Harmonic Generation via Phase-Mismatch in Quantum Plasma

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Abstract

We investigate the generation of third harmonic radiation through the propagation of a circularly polarized laser beam in under dense quantum magneto plasma. This investigation employs the Quantum Hydrodynamic (QHD) model, which has been recently developed. Notably, the analysis accounts for the influence of quantum Bohm potential, quantum statistical Fermi pressure, and the electron spin of -1/2. As the circularly polarized laser beam travels through the quantum plasma, it induces density oscillations at the second harmonic. These oscillations, combined with velocity oscillations, result in a third-order current density that gives rise to third harmonic radiation.

Keywords: Harmonic generation, Quantum plasma, Quantum diffraction effects, QHD model, Electron spin-1/2

1. Introduction

1.1: Plasma:

In 1920, Nobel laureate Irving Langmuir embarked on groundbreaking research, pioneering the systematic exploration of a luminescent ionized gas, generated through electric discharge within a confined tube. His efforts led to the conceptualization of this unique state of matter as "plasma," representing the fourth distinct phase. Remarkably, over 99% of the observable universe exists in the plasma state, underscoring its fundamental significance.

Presently, the realm of plasma physics stands as a burgeoning scientific field, gaining rapid traction in response to the pressing energy challenges of our time. With its interdisciplinary nature, plasma physics boasts a vast array of potential applications across laboratory settings, space exploration, and astrophysical contexts alike. In the industrial realm, laboratory-generated plasmas find practical implementation in diverse sectors, including metallurgy and industrial processes. Notable applications encompass plasma spraying, microelectronics etching, metalworking, welding, and enhancing surface durability through processes like nitriding to counteract corrosion and wear [1].

The influence of plasma extends to the realm of astroplasma physics, where it plays a pivotal role in unraveling the complexities of various celestial phenomena. These encompass the formation of dust clusters and intricate structures, the behavior of molecular clouds, the genesis of proto-stars and fully-fledged stars, the dynamics of cometary tails, the properties of nebulae, and the enigmatic behavior of magnetospheres, among others.

Yet another frontier in plasma physics involves addressing global energy scarcity. Scientists and researchers are diligently working towards devising solutions through controlled thermonuclear fusion. This ambitious pursuit employs cutting-edge experimental devices like ITER, JET, Tokamaks, and NOVA. These systems explore both inertial and magnetic confinement techniques, harnessing plasma to potentially revolutionize power generation and mitigate the energy crisis that looms on a global scale.

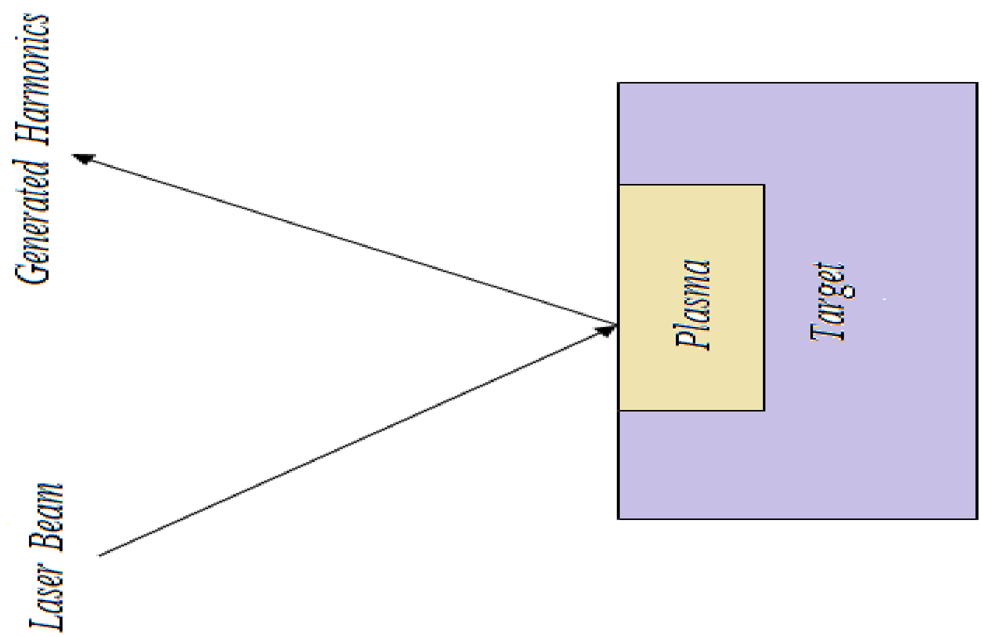
In general it is not easy to say that all ionized gases are plasmas. A more rigorous definition requires three criterion to be satisfied. Firstly, its dynamics is governed by long range electromagnetic forces rather than force due to local collisions i.e. collective behavior. Secondly, the plasma oscillation frequency must be greater than collision frequency of charged particle to neutrals so that dynamics is still governed by long range force. According to the third criterion, its ability to 'iron out' an external electric potential i.e. sparing (shielding out) the bulk of plasma from external field which leads to quasi neutrality condition i.e. $n_i = n_e = n.$ Microscopic variation of quasi-neutrality leads to plasma electron oscillations with a frequency $\omega_p (= (4\pi n_0 e^2 / m)^{1/2}$ where m is the electron mass), known as the plasma frequency. This characterizes plasma as an elastic medium [2-4]. The ionic mass is much greater than the electronic mass. Therefore, the ion oscillation frequency is very small as compared to that of the electrons. The ions can therefore be regarded as a stationary immobile background in the plasma system. The energy distributions of ions and electrons within a plasma typically exhibit distinct characteristics. These distributions are often quantified by ion temperature and electron temperature, which may not necessarily align or be equivalent. Moreover, it's important to note that various ion and electron species can coexist within the plasma, each potentially possessing disparate temperatures or diverse energy distribution patterns.

