Complex Fermatean pentapatitioned neutrosophic sets and its Applications

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Abstract:

Complex fuzzy sets and complex intuitionistic fuzzy sets cannot handle imprecise, indeterminate, inconsistent and incomplete information of periodic nature. To overcome this difficulty, we introduce complex fermatean pentapartitioned neutrosophic set whose complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function are the combination of real valued truth amplitude term in association with phase term, real-valued contradiction amplitude term in association with phase term, real valued ignorance amplitude term in association with phase term, real valued unknown amplitude term in association with phase term, real valued false amplitude term in association with phase term respectively, Further set theoretic operations such as complement, union, intersection,complex fermatean pentapatitioned neutrosophic product, Cartesian product, distance measure and – equalities of complex fermatean pentapartitioned neutrosophic sets are studied here. A possible application of complex fermatean pentapartitioned neutrosophic set is presented in this paper. The Cartesian product o CFPNSs and subsequently the CFPNR is formally defined. The notion of inverse, complement and composition of CFPNRs along with some related theorems and properties are introduced. The performance and utility of the composition concept in real-life situations is also demonstrated. Then we define the concept of projection for CFPNRs along with illustrative examples. Some interesting properties are also obtained in this paper.

Keywords:

complex fermatean pentapartitioned neutrosophic set, complex fermatean pentapartitioned neutrosophic relation, complex fermatean pentapartitioned neutrosophic applications.

Introduction:

Fuzzy sets were first proposed by Zedeh in the seminal paper. This novel concept is used successfully in modeling uncertainity in many fields of real life. A fuzzy set is characterized by a membership function with the range [0,1]. Fuzzy sets and their applications have been extensively studied in different aspects. Atanassov introduced intuitionistic fuzzy sets in 1986 which incorporate the hesitation degree called hesitation margin. The hesitation margin is defining as 1 minus the sum of membership and non-membership. Therefore, the intuitionistic fuzzy set is characterized by a membership function and non- membership function with range [0,1].

 Ramot et al. [6] proposed an innovative concept to the extension of fuzzy sets by initiating the complex fuzzy sets where the degree of membership is traded by a cimplex values of the form where and are both belongs to [0,1] and has the range in complex unit disk.

 Florentin Smarandache introduce the concept of Neutrosphic set in 1995 which provides the knowledge of neutral thought by introducing the new factor called indeterminacy in the set. Therefore neutrosophic set was framed and it includes the components of truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) respectively. Neutrosophic sets deals with non standard interval [0,1].

Wang[4](2010) introduced the concept of single valued nuetrosophic sets (SVNS) which is also known as an extension of intuitionistic fuzzy sets and it became a very new hot research topic now. Rajashi Chatterjee.,et al proposed the concept of Fermatean Pentapartitioned single valued neutrosophic sets which is based on Belnap’s four logic and Smarandache’s four numerical valued logic. In (FPSVNS) indeterminacy is splitted into two functions known as ‘Contradiction’ (both true and false) and ‘unknown’ (neither true nor false) so that (FPSVNS) has five components TA , CA , KA , UA , FA which also lies in the non-standard unit interval [0,1].

This paper is an extension of the work of Ramot et al. [6], Alkouri and Saleh [1], Cai and Zhang et al [8] to neutrosophic sets. Basically, we follow the philosophy of the work of Ramot et al. [6] to introduce complex neutrosophic set. The complex neutrosophic is characterized bu complex valued truth membership function, complex valued indeterminate memntership function, and complex valued falsehood membership function. Further,complex neutrosophic set is the mainstream over all because it not only is the generalization of all the current frameworks but also describes the information in a complete and comprehensive way.

Like a set, a relation is of fundamental importance in all engineering, science and mathematically based fields. Ralation are intimately involved in logic, approximate reasoning, rule-based systems, nonlinear simulation, synthetic evaluation, classification, pastern recognition, and control. The relations between fuzzy sets and intuitionistic fuzzy sets is a topic that has been extensively studied in the neutrosophic environment, Yang et al proposed single valued neutrosophic relationsSVRS) and study their properties.

