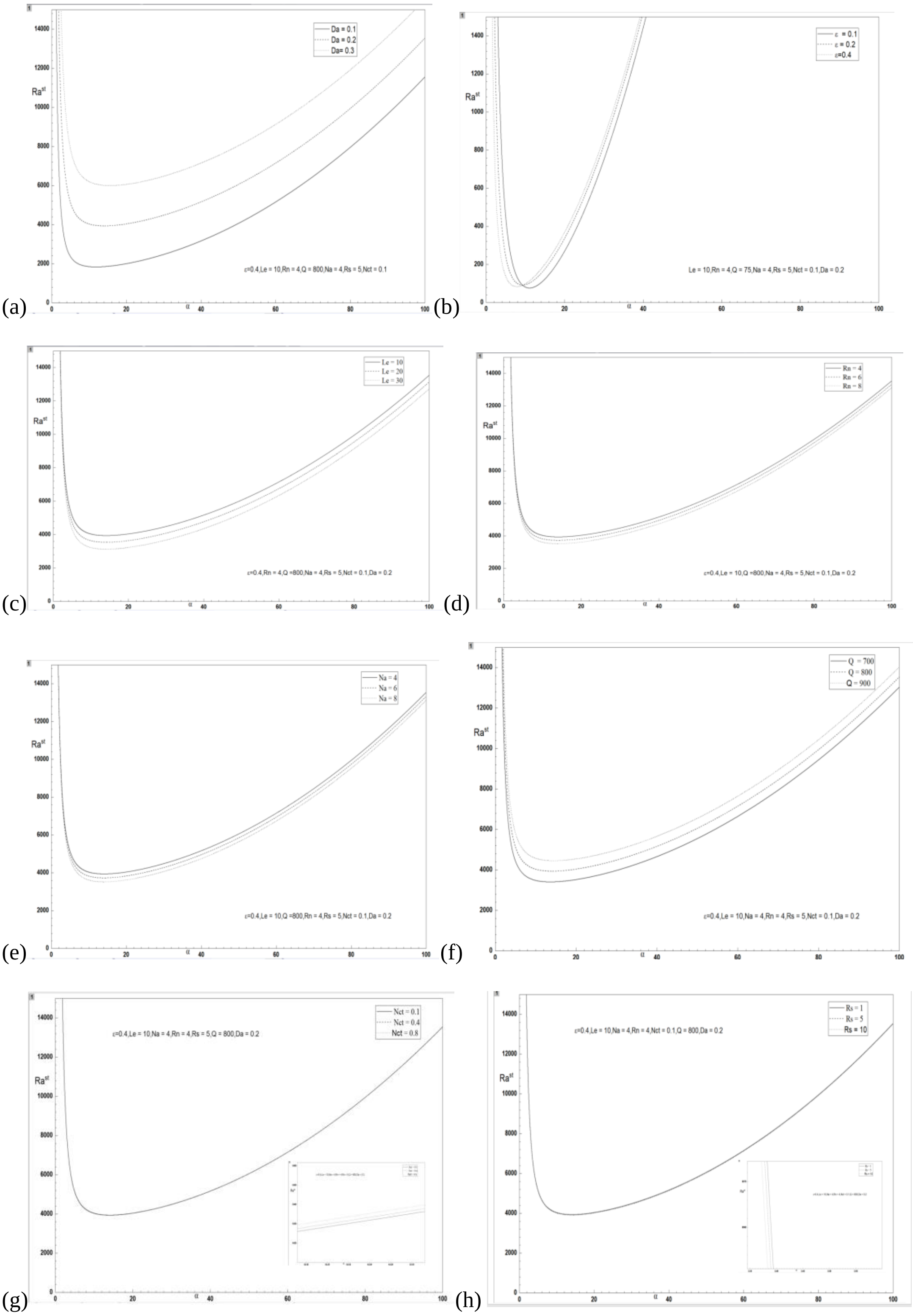
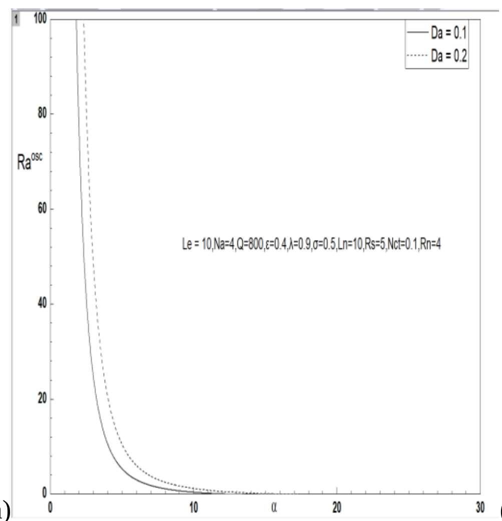
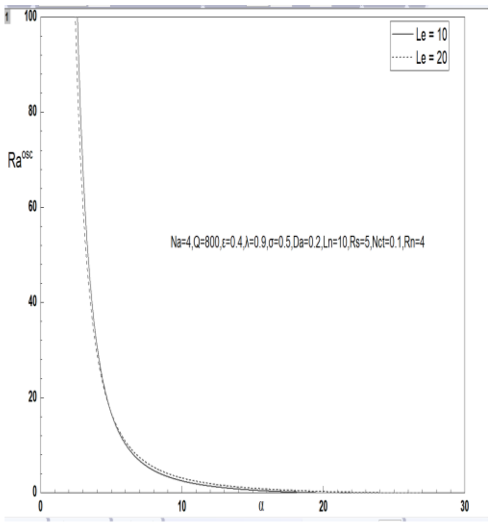