1.2: Quantum Plasma:

Plasma physics is conventionally regarded as a field governed by classical principles. Nonetheless, the past decade has witnessed a resurgent fascination with plasma systems wherein quantum effects wield significant influence, driven by their pivotal applications in densely packed astrophysical entities [5], such as the interiors of Jupiter, massive white dwarfs, magnetars, and neutron stars. This resurgence extends to intense experiments involving plasmas at solid density irradiated by lasers [6-8], as well as ultra-small electronic devices [9] encompassing microelectronics, semiconductor devices, quantum dots, nanowires [10], carbon nanotubes [11], quantum diodes [12], bio-photonics [13], ultra-cold plasmas [14], and micro-plasmas [15]. Quantum mechanics takes the forefront in plasmas when the quantum attributes of constituent particles distinctly shape their macroscopic characteristics. These quantum plasmas consist of ions, degenerate electrons, positrons, and charged nano-particles. The degeneracy of lighter plasma components emerges at notably high densities and relatively low temperatures, where the Wigner-Seitz radius equates to or dips below the De-Broglie thermal wavelength. In these equations, signifies the mass of the quantum particles (like degenerate electrons, ions, and nano-charged particles), denotes the thermal velocity of the quantum particles, represents temperature, and stands for the Boltzmann constant. The interplay of wave functions intensifies with overlapping or comparable values of the respective quantities, or when temperature approaches or falls below the Fermi temperature wherein denotes the Fermi energy. The transition from a Maxwell-Boltzmann distribution function to a Fermi-Dirac distribution function takes place as plasma particle temperature approaches. Consequently, defining a Quantum coupling factor becomes valuable for both electron-electron and ion-ion interactions. This factor for electron-electron Coulomb coupling hinges on the ratio of electrostatic interaction energy to the electron Fermi energy, taking into account as the electron charge magnitude and as the mean interaction separation. Notably, holds true for metallic plasmas, making it pertinent to scrutinize the role of inter-particle collisions in quantum plasma collective processes. Pauli blocking notably curtails collision rates in most practical scenarios, restricting collisions to an electron shell roughly around the Fermi surface. The collision frequency thus remains proportional to