The remainder of this paper is organized as follows. In section 2, we recapitulate some of the fundamental concepts related to complex neutrosophic sets and CNSs. Throughout section 3, we define the concept of the Complex Fermatean Pentapartitioned neutrosophic sets CFPNSs as forerunner of the concept of CNR. Subsequently, the concept of CNR will be defined. A decision- making algorithm is also generated in this section. This algorithm examines the affectivity of number of teaching techniques using the characteristics of the CFPNR. In section4, we define some basic operations on CFPNR such as the complement, inverse and composition of CFPNRs. This section also gives the definitions of projection for CFPNRs. Section 5 provides a comprehensive comparison among CFPNR and other recent approaches to the dominance of our proposed method. Section 6 outlines the conclusion of this paper and suggests directions for future research.

**Preliminaries :**

The definition of complex neutrosophic set is as follows.

**Definition 2.1:**

 A complex neutrosophic set A defined on a universe of course X, Which is characterized by a truth membership function TA(x ), an indeterminacy membership function

 IA(x ) and a falsity membership function FA(x) that assigns a complex-valued grade of TA(x ),

IA(x ) and FA(x) in S for any x X. The values TA(x ), IA(x ) and FA(x) and their sum may all within the unit circle in the complex plane and so is of the following form,

TA(x) =

IA(x) =

FA(x) = where ,, and ,, are respectively, real valued and ,, [0,1] such that 3.

 The complex neutrosophic set S can be represented in set form as

,

 where , and

 , and

**Complex Neutrosophic Relations**

In this section, we propose the definition of Cartesian product between two CNSs and subsequently, the formal definition of the CNR.

**Definition 2.2**

 Let X and Y be two complex neutrosophic sets over the U and V respectively. The Cartesian product of X and Y denoted by X Y is a CNS defined as

 , where is a complex valued truth membership function is a complex valued indeterminacy membership function and is a complex- valued falsity membership function and ,

,

,

.

**Definition 2.3: [7]**

Consider X a universe. An object of the form A Fermatean pentapartitioned neutrosophic set (FPN) A on X A = {< x, TA , CA , KA , UA , FA ,) >: x X }

 (TA)3 + (CA)3 + (KA)3+ (UA)3 + (FA)3 ≤ 3

Here, TA(x ) is the truth membership.

CA(x) is contradiction membership,

KA(x) is ignorance membership

UA (x) is unknown membership,

FA(x) is the false membership.

**Complex Fermatean Pentapartitioned neutrosophic sets:**

The definition of complex fermatean pentapartitioned neutrosophic set is as follows.

**Definition 3.1:**

A complex fermatean pentapartitioned neutrosophic set A defined on a universe of course X, Which is characterized by a truth membership function TA(x ), CA(x) is contradiction membership function, KA(x) is ignorance membership function, UA (x) is unknown membership function and FA(x) that assigns a complex-valued grade of TA(x ), CA(x), KA(x) , UA (x) ,and FA(x) in S for any x X. The values TA(x ), CA(x), KA(x) , UA (x) and FA(x)is the false membership function and their sum may all within the unit circle in the complex plane and so is of the following form,

TA(x) =

CA(x) =

KA(x) =

UA (x) =

FA(x)=where,,,,and ,,,,

are respectively, real valued and ,,,, [0,1] such that

 ,,,, 3.

 The complex fermatean pentapartitioned neutrosophic set S can be represented in set form as

 where , ,

 ,, and

 , and

**Definition 3.2.**

 If we consider X as a universal set, a complex fermatean pentapartitioned neutrosophic set A over the universal set X is defined as:

A = {< x, TA(x ) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } where TA(x ) is the degree of truth membership,CA(x) is called the degree of contradiction membership, KA(x) is called the degree of ignorance membership , UA (x) is called the degree of unknown membership and FA(x) is the degree of false membership that has a mapping TA : X → { z1 : z1 ∈ C : | z1| ≤ 1}

CA : X → { z2 : z2 ∈ C : | z2| ≤ 1}, KA : X → { z3 : z3 ∈ C : | z3| ≤ 1}, UA : X → { z4 : z4 ∈ C : | z4| ≤ 1}

And FA : X → { z5 : z5 ∈ C : | z5| ≤ 1}. For every x ∈ X, the degree of truth membership is

 TA(x ) = , the degree of contradiction membership is CA(x ) = , the degree of ignorance membership KA(x ) = , the degree of unknown membership

 UA(x ) = and the degree of false membership is FA(x ) = respectively. Where TA , CA , KA , UA , FA ∈ [0,1] , ,

 ,

++ + +.

