**Magnetohydrodynamic flow and heat transfer of Casson fluid over a stretching sheet in non-Darcy porous medium**

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**Abstract**

The current article investigates the effect of Lorentz force on Casson fluid flow and heat transfer over a stretching sheet embedded in non-Darcy porous medium with slip boundary condition. The governing partial differential equations (PDEs) are converted to ordinary differential equations (ODEs) by using suitable similarity transformations and then solved numerical solution by Runge-Kutta method with shooting technique. The impacts of various parameters such as magnetic parameter, porosity parameter, inertia parameter and slip parameter on velocity, temperature as well as skin friction coefficient and local Nusselt number are demonstrated through graphs and tables. The important findings are: slip parameter increases the skin friction coefficient but decreases the rate of heat transfer at the surface of the sheet and Casson parameter favours to increase the fluid temperature while declines the velocity profile .

**Keywords** MHD,Casson fluid, slip boundary condition, Darcy-Forchheimer model.

**1. Introduction**

 The boundary layer flow over a stretching/shrinking sheet has many industrial and technological applications such as paper production, extraction of polymer sheets, wire coating, polymer processing, cooling of electronic devices, and blood flow problems etc. Sakiadis [1] was first studied the boundary layer flow past a stretching sheet. Fang et al. [2] found an exact solution of MHD flow of viscous fluid past a stretching sheet. Mahantesh et al. [3] examined the slip effect over a stretching sheet with non-linear boundary condition. Hayat et al. [4] studied the MHD flow and heat transfer over permeable stretching sheet with slip conditions. Ibrahim and Shankar [5] explored the effects of slip boundary conditions on MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet. Makinde et al. [6] considered the MHD flow of a variable viscosity nanofluid over a radially stretching convective surface. Swain et al. [7-10] have contributed significantly to enrich the literature on stretching sheet by considering variable fluid properties. Abou-Zeid et al [11] studied the natural convective effects on gliding motion of bacteria on a power-lawnanoslime through a non-Darcy porous medium.

 Casson fluid is a non-Newtonian fluid with yield stress which represents blood flow in narrow arteries. When blood flows from larger diameter arteries at high shear rates it shows Newtonian character. But it exhibits non-Newtonian behaviour when it flows through small diameter arteries at low shear rates. Casson [12] studied the validity of Casson fluid model in his studies relating to the flow of blood and observed that at low shear rates the yield stress for blood is nonzero. Ibrahim et al. [13] numerical studied effects of chemical reaction and heat source on MHD stagnation point flow of Casson nanofluid over a nonlinear stretching sheet with slip boundary conditions. Bhattacharyya et al. [14] explored the analytic solution on MHD boundary layer flow of Casson fluid over a stretching/shrinking sheet with wall mass transfer. Mukhopadhyay [15] studied the Casson fluid flow and heat transfer over a nonlinearly stretching sheet. Recently, Senapati et al. [16] have numerically investigated the three dimensional MHD Casson nanofluid flow over a stretching sheet.

 The main objective of the present study is to analyse the effect of slip boundary condition on MHD Casson fluid over a stretching sheet in non-Darcy porous medium. The governing PDEs are converted to non-linear ODEs by using similarity transformations. The converted ODEs are solved by using effective shooting technique. The numerical results obtained are found in good agreement with the literature available. The effects of pertinent parameters are shown through graphs and tables.

**2. Mathematical Formulation**

Consider a steady two dimensional electrically conducting Casson fluid flow past a stretching sheet along positive *x*-direction. A transverse magnetic field of strength is applied along *y*-axis. The fluid flow is restricted towhich is caused by the linear stretching of the sheet with velocity  from a slit keeping the origin fixed as shown in Fig. 1. It is assumed that the magnetic Reynolds number of the fluid is very small so that the effect of induced magnetic field is neglected.



**Fig. 1** Flow geometry and coordinate system

The rheological equation of state for an isotropic and incompressible flow of Casson fluid is given by (Senapati et al. [16])



where is plastic dynamic viscosity of the non-Newtonian fluid, is the yield stress of the fluid, is the product of deformation rate with itself, is the component of the deformation rate and is a critical value of this product, based on the non-Newtonian model.

Under the above assumptions, the MHD boundary layer equations for steady flow of Casson fluid are given by

 (1)

 (2)

 (3)

The boundary conditions are

 (4)

where are velocity components in *x* and *y* directions respectively, is the magnetic field strength, is the kinematic viscosity, is the electrical conductivity, is the density, is the thermal conductivity, is the temperature of the fluid,  is the wall temperature, is the ambient temperature, is the specific heat, is the drag coefficient, is the permeability of the medium, and  is velocity slip factor .

Consider the stream function  such that and dimensionless variable is 

Therefore, the equation (1) is identically satisfied and the equations (2) - (4) become

 (5)

 (6)

 (7)

where is the magnetic parameter, is the porosity parameter, is the local inertia parameter, is the slip parameter, and is the Prandtl number.