These phenomena usher in a new realm of distinctions: (1) particles lack phase space localization, (2) distribution functions shift from Maxwell-Boltzmann to Fermi-Dirac, and (3) certain particles like electrons and protons possess intrinsic magnetic moments or spins, impacting dynamics through interactions with magnetic fields. A collision-free quantum plasma regime emerges, significant for phenomena transpiring within femtosecond time scales in metallic plasmas. In the context of astrophysical settings like white dwarf stars, the average electron separation aligns with the Compton length, leading to electron speeds at the Fermi surface comparable to the speed of light . A relativistic perspective becomes applicable, especially in the cores of massive stars where relativistic degenerate electrons prevail. As fermions, electrons adhere to the constraint of a single electron occupying a given quantum state (defined by position and spin). Consequently, the volume occupied by a lone electron amounts to. By virtue of the Heisenberg uncertainty principle, the mean momentum enters the equation. In scenarios featuring relativistic electrons, velocities approach the speed of light. Here, electron pressure in a simple gas arises from momentum, quantified as the momentum transfer per unit area: The manifestation of quantum effects can be quantified through the thermal De-Broglie wavelength, $\lambda_B = (\hbar / m v_T)$. In the classical regime, particles can be approximated as point-like ($\hbar$ tends to 0), precluding overlapping wave functions. This implies that the classical and quantum regimes do not typically coexist. However, recent investigations have unearthed the potential for a phase transition between these two regimes. Quantum corrections rise to prominence when the distance separating charged particles mirrors or exceeds inter-particle distances, and when temperature descends beneath a critical point termed the Fermi temperature.

1.3: Harmonic Generation:



The interaction of high intensity laser pulse with plasma leading to harmonic generation has been active area of research for last thirty years [16]. The physical phenomenon of interaction of intense laser pulse with plasma leads to number of parametric instabilities and nonlinear effects such as laser wakefield acceleration, inertial confinement fusion, Raman scattering, self phase modulation, ponderomotive self focusing and harmonic radiation generation. Generation of harmonic radiation in laser produced plasma and laboratory plasma is an important subject and also provides considerable potential for plasma diagnostics [17-23]. From few years a great deal of research has been focused on second and third harmonic in laser produced plasma [24-26]. In the process of harmonic generation, two photons of energy $\hbar\omega_1$ and momentum $\hbar k_1$ combines to produce a photon of energy $\hbar\omega_2$ and momentum $\hbar k_2$, where (ω_1, k_1) are the frequency and wave vector of fundamental wave and (ω_2, k_2) are the frequency and wave vector of second harmonic wave which satisfy dispersion relation for electromagnetic wave. During third harmonic generation phenomenon, the fundamental laser beams generates a beam of frequency with three times of the fundamental frequency. The interaction of circularly polarized intense laser with homogenous plasma induced transverse nonlinear plasma current, resulting in generation of odd harmonics of laser frequency in forward direction [27]. Although number of high order harmonics generation [28-30] has been analyzed but third harmonic generation [31-33] has its unique place in laser plasma interaction.

All previous research has centered around classical plasmas. However, when dealing with plasmas in which the de Broglie thermal wavelength of charge carriers is comparable to or exceeds the inter particle distance $n_e^{-1/3}$ i.e. $n_e\lambda_B \geq 1$ or temperature T is similar to or lower than the electron Fermi temperature, the introduction of degeneracy becomes relevant, leading to the adherence of plasma particles to the Fermi-Dirac distribution. At this point, quantum degeneracy effects become pivotal, highlighting the significance of studying quantum plasma. Over the past decade, there has been a growing fascination with exploring novel facets of quantum plasma due to its practical applications. Numerous authors have delved into the subject of harmonic generation within quantum magnetoplasmas [34-36]. While investigations have been conducted regarding phase-matched third harmonic generation of laser pulses in high-density quantum plasmas under the influence of a wiggler magnetic field [37], as of now, there is no recorded endeavor to examine the phase-mismatched third harmonic generation caused by circularly polarized lasers within densely magnetized quantum plasmas, accounting for electron spin-1/2 effects.

The objective of this chapter is to conduct a comprehensive analysis of the third harmonic radiation generated by a circularly polarized laser within a high-density, low-temperature quantum plasma. The investigation begins by assuming the plasma to be at a low temperature, allowing us to disregard the thermal motion of electrons. Within the framework of the mildly relativistic regime, a perturbative approach is employed alongside the newly developed Quantum Hydrodynamic (QHD) model. The QHD model extends the classical plasma model by expressing transport equations using conservation laws for particle quantities, momentum, and energy. One key advantage of the QHD model over kinetic models lies in its numerical efficiency and its direct utilization of macroscopic variables like momentum and energy. This facilitates the implementation of boundary conditions and enables a relatively simpler treatment of nonlinear phenomena. As a result, the QHD approach is particularly well-suited for describing such phenomena within quantum plasma. In the mildly relativistic regime, relativistic effects become significant in higher-order velocity components.