**Interpretation of complex fermatean pentapartitioned neutrosophic sets**

* The concept of complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function and complex valued is not a simple task in understanding. Real valued truth membership function, real-valued contradiction membership function, real-valued ignorance membership function, real valued unknown membership function, real-valued false membership function in the interval [0,1] can be easily perceptive.

 The notion of complex fermatean pentapartitioned neutrosophic set can be easily understood from the form of its truth membership function, contradiction membership function, ignorance membership function, unknown membership function, and false membership function which appears in above Definition 3.1.

TA(x) =

CA(x) =

KA(x) =

UA (x) =

FA(x) =

 It is clear that complex grade of truth membership function is defined by a truth amplitude

term and a truth phase term . Similarly the complex grade of contradiction membership function is defined as an contradiction amplitude term and a contradiction phase term , the complex grade of ignorance membership function is defined as an ignorance amplitude term and a contradiction phase term , the complex grade of unknown membership function is defined as an unknown amplitude term and a contradiction phase term , the complex grade of false membership function is defined as an false amplitude term and a contradiction phase term respectively. It should be noted that the truth amplitude term is equal to a , the amplitude term . The contradiction amplitude term is eual to, the ignorance amplitude term is equal to , the unknown amplitude term is equal and a the false amplitude term is equal to .

 Complex fermatean pentapartitioned neutrosophic sets are the generalization of neutrosophic sets. It is an easy task to represent a neutrosophic set in the form of complex fermatean pentapartitioned neutrosophic set. For instance, the neutrosophic set S is characterized by a real-valued truth membership function, contradiction membership function, ignorance membership function, unknown membership function and false membership function. By setting the truth amplitude term is equal to , and the truth phase term equal to zero for all x and similarly we can set the contradiction amplitude term equal to and a contradiction phase term equal to zero, the ignorance membership amplitude term equal to and a ignorance phase term equal to zero , the unknown amplitude term equal to and a unknown phase term equal to zero, while the false amplitude term equal to and a false phase term equal to zero for all x. Thus it has seen that a complex fermatean pentapartitioned neutrosophic set can be easily transformed into a fermatean pentapartitioned neutrosophic set. From this discussion, it is concluded that the truth amplitude term is equivalent to the real-valued grade of truth membership function, the contradiction amplitude term is equivalent to the real valued grade of contradiction membership function, the ignorance amplitude term is equivalent to the real-valued grade of ignorance membership function, the unknown amplitude term is equivalent to the real valued grade of unknown membership function, and the false amplitude term is equivalent to the real valued grade of false membership function. The only distinguishing factors are truth phase term,contradiction phase term, ignorance phase term, unknown phase term and false phase term. This differs the complex fermatean neutrosophic set from the ordinary neutrosophic set. In simple words, if we omit all the five phase terms, the complex fermatean pentapartitioned neutrosophic set will automatically convert into nfermatean pentapartitioned neutrosophic set effectively. All this discussion is supported by the reality that, , and have range [0,1] which is real-valued grade of truth membership, real-valued grade of contradiction membership, real-valued grade of ignorance membership, real-valued grade of unknown membership and real-valued grade of false membership.

 This means that complex fermatean pentapartitioned neutrosophic sets are the advance generalization to all the existence methods and due to this feature, the concept of complex fermatean neutrosophic sets is a distinguished and novel one.

3.3.1Numerical example of a complex fermatean pentapartitioned neutrosophic set

Example 3.2

Let be a universe of discourse. Then S be a complex fermatean pentapartitioned neutrosophic set in X as given below:

 +

Set theoretic operations on complex fermatean pentapartitined neutrosophic set

Ramot et al. [23], calculated the complement of membership phase term by several possible method such as. Zhang [39] defined the complement of the membership phase term by taking the rotation of by radian as.