Fig. 2: Stationary convection for different values of

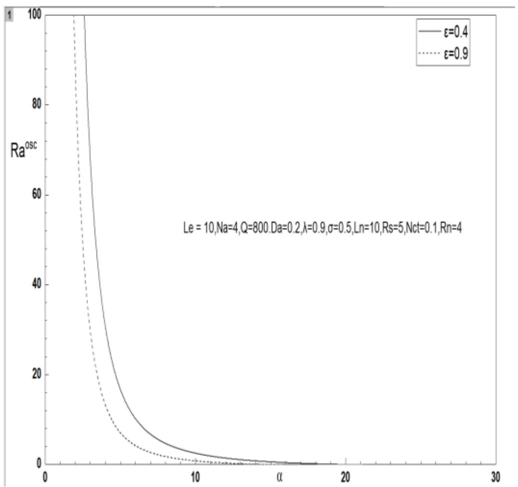
(a) D_a (b) ε (c) Le (d) R_n (e) N_a (f) Q (g) N_{ct} (h) R_s


Da = 0.1
Da = 0.2
Ra^osc
Le = 10,Na=4,Q=800,ε=0.4,λ=0.9,σ=0.5,Ln=10,Rs=5,Nct=0.1,Rn=4
α

(a)

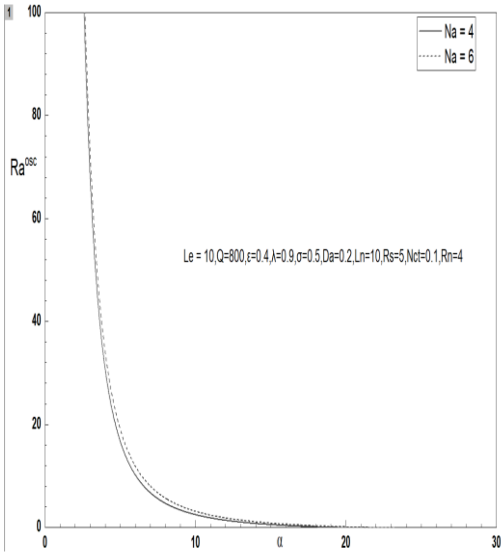

Le = 10
Le = 20
Ra^osc
Na=4,Q=800,ε=0.4,λ=0.9,σ=0.5,Da=0.2,Ln=10,Rs=5,Nct=0.1,Rn=4
α

(b)

