Weak $(\psi - \phi)$ hybrid contraction in metric like-spaces

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Abstract: In this manuscript, we shall introduce a weak $(\psi - \phi)$ hybrid contraction and prove fixed point theorem for such contractions in metric like spaces. In the end, the main result is supported by an example.

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1. Introduction:

In 1922, Banach [4] introduced a contraction principle to get a fixed point in complete metric space. In 2000, the concept of dislocated was introduced by Hitzler and Seda [3]. Metric like spaces were discovered by Amini and Harandi [1] in 2012. Till now many authors gave fixed point theorems in metric spaces see ([5]-[7], [12]-[18]). In 2018, Karapinar gave a new direction to the concept of contractions by introducing Jaggi type interpolative contraction. Since then, a numbers of fixed point results are proved for various type of hybrid contractions in various spaces see ([2],[8]-[10]). In this paper, we shall extend the concept of weak contraction given by Choudhary and Dutta [10] in metric like space.

Definition 1.1. [1] Let Ꝑ be a non empty set and $æ : \text{Ꝑ} \times \text{Ꝑ} \to [0, \infty)$ be a function such that

1. $æ(\beta, \gamma) = 0$ implies $\beta = \gamma$,
2. $æ(\beta, \gamma) = æ(\gamma, \beta)$,
3. $æ(\beta, \gamma) \leq æ(\beta, \delta) + æ(\delta, \gamma)$.

for all $\beta, \gamma, \delta \in \text{Ꝑ}$.

Then $æ$ is called dislocated metric and $(\text{Ꝑ}, æ)$ is called dislocated (metric like) spaces.

Definition 1.2. [1] Let $(\text{Ꝑ}, æ)$ be a metric like space.

1. A sequence $\{\Omega_n\}$ in Ꝑ is said to be Cauchy if $\lim_{m,n\to\infty} æ(\Omega_m, \Omega_n)$ exists and finite.
2. $(\text{Ꝑ}, æ)$ is said to be complete if every Cauchy sequence $\{\Omega_n\}$ in Ꝑ converges to some $\Omega \in \text{Ꝑ}$, that is $\lim_{n\to\infty} æ(\Omega, \Omega_n) = æ(\Omega, \Omega) = \lim_{m,n\to\infty} æ(\Omega_m, \Omega_n)$.
3. A mapping $\mathfrak{T}: (\text{Ꝑ}, æ) \to (\text{Ꝑ}, æ)$ is said to be continuous if for any sequence $\{\Omega_n\}$ in Ꝑ such that $æ(\Omega, \Omega_n) \to æ(\Omega, \Omega)$ as $n \to \infty$, we have $æ(\mathfrak{T}\mathfrak{x}, \mathfrak{T}\Omega_n) \to æ(\Omega, \Omega)$ as $n \to \infty$.

Lemma 1.1 [11] Let $(\text{Ꝑ}, æ)$ be a metric like space and $\{\Omega_n\}$ be a sequence in Ꝑ such that $\Omega_n \to \Omega$ in Ꝑ and $æ(\Omega, \Omega) = 0$. Then for all $\text{ꝏ} \in \text{Ꝑ}$, we have

$$\lim_{n\to\infty} æ(\Omega_n, \text{ꝏ}) = æ(\Omega, \text{ꝏ}).$$

2. Main Results:

In this section, we shall introduce weak $(\psi - \phi)$ hybrid contraction and prove fixed point theorem in metric like space for it.

Definition 2.1 Let $(\mathfrak{P}, æ)$ be a metric like space. Then $\mathfrak{T}: \mathfrak{P} \to \mathfrak{P}$ is said to be weak $(\psi - \phi)$ hybrid contractive mapping if

$$\psi(æ(\mathfrak{T}\Omega, \mathfrak{T}\varpi)) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega, \varpi)) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega, \varpi)), \tag{2.1}$$

for all distinct $\Omega, \varpi \in \mathfrak{P}$, where $s \geq 0$, $\lambda_i \geq 0$ for all $i = 1,2,3,4$ such that $\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$ and