 To define the complement of a complex fermatean pentapartitioned neutrosophic set, we simply take the neutrosophic complement [29] for the truth amplitude term ,contradiction amplitude term , the ignorance amplitude term , the unknown amplitude term , the false amplitude term . For phase terms, we take the complement defined in [23]. We now proceed to define the complement of complex fermatean pentapartitioned neutrosophic set.

Definition 3.5. Complement of complex fermatean pentapartitioned neutrosophic set.

Let A = {< x, TA(x),CA(x), KA(x), UA (x), FA(x)) >: x X} be a complex fermatean pentapartitioned neutrosophic set in X. Then the complement of a complex fermatean pentapartitioned neutrosophic set A is defined as c (A) and is defined by

 ,

Where = c ( , ,,

, in which c(= c( is such that c( and or .

Similarly, and or .

 and or .

 and or .

 Finally, where and

 or.

Proposition 3.6

Let A be a complex fermatean pentapartitioned neutrososphic set on X.Then, c(c(A)) = A.

Proof By definition 3.1. we can easily prove it.

Proposition 3.7.

Let A and B bee two complex fermatean pentapartitioned neutrososphic sets on X.

Then .

Definition 3.8. Union of complex fermatean pentapartitioned neutrososphic sets.

Ramot et al. [23] defined the union of two complex fermatean pentapartitioned neutrososphic sets A and B as follows. Let and be the complex-valued membership functions of A and B respectively. Then the membership union of is given by . Since and are real valued and belonging to [0,1], the operators max and min can be applied to them. For calculating phase term , they give several methods. Now we define the union of two complex fermatean pentapartitioned neutrososphic sets as follows:

Let A and B be two complex fermatean pentapartitioned neutrososphic sets X, where

A = {< x, TA(x ) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x) , CB(x) , KB(x) , UB (x) , FB(x)) >: x X }

Then the union of A and B is denoted as and is given as

Where the truth membership function, the contradiction membership function, the ignorance membership function, the unknown membership function and false membership function are defined by

Where and denote the max and min operators respectively. To calculate phase terms and we define the following:

Definitions 3.9

 Let A and B be two complex fermatean pentapartitioned neutrososphic sets in X with complex-valued truth membership functions TA(x ) and TB(x ), complex-valued contradiction membership function CA(x ) and CB(x ), complex-valued ignorance membership functions KA(x ) and KB(x ), complex-valued unknown membership functions UA(x ) and UB(x ) and complex-valued false membership functions FA(x ) and FB(x ) respectively. The union of the complex fermatean pentapartitioned neutrososphic sets A and B is denoted by which is associated with the function:

 X

.

 A complex value is assigned by, that is, for all x X,

 , ,,

and .

This function must be obey at least the following axiomatic conditions. For any :

Axiom 1: , , ,

 and ( boundary conditions).

Axiom 2 : , ,

 and (commutative condition).

Axiom 3 : If , then ,

, and

(Monotonic condition).

Axiom 4 : , ,

, ,

 and ) (associative condition).

 It may be possible in some cases that the following are also held:

Axiom 5: (continuity).

Axiom 6 : , ,

 , and (super idempotency).

Axiom 7: and , then ,

 and, then ,

 and, then

 and, then

and and , then

(Strict monotonicity).

The phase term of complex truth membership function, complex contradiction membership function ignorance membership function, unknown membership function, and false membership function belongs to (0,). To define the phase terms, we suppose that

,,

 ,,.

Now we take those forms which Ramot et al. presented in [23] to define the phase terms of

 respectively.

Sum :

.

Maximum :

.

Minimum :

.

“The game of winner, contradictor, ignorance person, unknown person, and loser”:

The game of winner, contradictor, ignorance person, unknown person, and loser is a novel concept,

and it is the generalization of the concept “winner take all” introduced by Ramot et. al.[23] for the union of phase terms.

Example 3.9.1

Let be a universe of discourse. Let A and B be two complex fermatean pentapartitioned neutrosophic set as given below:

 +,

and

 +

Then

Definition 3.10: Intersection of complex fermatean pentapartitioned neutrosophic set.