The structure of this chapter is organized as follows: It comprises four sections. Sec. 2 is dedicated to exploring the nonlinear current density associated with the generation of third harmonic radiation. Subsequently, the conversion efficiency is analyzed in Sec. 3. Finally, the Sec. 4 summarizes the findings and conclusions drawn from this study.

2: Formulation:

Let us take the propagation of a circularly polarized laser pulse of frequency ω_0 and wave number k_0 and constant amplitude E_0 in magnetized cold quantum plasma of uniform density n_0 along the direction of static magnetic field $\mathrm{B}_z \parallel \hat{z}.$ The fields of laser are

$$\vec{E}_{(x,y)} = E_0(\hat{x} + i\hat{y})e^{i(k_0 z - \omega t)} \tag{1}$$

and

$$\vec{B}_{(x,y)} = c\vec{k}_0 \times \vec{E}_{(x,y)} / \omega_0 .$$

We assume that the plasma is cold and there is a fixed ionic background to ensure charge neutrality and fast processes to be considered in quantum plasma. Response of electron to the electromagnetic field is governed by the set of QHD equations [34,35],

$$\frac{\partial(\gamma\vec{v})}{\partial t} = -\frac{e}{m\gamma}\left[\vec{E} + \frac{1}{c}(\vec{v}\times\vec{B})\right] - \frac{v_F^2}{3n_0^2}\frac{\nabla n^3}{n} + \frac{\hbar^2}{2m^2\gamma^2}\nabla\left(\frac{1}{\sqrt{n}}\nabla^2\sqrt{n}\right) - \frac{2\mu_B}{m\hbar}\vec{S}.(\nabla\vec{B}), \tag{2}$$

$$\frac{\partial\vec{S}}{\partial t} = \frac{2\mu_B}{\hbar}(\vec{B}\times\vec{S}), \tag{3}$$

$$\frac{\partial(\gamma n)}{\partial t} + \nabla(\gamma n\vec{v}) = 0. \tag{4}$$

where, m is the rest mass of electron, e is the electron charge, n is the electron density, γ is the relativistic factor, $\hbar$ is Planck's constant divided by 2π, S is the spin angular momentum with $S_0 = |S_0| = \hbar/2$, $\mu_B = e\hbar/2m$ is the Bohr magneton and $v_F = (\hbar/m)(3\pi^2 n)^{1/3}$ represents the Fermi velocity of electrons. On the right hand side of equation (2) the first term represents the Lorentz force, second term is the electron Fermi pressure, third term is the quantum Bohm potential produced due to density fluctuations and the last term denotes the force due to spin magnetic moment of plasma electrons and the classical case may be recovered in the limit of $\hbar = 0$.

On perturbation of eqs. (2) and (4) in orders of radiation field, the first order quiver velocity and density components are found to

$$v_x^{(1)} = \frac{1}{(\omega_c^2 - \omega_0^2)}\left[\frac{ie(\omega_0 - \omega_c)}{m} + \frac{k_0 Q(i\omega_c n_{y1} - \omega_0 n_{x1})}{n_0} + \frac{2i\mu_B k_0 S_0(\omega_c - \omega_0)}{m\hbar}\right]E_0 e^{i(k_0 z - \omega_0 t)}, \tag{5}$$

$$v_y^{(1)} = \left[\frac{-e}{m\omega_0} + \frac{i\omega_c v_{x1}}{\omega_0} + \frac{k_0 n_{y1} Q}{n_0 \omega_0} + \frac{2\mu_B k_0 S_0}{m\hbar\omega_0} \right] E_0 e^{i(k_0 z - \omega_0 t)}, \tag{6}$$