$$\mathfrak{R}_s^{\mathfrak{T}}(\Omega, \varpi)) =$$

$$\begin{cases} [\lambda_1(æ(\Omega, \varpi))^s + \lambda_2(æ(\Omega, \mathfrak{T}\Omega))^s + \lambda_3(æ(\varpi, \mathfrak{T}\varpi))^s + \lambda_4(\dfrac{æ(\varpi, \mathfrak{T}\varpi)(1 + æ(\Omega, \mathfrak{T}\Omega))}{1 + æ(\Omega, \varpi)})^s], & if \quad s > 0 \\ (æ(\Omega, \varpi))^{\lambda_1}(æ(\Omega, \mathfrak{T}\Omega))^{\lambda_2}(æ(\varpi, \mathfrak{T}\varpi))^{\lambda_3}(\dfrac{æ(\varpi, \mathfrak{T}\varpi)(1 + æ(\Omega, \mathfrak{T}\Omega))}{1 + æ(\Omega, \varpi)})^{\lambda_4}, & if \quad s = 0\,. \end{cases}$$

$\Omega, \varpi \notin \mathfrak{F}_{\mathfrak{T}}(\mathfrak{P})$, where

$\mathfrak{F}_{\mathfrak{T}}(\mathfrak{P}) = \{\Omega \in \mathfrak{P}: \mathfrak{T}\Omega = \Omega\}$, and $\psi, \phi: [0, \infty) \to [0, \infty)$ both are continuous and non-decreasing function with $\phi(t) = 0 = \psi(t)$ if and only if $t = 0$.

Theorem 2.1 Let $(\mathfrak{P}, æ)$ be a complete metric like space and $\mathfrak{T}$ be a continuous self map on $\mathfrak{P}$ satisfying equation (2.1). Then $\mathfrak{T}$ has a unique fixed point.

Proof. Let $\Omega_0 \in \mathfrak{P}$. Construct an iterative sequence $\{\Omega_n\}$ as $\Omega_n = \mathfrak{T}\Omega_{n-1}$ and $\mathfrak{T}\Omega_0 = \Omega_1$ for all $n \in \mathbb{N}$. $æ(\Omega_n, \Omega_{n+1}) > 0$ for all $n \in \mathbb{N}$. Because if for some $k \in \mathbb{N}$, $æ(\Omega_k, \Omega_{k+1}) = 0$ then $\Omega_k = \Omega_{k+1} = \mathfrak{T}\Omega_k$, this shows that Ω_k is the fixed point of $\mathfrak{T}$ and we are done.

We shall discuss the proof in two different cases: $s > 0$, $s = 0$.

Case (i). When $s > 0$.

Putting $\Omega = \Omega_{n-1}$ and $\varpi = \Omega_n$ in equation (2.1), we get

$$\psi(æ(\mathfrak{T}\Omega_{n-1}, \mathfrak{T}\Omega_n)) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n)) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n)), \tag{2.2}$$

where

$$\begin{aligned} \mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n) &= [\lambda_1(æ(\Omega_{n-1}, \Omega_n))^s + \lambda_2(æ(\Omega_{n-1}, \mathfrak{T}\Omega_{n-1}))^s + \lambda_3(æ(\Omega_n, \mathfrak{T}\Omega_n))^s \\ &\quad + \lambda_4(\frac{æ(\Omega_n, \mathfrak{T}\Omega_n)(1 + æ(\Omega_{n-1}, \mathfrak{T}\Omega_{n-1}))}{1 + æ(\Omega_{n-1}, \Omega_n)})^s] \\ &= [\lambda_1(æ(\Omega_{n-1}, \Omega_n))^s + \lambda_2(æ(\Omega_{n-1}, \Omega_n)^s \\ &\quad + \lambda_3(æ(\Omega_n, \Omega_{n+1}))^s + \lambda_4(æ(\Omega_n, \Omega_{n+1}))^s]. \end{aligned} \tag{2.3}$$

Now if possible suppose that $æ(\Omega_{n-1}, \Omega_n) \leq æ(\Omega_n, \Omega_{n+1})$,

From this, equations (2.2) and (2.3), we have

$$\begin{aligned} \psi(æ(\Omega_n, \Omega_{n+1})) &\leq \psi(æ(\Omega_n, \Omega_{n+1})) - \phi(æ(\Omega_n, \Omega_{n+1})) \\ &< \psi(æ(\Omega_n, \Omega_{n+1})). \end{aligned}$$

a contradiction.