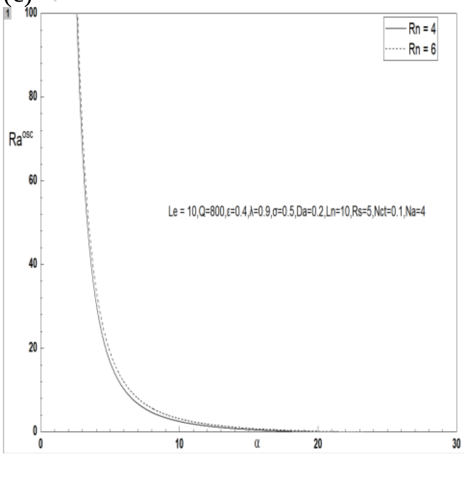

ε=0.4
ε=0.9
Ra^osc
Le = 10,Na=4,Q=800.Da=0.2,λ=0.9,σ=0.5,Ln=10,Rs=5,Nct=0.1,Rn=4
α

(c)

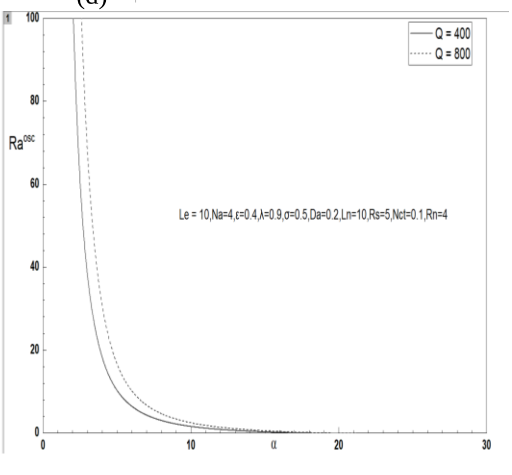
(d)


Na = 4
Na = 6
Ra^osc
Le = 10,Q=800,ε=0.4,λ=0.9,σ=0.5,Da=0.2,Ln=10,Rs=5,Nct=0.1,Rn=4
α

(e)


Rn = 4
Rn = 6
Ra^osc
Le = 10,Q=800,ε=0.4,λ=0.9,σ=0.5,Da=0.2,Ln=10,Rs=5,Nct=0.1,Na=4
α

(f)


Q = 400
Q = 800
Ra^osc
Le = 10,Na=4,ε=0.4,λ=0.9,σ=0.5,Da=0.2,Ln=10,Rs=5,Nct=0.1,Rn=4
α