$$n_x^{(1)} = \frac{n_0 k}{\omega_0 \left\{ k_0^2 Q + \left(\omega_c^2 - \omega_0^2 \right) \right\}} \left[\frac{ie(\omega_0 - \omega_c)}{m} + \frac{ik_0 Q \omega_c n_{y1}}{n_0} + \frac{2i\mu_B S_0 k_0 (\omega_c - \omega_0)}{m\hbar} \right] E_0 e^{i(k_0 z - \omega_0 t)}, \tag{7}$$

$$n_y^{(1)} = \frac{n_0 k_0 \left(\omega_c^2 - \omega_0^2 \right)}{\left\{ (\omega_0^2 - k_0^2)(\omega_c^2 - \omega_0^2) + k_0^2 Q \omega_c^2 \right\}} \left[\frac{-e}{m} + \frac{e\omega_c}{m(\omega_c + \omega_0)} - \frac{ik_0 Q \omega_0 \omega_c n_{x1}}{n_0 (\omega_c^2 - \omega_0^2)} - \frac{2\omega_c \mu_B S_0 k_0}{m\hbar(\omega_c + \omega_0)} + \frac{2\mu_B S_0 k_0}{m\hbar} \right] E_0 e^{i(k_0 z - \omega_0 t)}, \tag{8}$$

Quantum degenerate plasma has a crucial characteristic called spin. It is essential for exposing the plasma to the external magnetic field, whose impact is discernible in the perturbed spin magnetic moment for plasma electrons via the spin angular momentum,

$$S_x^{(1)} = \frac{2i\mu_B S_0 \left\{ \frac{2b\mu_B}{\hbar} - \omega_0 \right\}}{\hbar \left\{ \frac{4b^2 \mu_B^2}{\hbar^2} - \omega_0^2 \right\}} E_0 e^{i(k_0 z - \omega_0 t)}, \tag{9}$$

$$S_y^{(1)} = \frac{-2\mu_B (bS_{x1} - iS_0)}{i\hbar\omega_0} E_0 e^{i(k_0 z - \omega_0 t)}, \tag{10}$$

and

$$S_z^{(1)} = \frac{-2\mu_B S_0 (1 - i)}{\hbar\omega_0} E_0 e^{i(k_0 z - \omega_0 t)}. \tag{11}$$

By following similar steps for nth harmonic, the velocity, perturbed density and spin magnetic moment for electron can be obtained by substituting $\omega_0 \to n\omega_0$, $\vec{E}_0 \to \vec{E}_n$, $(k_0 z - \omega_0 t) \to (k_n z - n\omega_0 t)$, from equations (5)-(11). Hence, the linear part of induced current density for nth harmonic, $J^{(1)}(n\omega_0) = J_c^{(1)}(n\omega_0) + J_c^{(1)}(n\omega_0)$ can be written as,

$$J^{(1)}(n\omega_0)\left[J_c^{(1)}(n\omega_0) + J_s^{(1)}(n\omega_0) \right] E_n e^{i(k_n z - n\omega_0 t)} \tag{12}$$

where, $\vec{J}_c = -\frac{2\mu_B}{\hbar}\nabla(n.S)$ is the current density due to the spin magnetic moment and $\vec{J}_c = -e(\vec{n}.\vec{v})$ is the conventional current. The dispersion relation for n[th] harmonic is

$$c^2 k_n^2 = n^2\omega_0^2 + \omega_p^2 \frac{imn\omega_0\left\{J_c^{(1)}(n\omega_0`) + J_s^{(1)}(n\omega_0)\right\}}{n_0 e^2}. \tag{13}$$

The laser produces oscillatory velocity of electrons and exerts a ponderomotive force $F_2 = -(1/2)e\vec{v}_1 \times \vec{B}_1$ on them at $(2k_0, 2\omega_0)$, which gives rise to oscillatory velocity $v^{(2)}$, which couples with density perturbation at laser frequency through equation of continuity to produce density perturbation at $(2k_0, 2\omega_0)$ and $n^{(2)}$ couples with $v^{(1)}$ to produce nonlinear current density at the third harmonic of frequency.