Let A and B be two complex fermatean pentapartitioned neutrosophic sets in X, where

 A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x),CB(x) , KB(x) , UB (x) , FB(x)) >: x X }

Then the intersection of A and B is denoted as and is defined as

Where the truth membership function, the contradiction membership function, the ignorance membership function, the unknown membership function and false membership function are defined by

Where and denote the max and min operators respectively. To calculate phase terms and we define the following:

Definitions 3.11

Let A and B be two complex fermatean pentapartitioned neutrososphic sets in X with complex-valued truth membership functions TA(x ) and TB(x ), complex-valued contradiction membership function CA(x ) and CB(x ), complex-valued ignorance membership functions KA(x ) and KB(x ), complex-valued unknown membership functions UA(x ) and UB(x ) and complex-valued false membership functions FA(x ) and FB(x ) respectively. The intersection of the complex fermatean pentapartitioned neutrososphic sets A and B is denoted by which is associated with the function:

 X

.

 A complex value is assigned by, that is, for all x X,

 , ,,

and .

This function must satisfy at least the following axiomatic conditions. For any :

Axiom 1: , , ,

 and ( boundary conditions).

Axiom 2 : , ,

 and (commutative condition).

Axiom 3 : If , then ,

, and

(Monotonic condition).

Axiom 4 : , ,

, ,

 and ) (associative condition).

 It may be possible in some cases that the following are also held:

Axiom 5: (continuity).

Axiom 6 : , ,

 , and (super idempotency).

Axiom 7 : and , then ,

 and, then ,

 and, then

 and, then

and and , then

(Strict monotonicity).

We can easily calculate the phase terms,,, and on the same lines by winner, contradiction person, ignorance person, unknown person and loser game.

Preposistion 3.12

Let A, B, C be three complex fermatean pentapartitioned neutrososphic sets on X. Then,

Proof: Here we only prove that the part 1. Let A,B,C be three complex fermatean pentapartitioned neutrososphic sets in X and A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } ,

B = {< x, TB(x),CB(x), KB(x), UB (x), FB(x)) >: x X} and

C = {< x, TC(x) , CC(x) , KC(x) , UC (x) , FC(x)) >: x X } respectively be their complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function then we have

=

 = min (max (

 = max (min (

 =max ((

 = (=

Similarly, on the same lines, we can show it for

and respectively.

Preposition 3.13

Let A and B be three complex fermatean pentapartitioned neutrososphic sets on X. Then,

Proof: We prove it for part 1. Let A and B be two complex fermatean pentapartitioned neutrosophic sets in X and A = {< x, TA(x) , CA(x) , KA(x) , UA (x) , FA(x)) >: x X } and

B = {< x, TB(x),CB(x), KB(x), UB (x), FB(x)) >: x X} respectively be their complex valued truth membership function, complex-valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function. Then,

=

 = min (max (

 = .

Similarly,we can show it for,

and respectively.

Definition 3.14.

Let A and B be three complex fermatean pentapartitioned neutrososphic sets on X, and

,,,

 , and

,,

 respectively, be their complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function. The complex fermatean pentapartitioned neutrososphic product A and B denoted as

A o B and is specified by the functions, ,

,

,

,

,

Example 3.14.1. Let be a universe of discourse. Let A and B be two complex fermatean pentapartitioned neutrosophic set as given below:

 +

And

 +

Then

 ,

Definition 3.15.

Let be N complex fermatean pentapartitioned neutrosophic sets on X (n=1, 2, 3….N), and

,,,

 , be their complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function. The Cartesian product of denoted as, specified by the function

 = min ((

 = min (()

 = max ((

 = max ((

And

= max ((

Where. (N times X)

4. Distance measure and -equalities of complex fermatean pentapartitioned neutrosophic sets

In this section, we introduced distance measure and other operational properties of complex feramtean pentapartitioned neutrosophic sets.

Definition 4.1

Let CN(X) be the collection of all complex feramtean pentapartitioned neutrosophic sets on X and A, B. Then B if and only if such that the amplitude terms and a truth phase terms and such that the amplitude terms and a contradiction phase terms , amplitude terms and the phase terms , amplitude terms and a contradiction phase terms, amplitude terms and a contradiction phase terms respectively.

Definition 4.2

Two complex fermatean pentapartitioned neutrosophic sets A and B are said to equal if and only if

 ,,, and for amplitude terms and ,,, and for phase terms (arguments).