So, $æ(\Omega_{n-1}, \Omega_n) \geq æ(\Omega_n, \Omega_{n+1})$.

Using this in equation (2.3), we get

$$\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n) = æ(\Omega_{n-1}, \Omega_n). \tag{2.4}$$

It follows that the sequence $\{æ(\Omega_n, \Omega_{n+1})\}$ is a monotonically decreasing sequence of positive reals. So, the sequence $\{æ(\Omega_n, \Omega_{n+1})\}$ converges to some $r \geq 0$.

Taking limit as $n \to \infty$ in equation (2.1) and using above, we obtain that $\psi(r) \leq \psi(r) - \phi(r)$, which holds only when $r = 0$.

So,

$$æ(\Omega_n, \Omega_{n+1}) \to 0, \tag{2.5}$$

as $n \to \infty$.

Now, we shall show that $\{\Omega_n\}$ is a Cauchy sequence. If possible, suppose that $\{\Omega_n\}$ is not a Cauchy sequence. Then there exist $\epsilon > 0$ for which we can find two subsequences $\{\Omega_{m(k)}\}$ and $\{\Omega_{n(k)}\}$ of $\{\Omega_n\}$ with $n(k) > m(k) > k$, such that

$$æ(\Omega_{m(k)}, \Omega_{n(k)}) \geq \epsilon. \tag{2.6}$$

Further for $m(k)$ we can choose $n(k)$ in such a way that it is the smallest integer with $n(k) > m(k)$ and satisfying equation (2.6).

Then

$$æ(\Omega_{m(k)}, \Omega_{n(k)-1}) < \epsilon. \tag{2.7}$$

Then using triangle inequality, equations (2.6) and (2.7), we have

$$\begin{aligned} \epsilon &\leq æ(\Omega_{m(k)}, \Omega_{n(k)}) \\ &\leq æ(\Omega_{m(k)}, \Omega_{n(k)-1}) + æ(\Omega_{n(k)-1}, \Omega_{n(k)}) \\ &\leq \epsilon + æ(\Omega_{n(k)-1}, \Omega_{n(k)}). \end{aligned} \tag{2.8}$$

Making $k \to \infty$ and using equation (2.5) in (2.8), we get

$$\lim_{k\to\infty} æ(\Omega_{m(k)}, \Omega_{n(k)}) = \epsilon. \tag{2.9}$$

Again

$$æ(\Omega_{m(k)}, \Omega_{n(k)}) \leq æ(\Omega_{m(k)}, \Omega_{m(k)-1}) + æ(\Omega_{m(k)-1}, \Omega_{n(k)-1}) + d(x_{n(k)-1}, x_{n(k)}). \tag{2.10}$$

$$æ(\Omega_{m(k)-1}, \Omega_{n(k)-1}) \leq æ(\Omega_{m(k)-1}, \Omega_{m(k)}) + æ(\Omega_{m(k)}, \Omega_{n(k)}) + æ(\Omega_{n(k)}, \Omega_{n(k)-1}). \tag{2.11}$$

Taking $k \to \infty$ and using equations (2.5), (2.9) in equations (2.10) and (2.11), we get

$$\lim_{k\to\infty} æ(\Omega_{m(k)-1}, \Omega_{n(k)-1}) = \epsilon. \tag{2.12}$$

Putting $\Omega = \Omega_{m(k)-1}, \varpi = \Omega_{n(k)-1}$, in the equation (2.1), we get

$$\psi(æ(\mathfrak{T}\Omega_{m(k)-1}, \mathfrak{T}\Omega_{n(k)-1})) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{m(k)-1}, \Omega_{n(k)-1})) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{m(k)-1}, \Omega_{n(k)-1})), \tag{2.13}$$

where

$$\begin{aligned} \mathfrak{R}_s^{\mathfrak{T}}(\Omega_{m(k)-1}, \Omega_{n(k)-1}) = {} & [\lambda_1(æ(\Omega_{m(k)-1}, \Omega_{n(k)-1}))^s + \lambda_2(æ(\Omega_{m(k)-1}, \mathfrak{T}\Omega_{m(k)-1}))^s \\ & + \lambda_3(æ(\Omega_{n(k)-1}, \mathfrak{T}\Omega_{n(k)-1}))^s + \lambda_4(\tfrac{æ(\Omega_{n(k)-1}, \mathfrak{T}\Omega_{n(k)-1})(1+æ(\Omega_{m(k)-1}, \mathfrak{T}\Omega_{m(k)-1}))}{1+æ(\Omega_{m(k)-1}, \Omega_{n(k)-1})})^s], \end{aligned} \tag{2.14}$$