The third harmonic velocity and density components are obtained as,

$$v_x^{(3)} = \frac{1}{(\omega_c^2 - 9\omega_0^2)}\left[3\omega_0(3\omega_0 v_{x1} - i\omega_c v_{y1}) + \frac{iev_{z2}(\omega_c - 3\omega_0)}{mc} + \frac{2i\mu_B k_0}{m\hbar}(\omega_c S_{y2} - 3\omega_0 S_{X2})\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{14}$$

$$v_y^{(3)} = \left[\frac{i\omega_c v_{x3}}{3\omega_0} + \frac{ev_{z2}}{3mc\omega_0} - v_{y1} + \frac{2\mu_B k_0 S_{y2}}{3m\hbar\omega_0}\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{15}$$

$$v_z^{(3)} = \left[\frac{ie}{3m\omega_0 c}(iv_{y2} - v_{x2})\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{16}$$

$$n_x^{(3)} = \frac{k_0}{\omega_0}\left[v_{x2}n_{x1} + v_{x1}n_{x2} + n_0 v_{x3}\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{17}$$

$$n_y^{(3)} = \frac{k_0}{\omega_0}\left[v_{y2}n_{y1} + v_{y1}n_{y2} + n_0 v_{y3}\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{18}$$

$$n_z^{(3)} = \frac{k_0}{\omega_0}\left[v_{z2}n_{z1} + v_{z1}n_{z2} + n_0 v_{z3}\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{19}$$

where,

$$v_{x2} = \left[\frac{2i\mu_B k_0(\omega_c S_{y1} - 2\omega_0 S_{x1})}{m\hbar(\omega_c^2 - 4\omega_0^2)}\right],\quad v_{y2} = \left[\frac{i\omega_c v_{x2}}{2\omega_0} + \frac{\mu_B k_0 S_{y1}}{m\hbar\omega_0}\right],\quad v_{z2} = \left[\frac{-ie(v_{x1} - iv_{y1})}{2m\omega_0 c}\right],$$

$$n_{x2} = \left[\frac{k_0 n_0 v_{x2}}{\omega_0} + \frac{k_0 n_{x1} v_{x1}}{\omega_0}\right],\quad n_{y2} = \left[\frac{k_0 n_0 v_{y2}}{\omega_0} + \frac{k_0 n_{y1} v_{y1}}{\omega_0}\right]$$

and

$$n_{z2} = \left[\frac{k_0 n_0 v_{z2}}{\omega_0} + \frac{k_0 n_{z1} v_{z1}}{\omega_0}\right].$$

The spin angular momenta also contributes to source current thus we need to evaluate the spin magnetic moment plasma electron at third harmonic,

$$S_x^{(3)} = \frac{2i\mu_B}{3\hbar\omega_0}(S_{z3} + iS_{z2})E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{20}$$

$$S_y^{(3)} = \frac{-2i\mu_B S_{z2}}{3\hbar\omega_0} E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{21}$$

$$S_z^{(3)} = \frac{1}{\left(\frac{4\mu_B^2 b^2}{\hbar^2} - 9\omega_0^2\right)}\left[\frac{-4i\mu_B^2 b S_{z2}}{\hbar^2} + \frac{6i\mu_B\omega_0}{\hbar}(S_{y2} - iS_{x2})\right]E_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{22}$$

where,

$$S_{x2} = \frac{\left[\frac{2b\mu_B}{\hbar}\left\{\frac{2i\mu_B}{\hbar} + ik_0 s_{y1} v_{y1}\right\} - 2i\omega_0\left\{\frac{2\mu_B S_{z1}}{\hbar} - ik_0 v_{x1} S_{x1}\right\}\right]}{\left\{\frac{4\mu_B^2 b^2}{\hbar^2} - 4\omega_0^2\right\}},$$

$$S_{y2} = \left[\frac{ib\mu_B S_{x2}}{\hbar\omega_0} + \frac{\mu_B S_{z1}}{\hbar\omega_0} + \frac{k_0 S_{y1} v_{y1}}{2\omega_0}\right]$$

and

$$S_{z2} = \frac{i\mu_B(iS_{y1} - S_{x1})}{\hbar\omega_0}.$$

The third harmonic source current is

$$\vec{J}_{NL}^{(3)} = (J_{S3} + J_{c3})\mathrm{E}_0^3 e^{3i(k_0 z - \omega_0 t)} \tag{23}$$

where, J_{S3} and J_{c3} are the magnetization due to spin effect of electron and conventional current density,

