**Local Isolate Domination In Graphs**

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**Abstract**

In this Communication a new parameter called Local Isolate Domination in graphs is defined and Studied. A Dominating Set S is a Local Dominating set iff for each u in S ,< N(u) > has an isolate.

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**Key Words:** Dominating Set, Isolate Dominating Set, Local Isolate Dominating Set, Local Isolate Domination Number.

# Introduction

Throughout this paper , Simple Finite Graphs without loops and multiple edges are considered. For terminologies and notations refer Chartrand And Lesniak[3]. Domination and related topics are dealt in [1 , 4, 5, 6].

A subset S of the vertex set V(G) is a Dominating set if each vertex in the set V \ S is adjacent to a vertex in S. Minimum cardinality of a minimal dominating set is the Domination number of a Graph denoted byγ(G).

It is an isolate dominating set if the induced graph<S> contains an isolate and is introduced and studied in [7].

A Dominating set S is called a Doubly Isolate Dominating set if both the induced graphs < S > and < V \ S > have isolates. Doubly Isolate Dominating set is introduced and studied in [2].

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Also when the concept of Isolate Dominating set is localized to the Neighbour set we arrive at a new variant called Local Isolate Dominationin Graphs. This motivated us to define a new parameter that is introduced and studied in this communication.

# Preliminary Results

**Theorem2.*1 “***For a Graph G with order atleast 3, ∆(G)= n−1 and minimum degree atleast 2, G has no Local Isolate Dominating Set.”

**Proof:**From the hypothesis we observe that G is a graph without isolates and with Domination number as one and hence this dominating set is not a Local Isolate Dominating set of G. Suppose S is any dominating set of G and

S \ {v} ≠ φ where {v} is a full degree vertex of G.Now for each u in {S \ {v}} the induced graph < N(u) > has no isolated vertex. Hence S is not a Local Isolate Dominating Set of G.

**Corollary2.2 “**The Local Isolate Dominating Set Does Not Exist For The Following Graphs:

* 1. Complete Graph Kn.
  2. Wheel Graph Wn.
  3. Fan Graph Fn.”

**Observation2.3 “**The Local Isolate Dominating Set Does Not Exist For Complete r-Partite Kn1,n2,…nr, r ≥ 3 Graph.”

# 3. Main results

**Proposition3.1**

“(i) For the Paths Pn and the Cycles Cn we have and 

(ii) If G is a Graph of Order n, Then 𝛾(𝐺+)= Γ𝑙o(𝐺+)=𝑛, Where G+ is the Graph that was produced from G by attaching at each of G's vertex's e Edges.”

**Proof.**(i) “Obviously and when n≠4, any γ-set of Pn is a local isolate dominating set as well, so that .” Every Local Isolate D dominating set is a dominating set so  thusand so as . Now if Pn = {v1,v2,v3,…vn}then the set s = is a minimal isolate dominating set so that . Additionally, since any set that contains more than vertices of Pn  is no longer able to be a minimal isolation dominating, we have  Similar to this, one may  prove and 

(ii) Each pendant vertex is required to be present in any minimal isolate-dominating set S of G+ or one of its neighbours, in order to have at least n vertices. “Further, if |𝑆|>𝑛, S must consequently include a pendant vertex along with its support and so 𝑆−{𝑣}, where v is the support, is an isolate dominating set of 𝐺+,a contradiction to the minimality of S.” Hence |𝑆|=𝑛.

**Theorem3.2 “**For a Graph G of order at least 2, γlo(G)=1 iff there exists a pair u,vin V(G), degG(u) = 1 and degG(v) =n−1.”

**Proof:** Let G be a graph with n≥2. Suppose γlo(G)=1. Let S={v} be a Local Isolate Dominating set of G. “Since S is a dominating set and |V(G)\ S|=n−1, degG(v)=n−1.” Also since S is a γlo-set of G, <n(v) > has an isolate vertex, say u. Therefore u is a pendent vertex of G. Hence degG(u)=1. Conversely, {V} is a dominant set of G since there is a vertex v with degG(v)=n. Since degG(u)=1, u is a isolate vertex in <n(v)>, thus γlo(G)=1.

**Corollary3.3** “For a Star Graph Sn with n≥2, γlo(Sn) = 1.”

**Theorem 3.4 “**If G is a Tree with n ≥ 2 then G has a Local Isolate Dominating Set.”

**Proof :**Let G be a Tree of Order n ≥2 and S be any Dominating Set of G. “Suppose G has no Local Isolate Dominating Set, there exist a vertex v ∈S, and < n(v)>has no isolate vertex.” Thus <n(v)> is a connected Graph. This implies < n[v] > contains a cycle, which contradicts that G is a Tree. Therefore G has a Local Isolate Dominating Set.