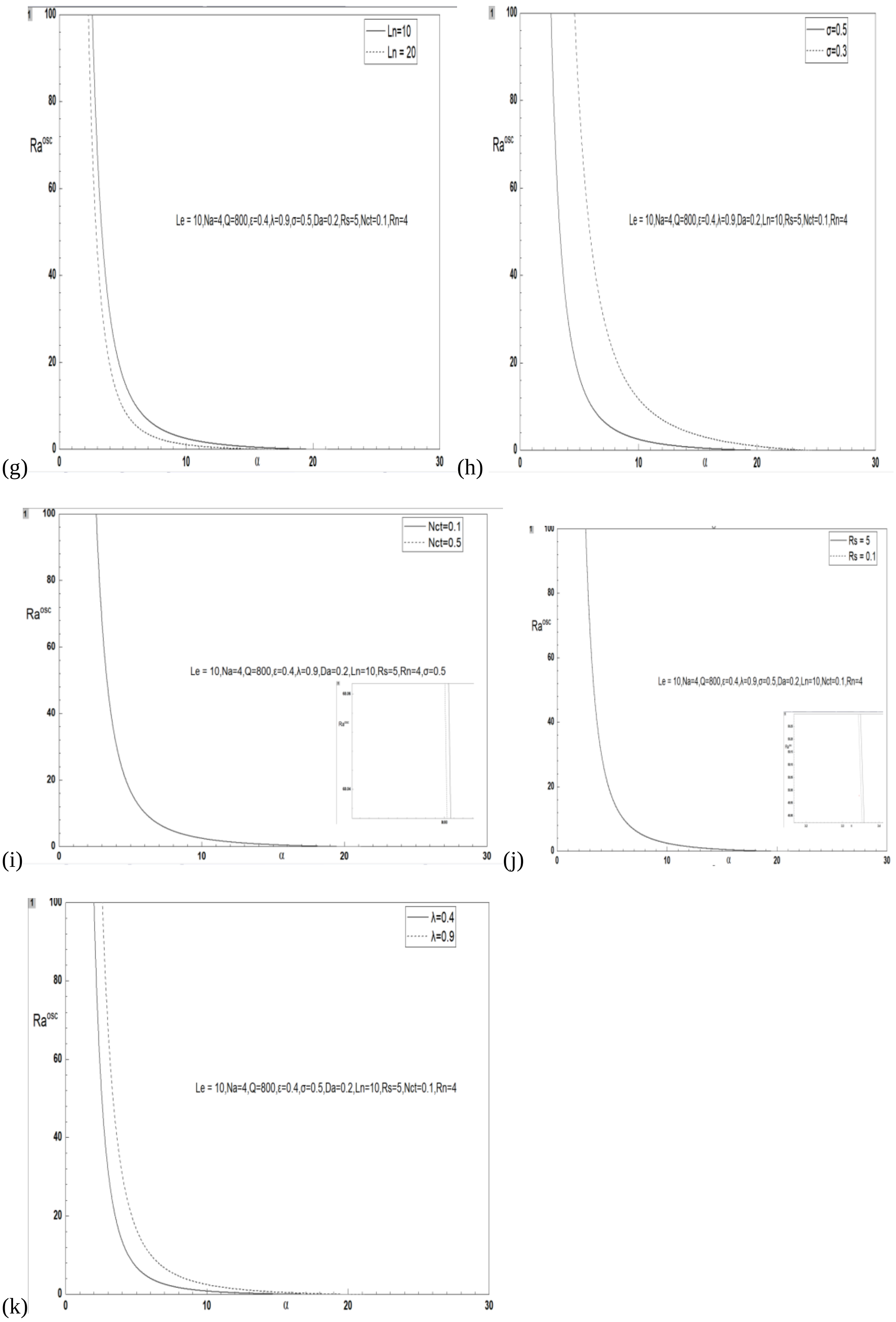


Fig. 3: Oscillatory convection for different values of

(a) D_a (b) ε (c) Le (d) N_a (e) R_n (f) Q (g) Ln (h) σ (i) N_{ct} (j) R_s (k) λ

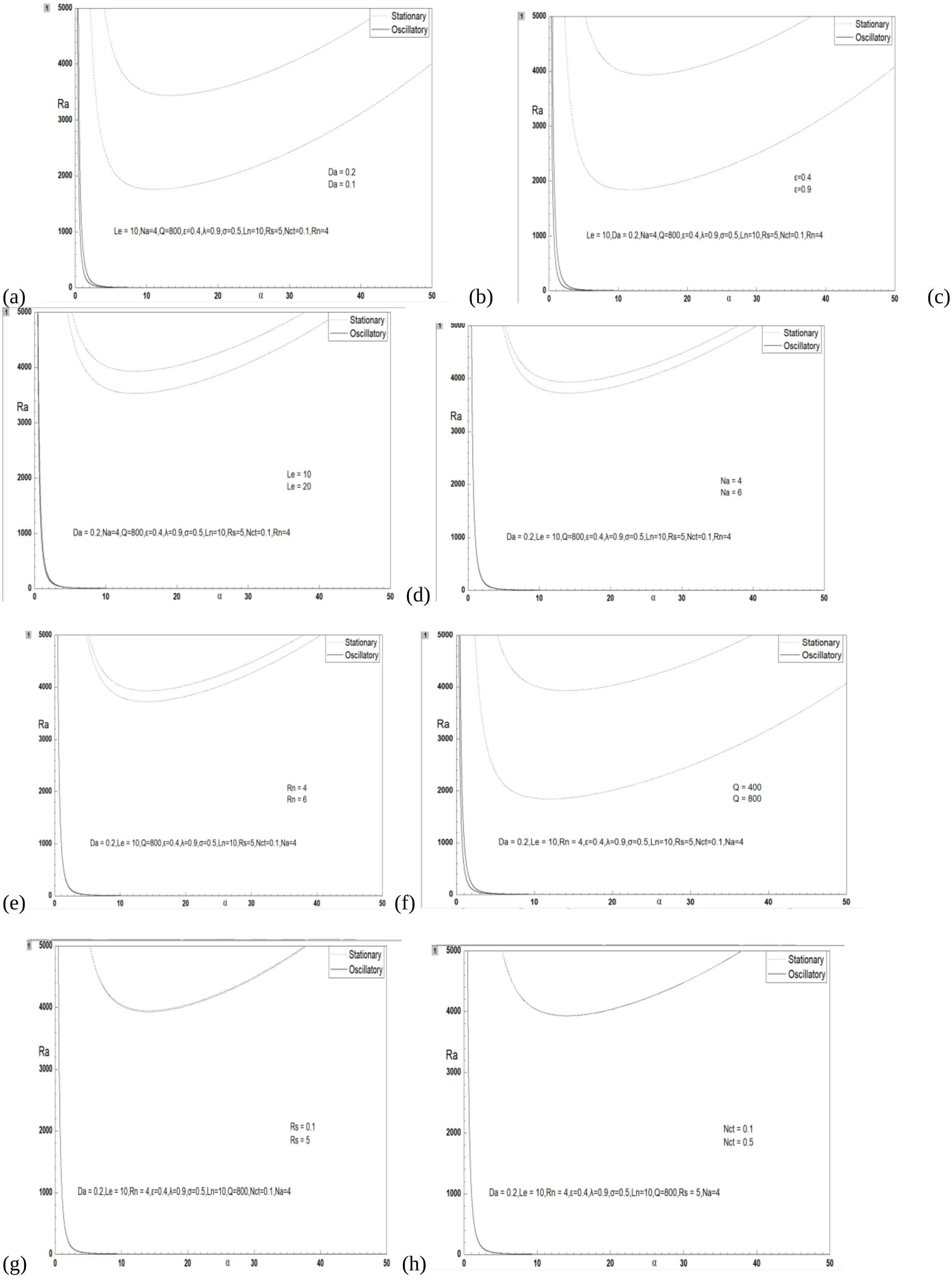


Fig. 4: Comparison of both types for different values of

(a) D_a (b) ε (c) Le (d) N_a (e) R_n (f) Q (g) R_s (h) N_{ct}

V. CONCLUSION

In this paper, we have determined how the presence of the magnetic field affects double diffusive convection in Soret induced Darcy Maxwell nanofluid layer. The layer was soluted and heated from below and uniform magnetic field was applied in vertical direction. The comparison of results obtained has been done with the existing relevant studies. The main conclusions of the present analysis are as follows:

- R_a^{st} has been observed to be function of parameters D_a, ε, Le, R_n, N_a, Q, N_{ct}, R_s whereas R_a^{osc} is function of Ln, σ and λ in addition to above parameters.
- The effect of Lewis number Le is to decrease R_a^{st} but has twin effect on oscillatory convection.
- An increase in porosity decreases R_a^{osc} but dual effect on stationary Rayleigh number.
- A positive Soret coefficient N_{ct} has stabilizing effect on convection as obtained by Gaikwad, Malashetty and Prasad [32] for a regular fluid as well as obtained by Singh et al. [23] for a nanofluid but here in presence of magnetic field the effect is found to destabilize the oscillatory Rayleigh number as increase in N_{ct} decreases R_a^{osc}.
- The influence of magnetic field is to stabilise the Soret induced double diffusive convection as was found by Yadav [13] in nanofluid convection induced by internal heating.
- In this convection under magnetic field, Darcy number also comes into play and has been observed to provide stabilizing effect on stationary and oscillatory modes.
- An increase in Solutal Rayleigh Darcy number R_s was observed to cause increase in R_a^{st} and R_a^{osc} by Singh, Bishnoi and Tyagi [23] but the presence of magnetic field here causes the effect of R_s to be stabilize the oscillatory convection.
- The increase in parameters Ln, σ increases the oscillatory Rayleigh number. This behaviour is opposite to be observed in absence of magnetic field [23].
- Parameters R_n and N_a destabilizes the stationary mode but are found to stabilize the oscillatory convection.

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