$$J_{S3} = -\frac{6ik_0\mu_B}{\hbar}\left[n_0 S_{x3} + S_0 n_{x3} + n_{x1}S_{x2} + n_{x2}S_{x1} + n_0 S_{y3} + S_0 n_{y3} + n_{y1}S_{y2} + n_{y2}S_{y1} + n_0 S_{z3} + S_0 n_{z3} + S_{z1}n_{z2} + S_{z2}n_{z1}\right]$$

and

$$J_{c3} = -e.\left[n_0 v_{x3} + n_{x1}v_{x2} + n_{x2}v_{x1} + n_0 v_{y3} + n_{y1}v_{y2} + n_{y2}v_{y1} + n_0 v_{z3} + n_{z2}v_{z1} + n_{z1}v_{z2}\right]$$

3. Third harmonic generation

The non-linear component of the third source current $\vec{J}_{NL}^{(3)}$, can be used with the wave equation, to analyze the growth of harmonic radiation,

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_p^2}{c^2}\right)\vec{E}^{(3)} = \frac{4\pi}{c^2}\frac{\partial \vec{J}^{(3)}}{\partial t}. \tag{24}$$

The procedure to derive the amplitude of the phase mismatched third harmonic involves considering the harmonic field's variationis $\vec{E}^{(3)} = E_3(z)e^{i(k_3 z - 3\omega_0 t)}$ as described by the assumption that $\partial^2 E_3(z)/\partial z^2 << k_3\, \partial E_3(z)/\partial z$. This implies that $\partial E_3(z)/\partial z$ changes in are significantly larger than the wavelength $2\pi/k_3$. Consequently, we can deduce the normalized amplitude for the third harmonic under phase-mismatched conditions

$$E_3 = \frac{-4\pi a_0^2 m^2 c^2 (J_{c3} + J_{s3})E_0}{\frac{\omega_p^2}{9\omega_0^2}.e\left\{\frac{3\omega_0 m}{n_0 e^2}[J_c(3\omega_0) + J_s(3\omega_0)] - 1\right\}} e^{i(\Delta k.z)} \tag{25}$$

where,

$\Delta k = (k_3 - 3k_0)$ is the wave vector.

$$J_s(3\omega_0) = -\frac{2ik_0\mu_B}{\hbar}[n_0 S_{x1} + S_0 n_{x1} + n_0 S_{y1} + S_0 n_{y1} + n_0 S_{z1} + S_0 n_{z1}](3\omega_0)$$

$J_c(3\omega_0) = -en_0[v_{x1} + v_{y1} + v_{z1}](3\omega_0)$ and $a_0 = eE_0/mc\omega$.

From eq. (25), it is found that the harmonic oscillate in magnitude due to the de-phasing between pump laser and the radiation harmonics. The third harmonic radiation is proportional to the plasma electron density, propagation distance and the intensity of laser pulse.

4: **Conclusion:**

The study focused on generating the third harmonic, an outcome of the interaction between a circularly polarized laser pulse and a dense, uniform quantum magnetoplasma. The magnetization process applied a steady longitudinal magnetic field. Employing the newly developed quantum hydrodynamic (QHD) model, the investigation initially derived self-consistent QHD equations. This analysis considered the influence of distinct factors, including the quantum Bohm potential, Fermi statistical pressure, and electron spin. Subsequently, employing a perturbative expansion technique for QHD equations, the calculations yielded third-order velocities, electron densities, and spin angular momenta. Two primary quantum corrections emerged for electrons, stemming from density fluctuations and magnetization energy. Quantum processes and electron spin induced variations in plasma current density, thereby introducing correction terms to the harmonic field amplitude. The quantum diffraction's impact significantly intensified nonlinear third harmonic radiation production. Interestingly, the research revealed a direct correlation between escalating third harmonic output and plasma density, along with magnetic field potency, up to saturation levels. However, harmonic generation ceased beyond these saturation points. Additionally, higher magnetic field strengths led to earlier plasma density saturation due to the influence of the dense and robustly magnetized polarization field effect. Notably, within interactions involving laser plasma and gas jet in clustered plasma, the enhanced efficiency of the third harmonic could serve as a diagnostic tool for detecting cluster presence and evaluating their dimensions.

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