Definitions: 4.3

A distance of complex fermatean pentapartitioned neutrosophic sets is a function

, such that for any A, B, C

,

 ,

.

Let be a function which is defined as

Theorem 4.4

The function defined above is a function of complex fermatean pentapartitined neutrosophic sets on X.

Proof: The proof is straight forward.

Definition 4.5

Let A and B be two complex fermatean pentapartitioned neutrosophic sets on X, and

,,,

 , and

,,

 are their complex-valued truth membership function, complex-valued contradiction membership function, complex-valued ignorance membership function, complex-valued unknown membership function, complex-valued false membership function.

Then A and B are said to be -equal, if and only if, where it is denoted by

.

Preposition 4.6

For complex fermatean pentapartitioned neutrosophic sets A, B and C. the following holds.

A =(0)B

A = (1) B if and only if A = B.

If if and only if

 andthen

If , then for all ,where J is an index set,

If andthere exist a unique such that , then for all A,B

If and, then where.

Proof 4.7 Properties 1-4, 6 can be proved easily. We only prove 5 and 7.

Since , for all , we have

Therefore,

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, and

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Thus,

Hence, .

7. Since, we have

Which implies

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Also we have ,so

Which implies

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Now

Now

 = = +-1),

From definition 4.3,

 . Therefore, -\* = 1- , where =\*. Thus, .

Theorem 4.7

If , then, where c(A) and c(B) are the complement of the complexfermatean pentapartitioned neutrosophic sets A and B.

Proof: Since

 = .

5. Relations between the CFPNS with their applications in Decision Making

Definition 5.1 Let AN be N CNSs on X, (n = 1,2,3,….,N), and be a complex-valued truth membership function, CA(x) = be a complex-valued contradiction membership function, KA(x) = be a complex-valued ignorance membership function, UA (x) = be a complex-valued unknown membership function and FA(x) = be a complex-valued false membership function. The Cartesian products of denoted as , are specified by the functions:

,

,

,

,

5.2. Complex fermatean pentapartitioned neutrosophic relation

 In this section, we propose the definition of Cartesian product between two CFPNSs and subsequently, the formal definition of the CFPNR. We also apply the CFPNR method to solve a MCDM problem.

The following is the definition of the Cartesian product between two CFPNSs:

Definition 5.3. Let X and Y be two CFPNSs over the universal sets U and V respectively. The Cartesian product of X and Y, denoted by X Y is a CFPNS defined as

,

where is a complex valued truth membership function is a complex valued contradiction membership function, is a complex valued ignorance membership function, is a complex valued unknown membership function, and is a complex- valued falsity membership function and ,

,

,

,

,

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We will now define the concept of CFPNR as follows:

Definition 5.4. Let X and Y be two CFPNSs over the universe U and V, respectively. A complex fermatean pentapartitioned neutrosophic relation from X to Y is a complex fermatean pentapartitioned neutrosophic subset of. Thus, a CFPNS from X to Y, denoted by R(X, Y), where . As always, may be represented as the set of ordered sequence

, where,, TR(u,v) =

CR(u,v) =

KR(u,v) =

UR (u,v) =

FR(u,v) = .

The values TR(u,v) ,CR(u,v) ,KR(u,v) ,UR (u,v) ,FR(u,v) are within the unit circle in the complex plane and both the amplitude terms and the phase terms are real valued such that

TR(u,v) ,CR(u,v) ,KR(u,v) ,UR (u,v) ,FR(u,v) and

.

Now, we put forward a real-life application of CFPNR to reveals its ability to describe and analyses real events.

6 Complex fermatean pentapartitioned neutrosophic relation Education:

CFPNR can be effectively used to measure the interaction between several education variables where the factor of time plays a key role and the indeterminacy is unavoidable.

 Now, we give an example of a relation between two CFPNSs.

Example 6.1. Suppose a study us conducted to determine the most influential teaching strategy contributing to student achievement. Let U be the set of teaching techniques in education applied on a certain group of students, where

 U = {.

Let V be a set of indicators that measure the student’s achievement and his interaction, where

V= {.