The surface condition of practical interest is the local skin friction coefficient and local Nusselt number which are given by and respectively.

Here wall shear stress and wall heat flux  is the local Reynolds number.

**3. Results and Discussion**

 The dimensionless coupled nonlinear ODEs (4) and (5) are solved numerically by Runge-Kutta fourth order method with shooting technique using MATLAB software with step length and the error tolerance. In this method, the equations are reduced to a system of first order differential equations:











with the initial conditions

 

Now, the initial value problem is solved by suitably predicting the missing initial values by shooting technique. During calculation we fix the parameters as unless otherwise the values are mentioned. A comparison is made with previously published works of Hayat et al. [4] and Ibrahim and Shankar [5] as shown in Table 1. It is found that our numerical results are in good agreement.

**Table 1** Comparison of  for various values of when 

|  |  |  |  |
| --- | --- | --- | --- |
|  | Hayat et al. [4] | Ibrahim and Shankar [5] | Present results |
| 0.0 | 1.000000 | 1.0000 | 1.00000 |
| 0.1 | 0.872082 | 0.8721 | 0.87344 |
| 0.2 | 0.776377 | 0.7764 | 0.77770 |
| 0.5 | 0.591195 | 0.5912 | 0.71827 |
| 2 | 0.283981 | 0.2840 | 0.28493 |
| 5 | 0.144841 | 0.1448 | 0.14553 |
| 10 | 0.081249 | 0.0812 | 0.08596 |
| 20 | 0.043782 | 0.0438 | 0.04428 |
| 50 | 0.018634 | 0.0186 | 0.01875 |

Figs. 2 and 3 display the effects of magnetic parameter () and porosity parameteron velocity and temperature distribution respectively. It is observed that an increase  and  give rise to low flow rates due to additional resistive forces and consequently, boundary layer thickness decreases. But the opposite effects are observed in case of temperature distribution. The reason for this is that an increase in magnetic parameter causes an increase in electromagnetic force that restrains the fluid motion which in turn brings about the temperature rise leading to thicker thermal boundary layer.

Figs. 4 and 5 represent influences of inertia parameter  and Casson parameteron velocity and temperature distribution respectively. It is seen that the velocity profile declines with an increase in both the parameters whereas reverse trend is observed in case of temperature profile. Since, higher values of lead to decrease the yield stress.

Figs. 6 and 7 depict the impact of slip parameter on velocity and temperature distribution respectively. The velocity profile deceases with an increase in slip parameter but the fluid temperature increases. This result is well established with the work of Ibrahim and Shankar [5]. Fig. 8 shows the effect of Prandtl number on temperature profile. The higher values of, having lower thermal diffusivity contributes a reduction in fluid temperature and consequently, thermal boundary layer shrinks.



Fig. 2 Influences of and on velocity profile



Fig. 3 Influences of and on temperature profile



Fig. 4 Influences of and on velocity profile



Fig. 5 Influences of and on temperature profile



Fig. 6 Influence of  on velocity profile



Fig. 7 Influence of  on temperature profile



Fig. 8 Influence of  on temperature profile

**Table 2** Values of skin friction coefficientand local Nusselt number 

when 

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| 0.1 | 0.1 | 0.1 | 0.1 | 1 | -2.817804 | 0.598782 |
| 0.5 | 0.1 | 0.1 | 0.1 | 1 | -3.056013 | 0.578155 |
| 1 | 0.1 | 0.1 | 0.1 | 1 | -3.309591 | 0.555186 |
| 1 | 0.5 | 0.1 | 0.1 | 1 | -3.389258 | 0.548616 |
| 1 | 1 | 0.1 | 0.1 | 1 | -3.480784 | 0.5409936 |
| 1 | 1 | 0.3 | 0.1 | 1 | -1.984094 | 0.412997 |
| 1 | 1 | 0.5 | 0.1 | 1 | -1.403070 | 0.346950 |
| 1 | 1 | 0.5 | 0.5 | 1 | -1.127293 | 0.355697 |
| 1 | 1 | 0.5 | 2 | 1 | -0.969014 | 0.338624 |
| 1 | 1 | 0.5 | 2 | 2 | -0.969014 | 0.551084 |
| 1 | 1 | 0.5 | 2 | 3 | -0.969014 | 0.728621 |

Table 2 is computed to know the effects of different physical parameters on the skin friction coefficientand local Nusselt number. It is observed that skin friction coefficient increases as and increase whereas it decreases with an increase in and. Further, the rate of heat transfer declines with an increase in and, while it boosts as and increase.

**4. Conclusions**

From the present analysis the following conclusions are drawn:

* The thickness of momentum boundary layer decreases with an increase in magnetic field parameters  and.
* The slip parameter increases the skin friction coefficient but decreases the rate of heat transfer at the surface of the sheet.
* Thermal boundary layer thickness decreases with an increase in values of Prandtl number while reverse tend is observed in case of slip parameter.

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