Taking $k \to \infty$ and using (2.5), (2.9) (2.12) and (2.14) in (2.13), we get

$$\psi(\epsilon) \leq \psi(\lambda_1^{\frac{1}{s}}\epsilon) - \phi(\lambda_1^{\frac{1}{s}}\epsilon). \tag{2.15}$$

Subcase (i): When $\lambda_1 = 0$.

From equation (2.15), we have

$\psi(\epsilon) \leq 0$, but ψ is a non-negative function,

so $\psi(\epsilon) = 0$ and this holds only when $\epsilon = 0$.

Subcase (ii): When $\lambda_1 = 1$.

From equation (2.15), we have

$\psi(\epsilon) \leq \psi(\epsilon) - \phi(\epsilon)$,

but ψ is non-decreasing function, so this holds only when $\epsilon = 0$.

Subcase (iii): When $0 < \lambda_1 < 1$.

Clearly, $\epsilon > \lambda^{\frac{1}{s}}\epsilon$, so equation (2.15) implies that $\psi(\epsilon) \leq \psi(\lambda^{\frac{1}{s}}\epsilon)$ but as ψ is non-decreasing function, so this holds only when $\epsilon = 0$.

From all the above discussed three subcases it is clear that $\epsilon = 0$.

a contradiction to our assumption.

So, $\{\Omega_n\}$ is a Cauchy sequence.

Now as $(\mathfrak{P}, \text{æ})$ is a complete metric like space so there exists $\mathfrak{u} \in \mathfrak{P}$ such that

$$\lim_{n\to\infty} \text{æ}(\mathfrak{u}, \Omega_n) = \text{æ}(\mathfrak{u}, \mathfrak{u}) = \lim_{m,n\to\infty} \text{æ}(\Omega_m, \Omega_n) = 0. \tag{2.16}$$

Since $\mathfrak{T}$ is continuous, from equation (2.16), we have

$$\lim_{n\to\infty} \text{æ}(\mathfrak{T}\mathfrak{u}, \Omega_{n+1}) = \text{æ}(\mathfrak{T}\mathfrak{u}, \mathfrak{T}\mathfrak{u}). \tag{2.17}$$

On the other hand, by Lemma 1.1 and equation (2.16), we have

$$\lim_{n\to\infty} \text{æ}(\mathfrak{T}\mathfrak{u}, \Omega_{n+1}) = \text{æ}(\mathfrak{u}, \mathfrak{T}\mathfrak{u}). \tag{2.18}$$

Using equations (2.1), (2.17) and (2.18), we get

$\psi(\text{æ}(\mathfrak{T}\mathfrak{u}, \mathfrak{T}\mathfrak{u})) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\mathfrak{u}, \mathfrak{u})) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\mathfrak{u}, \mathfrak{u}))$,

Now by simple calculations, we get

$\psi(\text{æ}(\mathfrak{u}, \mathfrak{T}\mathfrak{u})) \leq \psi(\text{æ}(\mathfrak{u}, \mathfrak{T}\mathfrak{u}))$,

which holds only when $\text{æ}(\mathfrak{u}, \mathfrak{T}\mathfrak{u}) = 0$, that is

$\mathfrak{u} = \mathfrak{T}\mathfrak{u}$.

So, $\mathfrak{u}$ is the fixed point.