Let X and Y be two CFPNSs over U and V respectively defined as follows:

Now, we compute the relation between the two CFPNSs X and Y to investigate the effect of modern methods in education on the student’s performance. Our CFPNS denoted by R(X, Y) such that , is

Suppose that the relation between X and Y are measured within a time frame of 12 months. In our example, the terms of truth amplitude, contradiction amplitude, ignorance amplitude, unknown amplitude and false amplitude, of R(X, Y) measure the truth membership degree of the impact of the modern methods in education on the student’s performance, the contradiction membership degree of the impact of the modern methods in education on the student’s performance, the ignorance membership degree of the impact of the modern methods in education on the student’s performance, the unknown membership degree of the impact of the modern methods in education on the student’s performance and false membership degree of the impact of the modern methods in education on the student’s performance respectively, whereas the truth phase term, the contradiction phase term, the ignorance phase term, the unknown phase term and false phase term of R(X, Y) represent the period of time I which the modern methods influence the student’ performance, the period of time in which we are unable to determine if the modern methods influence the student’s performance or does not influence the student’s performance and the period of time in which the modern methods does not influence the student’s performance, respectively. Since the phase terms in R(X, Y) represent periods of time and R(X, Y) represents the relation between the modern methods in education and the student’s performance within the time frame of 12 months, then, each complex fermatean pentapartitioned neutrosophic value, the range value of each of the truth, contradiction, ignorance, unknown ad false phase terms should be between 0 and 1.

6.2. Operations on complex fermatean pentapartitoned neutrosophic relation

We will now introduce some basic operation on CFPNS such as complement, inverse and composition of CFPNRs. We will begin by first proposing the definition of the complement of CFPNR.

Definition 6.3

Let R be a CFPNR on, where

.Then, the complex fermatean pentapartitioned neutrosophic complement relation R denoted by, defined as

, where

 = = .

 = =

 = =

 = =

 = .=

Next, we will give the definition of the inverse of a CFPNR and a give a preposition on the inverse of CFPNR.

Definition 6.4. Let R be a CFPNR from X to Y. The inverse of R is denoted by and is a CFPNR from Y to X defined as

 .

Where and.

.

Preposition 6.5

 Let X and Y be two CFPNSs over U and V respectively. Suppose that R and S are two CFPNS from X to Y. Then, the following results hold true:

If then.

Proving and, we have

,

Where,

,,

,. This implies that.

If , then

,

Similarly we can show that

,

,

.

In this part, we propose the axiomatic definition of the compositiuon of CFPNRs followed by an example illustrates application of this concept in real life. We then give two theorems on the composition concept.

Definition 6.6. Let X and Y and Z be three CFPNSs over the universes U, V and W respectively. Let be a CFPNR from X to Y and S a CFPNR form Y to Z. The composition of the CFPNRs R and S is a CFPNR from X to Z, defined as:

 and, = ,

Where and ,

= ,

Where and,

= ,

Where and ,

= ,

Where and ,

= ,

Where and ,

This relationship can be written as.

The following example demonstrate the utilization of the composition of the CFPNRs in real life.

Example 6.7. Suppose X, Y and Z are three CFPNSs that represent the sets of Chinese financial indicators, Malaysian financial indicators and Malaysian public opinion indicators, respectively. Suppose the interactions among these sets are measured over the limited time frame of 12 months using the CFPNRs R and S, where R(X,Y) represents the effect of the Chinese financial indicators to Malaysian financial indicators, and S(Y,Z) represents the effect of the Japanese financial indicators to Malaysian public opinion indicators.

 The composition of CFPNRs R(X, Y) and S(Y, Z) obtains a new CFPNR T(X, Z), which represents the effect of the Chinese financial indicators to the Malaysian public opinion indicators.

 For the sake of illustration, it will suffice in this example to consider the composition of the following two approximations in the CFPNRs R(X, Y) and S(Y, Z).

(,

Where and represent respectively, the exchange rate of the Chinese Yuvan and the inflation rate in Malaysia. This approximation measures the truth, contradiction, ignorance, unknown and false for both degree and phase (period) of the influence of Chinese Yuvan’s exchange rate on the inflation rate in Malaysia.