**Corollary3.5 “**For any Tree T, γ(T)=γo(T)=γlo(T)”.

**Theorem 3.6 “**Let S be any Local Isolate Dominating Set of a Graph G and

U ∈ S. Then there exist a vertex v ∈V(G) such that uv ∈ E(G) and

N(U) ∩ N(V) = φ.”

**Theorem3.7 “**For a Complete bipartite Graph Km,n , γlo(Km,n)=2, m ≥2, n ≥2.”

**Theorem3.8 “**A Local isolate dominating set S of a graph G is minimal iff it is 1-minimal.”

**proof :** “Let S be a 1-minimal Local Isolate Dominating Set of a graph G. Suppose there exists a S’⊂ S that is also a Local Isolated Dominating Set of G, then for all v in S’, <n(v) >has an isolate vertex”. Since S’ is a Dominating Set, for all vertex in u in S \ S’ is adjacent to at least one vertex in S’and either u is an isolate vertex in <n(v)>,v ∈ S’ or <n(v) >has an isolated vertex in V \ S.

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**case(i):** u is an isolate vertex in <n(v)>, v∈ S’ then S\{v} is Local Isolate Dominating Set of G which contradicts the 1-minimality of S.

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**case(ii):** <n(v)>has an isolated vertex in V \ S. Let w∈<n(v)>be isolate vertex in V \ S then S \{u} is Local Isolate Dominating Set of G which contradicts the 1-minimality of S. Hence S is minimal.

Converse is obvious.

**Theorem3.9 “**A Local Isolate Dominating Set S of a Graph G is Minimal iff every vertex in S has a Private Neighbor with respect to S.”

**Proof: “**Let S be a minimal Local Isolate Dominating set and u be a vertex of S.” u is a private neighbour of itself if it is an “isolate” in <S>. “Suppose u is not an isolate of < S >”. “The set S \ {u} will be a Local Isolate Dominating set if u has no private neighbours with regard to S.” “This contradicts the minimality of S”, hence “u must have a private neighbour with regard to S”. Conversely, “suppose S is a Local Isolate Dominating set of G and every vertex of S has a private neighbor with respect to S”. If S is not minimal ,then by theorem 3.8, as a result of S's inability to be a “1-minimal Dominating Set” of G,  S has a vertex u that makes S\{u} a Local Isolate Dominating Set of G. “Each vertex in V \(S \u) must thus have at least one neighbour in S \ u, and as a result, the vertex u cannot have any private neighbour with regard to S.” This is a contradiction to our assumption ,therefore S is minimal.

**Corollary 3.10 “**A minimal Local Isolate Dominating Set S of a Graph G is also a minimal Dominating Set of a Graph G.”

# 4. Join of Graphs

**Observation 4.1 “**Let G and H be any two Graphs of order m , n ≥3 with isolate vertex and S be a Local Isolate Dominating Set of G+H. Then S∩V(G) ≠ ≠φ and S∩V(H) ≠ φ.”

**Theorem 4.2 “**Let G and H be any two Graphs. Then S a subset of V(G+H) is a Local Isolate Dominating Set of G+H iff G and H have isolated vertices.”

**Proof “**Let G and H be any two Graphs and S ⊆V(G+H) be a Local Isolate Dominating Set of G+H.” Suppose G and H have no isolated vertex then for each u ∈ S, < n(u)>is connected, which is a contradiction. Therefore there are isolated vertex in both G and H.

Conversely, U and V be isolated vertices of G and H respectively, Then S={u,v} is a Dominating Set of G + Hand also n(u) ≥V(H) and n(v)≥V(G). Thus <n(u)> and < n(v)> have isolated vertex.

Therefore “S is a Local Isolate Dominating Set of G+H.”

**Corollary4.3 “**LetG and H be any Graphs with isolated vertex, Then

γlo(G+H)≤2.”

**Proof. “**Let G and H be Graphs with isolated vertex.” Suppose either G=K1 or H= K1or G= H= K1 Then Clearly, γlo(G +H) = 1. Suppose G≠K1 and H ≠ K1, by the theorem 4.2, γlo(G +H) = 2. Thus γlo(G +H)≤ 2.

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