Uniqueness: If possible suppose that $\mathfrak{u}$ and $\mathfrak{v}$ be two distinct fixed point of $\mathfrak{T}$, then from equation (2.1), we obtain that

$\psi(\text{æ}(\mathfrak{T}\mathfrak{u}, \mathfrak{T}\mathfrak{v})) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\mathfrak{u}, \mathfrak{v})) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\mathfrak{u}, \mathfrak{v}))$,

Where

$$\mathfrak{R}_s^{\mathfrak{T}}(\mathfrak{u}, \mathfrak{v}) = [\lambda_1(\text{æ}(\mathfrak{u}, \mathfrak{u}))^s + \lambda_2(\text{æ}(\mathfrak{u}, \mathfrak{T}\mathfrak{u}))^s + \lambda_3(\text{æ}(\mathfrak{v}, \mathfrak{v}))^s + \lambda_4(\tfrac{\text{æ}(\mathfrak{v},\mathfrak{v})(1+\text{æ}(\mathfrak{u},\mathfrak{u}))}{1+\text{æ}(\mathfrak{u},\mathfrak{v})})^s],$$

By equation (2.16), we get

$\psi(\text{æ}(\mathfrak{u}, \mathfrak{v})) \leq 0$, which implies that $\mathfrak{u} = \mathfrak{v}$.

Hence $\mathfrak{T}$ has a unique fixed point.

Case (ii). When $s = 0$. Taking $\Omega = \Omega_{n-1}$ and $\varpi = \Omega_n$ in equation (2.1), we get

$$\psi(\text{æ}(\mathfrak{T}\Omega_{n-1}, \mathfrak{T}\Omega_n)) \leq \psi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n)) - \phi(\mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n)), \tag{2.19}$$

where

$$\begin{aligned} \mathfrak{R}_s^{\mathfrak{T}}(\Omega_{n-1}, \Omega_n) &= (\text{æ}(\Omega_{n-1}, \Omega_n)^{\lambda_1}(\text{æ}(\Omega_{n-1}, \Omega_n))^{\lambda_2}(\text{æ}(\Omega_n, \Omega_{n+1}))^{\lambda_3}(\frac{\text{æ}(\Omega_n, \Omega_{n+1})(1 + \text{æ}(\Omega_{n-1}, \Omega_n))}{1 + \text{æ}(\Omega_{n-1}, \Omega_n)})^{\lambda_4} \\ &= (\text{æ}(\Omega_{n-1}, \Omega_n)^{\lambda_1}(\text{æ}(\Omega_{n-1}, \Omega_n))^{\lambda_2}(\text{æ}(\Omega_n, \Omega_{n+1}))^{\lambda_3}(\text{æ}(\Omega_n, \Omega_{n+1}))^{\lambda_4} \end{aligned}$$

Using this in equation (2.19), we have

$(\text{æ}(\Omega_n, \Omega_{n+1}))^{1-\lambda_3-\lambda_4} \leq (\text{æ}(\Omega_{n-1}, \Omega_n)^{\lambda_1}(\text{æ}(\Omega_{n-1}, \Omega_n))^{\lambda_2}$

$\text{æ}(\Omega_n, \Omega_{n+1}) \leq \text{æ}(\Omega_{n-1}, \Omega_n)$.(as $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$)

Now by repeating the same steps as in case (i), one can get the proof of unique fixed point for $s = 0$.

Example 2.1 Let $\mathfrak{P} = [0, \infty)$ and $\text{æ}(\Omega, \text{ꝏ}) = \max\{\Omega, \text{ꝏ}\}$.

Clearly, $(\mathfrak{P}, \text{æ})$ is a complete metric like space.

Define $\mathfrak{T}\mathfrak{x} = \frac{\Omega}{2}$ and $\psi(t) = 2t$, $\phi(t) = 0$ and $s = 1$.

Without loss of generality assume that $\Omega \geq \text{ꝏ}$ for all $\Omega, \text{ꝏ} \in \mathfrak{P}$.

Now $\psi(\text{æ}(\mathfrak{T}\mathfrak{x}, \mathfrak{T}\mathfrak{y})) = \Omega$.

Similarly, right hand side of equation (2.1) is equal to $\Omega + \text{ꝏ}$.

$\mathfrak{T}$ is continuous also.

So, all the conditions of Theorem 2.1. are satisfied. Hence $\mathfrak{T}$ has a unique fixed point.

Clearly, 0 is the fixed point.

3. Conclusion:

In this manuscript,we have introduced a new notion of weak $(\psi - \phi)$ hybrid contraction in metric like space and proved a fixed point theorem in it. We have supported our result by a non trivial example. It is an open problem to check whether the condition of continuity on the mappings can be relaxed by adding another condition. They can also extend our proved result for two or four mappings in metric like or in c*-algebra valued, bipolar, fuzzy, soft metric spaces etc.

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