(,

Where and represent, respectively, the inflation rate in Malaysia and confidence in the Malaysian economy. This approximation measures the truth, contradiction, ignorance, unknown and false for both degree and phase (period) of the influence of exchange rate of Chinese Yuvan on the confidence in the Malaysia economy.

The result of this composition is:

The components measure respectively the truth, the contradiction, ignorance, unknown and falsity for both degree and phase (period) of the influence of the inflation rate in Malaysia on the Confidence in the Malaysian economy.

 In view of Definition 10, we prove the following results.

Theorem 6.8. Let X,Y and Z be three complex fermatean pentapartitioned neutrosophic sets over the universes U,V and W respectively. Let R be a CFPNR from X to Y and S a CFPNR from Y to Z. Then

.

Proof: For all and, Let

,

and

To prove the equality, we have to show that , ,

, and .

Therefore ,

 = max [ = max [

 = max [min (, min (

 = max [min (, min (

 = max [

 =,

Which implies =, similarly we can show that =, proving that

=. Similarly, we can show it also holds for the contradiction, ignorance, unknown and falsity terms and this completes the proof.

Theorem 6.9 Let X, Y, Z and W be CFPNSs over the universes U, V, L and M respectively. Let R be a CFPNR from X to Y, S a CFPNR from Y to Z and T a CFPNR from Z to W. Then.

Proof. For all and , . Let

.

To prove the equality, we have to show that,

, and.

Therefore , = max [ = max [

 = max [min (, max [min (,

 = max [min (, min (

 = max [max [min (

 = max [max [,

 = max [,

 = ,

Which implies that. Similarly, we can show that , proving that .

The proofs for the contradiction, ignorance, unknown and falsity terms can be similarly proven and this completes the proof.

 In order to give a deeper insight into this issue, we introduce the definitions of projection and cylindrical extension for CFPNSs.

Definition 6.10. Let U and V be two universe and R be a CFPNR on. Then for all and

The projection of R on U is a CFPNS. Defined respectively, by the complex valued truth, contradiction, ignorance, unknown and falsity membership functions:

(u) =,

(u) =

(u) =

(u) =

(u) =

The projection of R on V is a CFPNS , defined respectively by the complex valued truth, contradiction, ignorance, unknown and falsity functions:

(v) =,

(v) =

(v) =

(v) =

(v) =

Example 6.11. Let and be two universes. Let R be a CFPNR on defined as follows:

The projection of the CFPNR R on U is given by:

(u)=

(u)=

The projection of the CFPNR R on V is given by:

(v)=

(v)=

7.Conclusion:
A Complex fermatean pentapartitioned neutrosophic set is defined by complex valued truth membership function, complex valued contradiction membership function, complex valued ignorance membership function, complex valued unknown membership function, complex valued false membership function. Therefore, a complex valued truth membership function is a combination of traditional truth membership function with the addition of an extra term. The traditional truth membership function is called truth amplitude term, and the additional term is called phase term. Thus, in this way the truth amplitude term represents uncertainity and the phase term represents periodity in the uncertainity. Thus, a complex-valued truth membership function represents uncertainity with periodicity as a whole. Similarly, a complex-valued contradiction membership function represents contradiction with periodicity and complex-valued ignorance membership function represents ignorance with periodicity, complex-valued unknown membership function represents unknown with periodicity and complex-valued falsehood memberhip function represents falsity with periodicity. Further, we presented an interpretation of complex fermatean pentapartitioned neutrosophic set and also discussed some of the basic set theoretic properties such as complement, union, intersection, complex fermatean pentapartitioned neutrosophic product, cartesean product in this paper. We also presented – equalities of complex fermatean pentapartitioned neutrosophic sets are studied here. This paper derives and investigated the relation among the CFPNSs and used these relations to describe and handle a real decision making problem. The Cartesian product between two CFPNSs is defined as a prerequisite to define the CFPNR. We then presented some fundamental operators on the CFPNR such as complement and inverse of CFPNR. The axiomatic definition of the composition of CFPNRs is also defined along with an example of illustrate the application of this concept to derive useful information in reality by combining two CFPNRs. We provided some theorems on the previous operation and derived some properties with illustrative example. The concept of projection for CFPNRs is also defined and illustrated by examples.

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