**SIMILARITY MEASURES OF FERMATEAN NEUTROSOPHIC SETS BASED ON THE COSINE FUNCTION AND THEIR APPLICATIONS**

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**ABSTRACT:**

In this paper, we present similarity measures between Fermatean Neutrosophic sets (FNSs) based on the cosine function by considering the degree of membership, degree of non-membership and the degree of hesitation in FNSs. Then, we apply these similarity measures and weighted similarity measures between FNSs to pattern recognition and medical diagnosis. Finally, two illustrative examples are given to demonstrate the efficiency of the similarity measures for pattern recognition and medical diagnosis.

1. **INTRODUCTION**

The similarity measures are important and useful tools for determining the degree of similarity between two objects. Measures of similarity between fuzzy sets have gained attention from researchers for their wide application in various fields, such as pattern recognition, machine learning, decision making and image processing, many measures of similarity between fuzzy sets have been proposed and researched in recent years[6-8,15]**.** Fuzzy set theory, introduced by Zadeh[31], has been widely used to model uncertainty present in real-world applications. Atanassov[3,4] extended fuzzy sets to Atanassov’s intuitionistic fuzzy sets (IFSs), many different similarity measures between IFSs have been investigated in the literature[17]. Li and Cheng [16] proposed a suitable similarity measure between IFSs and applied it to pattern recognition problems. Liang and Shi [16]defined some similarity measures to differentiate different IFSs and discussed the relationships between them. Furthermore, Mitchell [19] modified Li and Cheng’s measures. Based on the extension of Hamming distance on fuzzy sets, Szmidt and Kacprzyk [24,25] developed a similarity measure between IFSs based on the Hamming distance. Hung and Yang[12] calculated the distance between IFSs based on the Hausdorff distance and generated some similarity measures between IFSs. Liu[18]developed some new similarity measures between IFSs and between elements. Hung and Yang [13]proposed a similarity measures between IFSs based on the Lp metric. Xu and Xia [28] defined the geometric distance and similarity measures of IFSs for group decision-making problems. Ye[29]proposed the cosine similarity measure between IFSs. Hung [14] developed the likelihood-based measurement of IFSs for the medical diagnosis and bacteria classification problems. Shi and Ye [22] further improved the cosine similarity measure of IFSs. Tian [27]proposed the cotangent similarity measure between IFSs for medical diagnosis. Rajarajeswari and Uma[20] further introduced the cotangent similarity measure, which considered membership, non-membership and hesitation degrees described in IFSs. Furthermore, Szmidt [26] discussed distances between IFSs and introduced a family of similarity measures which considered membership, non-membership and hesitation degrees described in IFSs. Ye [30] proposed two new cosine similarity measures and weighted cosine similarity measures based on cosine function and the information carried by the membership degrees, non-membership degrees and the hesitation degrees in IFSs. Son and Phong [23] gave the intuitionistic vector similarity measures for medical diagnosis. Fermatean Neutrosophic Sets was proposed by Antony, Jansi[1]

 This paper is set out as follows: In the next section, we introduce some basic concepts related to IFS and FNSs and some similarity measure between IFSs. In Section 3, we shall propose some similarity measure and weighted similarity measure between FNSs based on the concept of the cosine function. In Section 4, the similarity measure of FNSs are applied to pattern recognition and medical diagnosis. Section 5 deals with the advantages of the proposed similarity measures and the last section concludes the paper with some remarks.

1. **PRELIMINARIES**

In the following, we introduce some basic concepts related to IFSs and some similarity measure between IFSs.

**Definition 2.1 [3,4] :**

An IFS in is given by,

 ------------(1)

Where and where The number and represents, respectively the membership degree and non-membership degree of the element to the set .

**Definition 2.2 [3,4]:**

For each IFS in , if

 ----------(2)

Then is called the degree of indeterminacy of to .

Suppose that there are two IFSs and in the universe of discourse

Ye[30] proposed the cosine similarity measure between IFSs and as following:

 ---------------(3)

Shi and Ye [22] further presented the cosine similarity measure by considering membership degree , non-membership degree, and hesitancy degree in IFSs as the vector space of the three terms:

 ------------(4)

Based on the cosine function, Ye [30] proposed two cosine similarity measures between IFSs A and B.

 -------------(5)

 --------------(6)

On the other hand, Tian [27] proposed a cotangent similarity measure between IFSs A and B as following:

 -------------(7)

Where the symbol is the maximum operation.

 When the three terms such as the membership degree, non-membership degree, and hesitancy degree are considered in IFSs, Rajarajeswari and Uma [20] defined the cotangent similarity measure of IFSs:

 ------------(8)

In the following, we introduce the weighted cosine and cotangent similarity measures between IFSs A and B, respectively [29, 22, 27, 20, 30]**.**

 --------------(9)

 ------------(10)

 -------------(11)

 ----------------(12)

 ---------------(13)

 ---------------(14)

Where is the weight of an element and and the symbol is the maximum operation.

**Definition 2.3[2]:**

Let X be a universe of discourse. A Fermatean Neutrosophic set [FN Set] A on X is an object of the form: 𝐴 = , where and .

Then, for all .

 is the degree of membership function, is the degree of indeterminacy and is the degree of non- membership function. Here and are dependent components and is an independent components.

**Definition 2.4[2]:**

Let X be a non-empty set and I the unit interval [0,1]. A Fermatean Neutrosophic sets M and N of the form and

1. .
2. = .
3. =
4. **SOME SIMILARITY MEASURE BASED ON THE COSINE FUNCTION FOR FERMATEAN NEUTROSOPHIC SETS**

**3.1 Cosine Similarity Measure for FNSs**

Let be a FNS in an universe of discourse , the FNS is characterized by the degree of membership , the degree of non-membership , and the degree of hesitation , , which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure and a weighted cosine similarity measure for FNSs are proposed in an analogous manner to the cosine similarity measure based on Bhattacharya’s distance [21, 6]and cosine similarity measure for IFS [28]**.**

 Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

 ------------(17)

If we take n=1, then the cosine similarity measure between FNSs and becomes the correlation coefficient between FNSs and . Therefore, the cosine similarity measure between and also satisfies the following properties:

1. .
2. if

Proof:

1. It is obvious that the proposition is true according to the cosine value.
2. It is obvious that the proposition is true.
3. When there are , and So there is .

Therefore, we have finished the proofs.

If we consider the weights of , a weighted cosine similarity measure between FNSs and is proposed as follows:

 --------------(18)

 Where is the weight vector of , with , In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is . Obviously, the weighted cosine similarity measure of two FNSs and also satisfies the following properties:

1. .
2. if .

Similar to the previous proof method, we can prove the above three properties.

In the following, we shall investigate the distance measure of the angle as

 It satisfies the following properties:

1. if ;
2. , if
3. if
4. if for any FNS .

Proof:

Obviously, satisfies the property (1) - (3). In the following, will be proved to satisfy the property (4).

 For any , , Since Equation (16) is the sum of terms, let us investigate the distance measures of the angle between the vectors:

 and

 where

 For three vectors ,

 in one plane, if Then, it is obvious that according to the triangle inequality. Combining the inequality with Equation (16), we can obtain Thus satisfies the property (4). So we finished the proof.

* 1. **Similarity measures of FNSs based on cosine function:**

Based on the cosine function, in this section, we shall propose two cosine similarity measures between FNSs and analyse their properties.

**Definition 3.2.1:**

Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

 --------------(19)

 --------------(20)

Where the symbol is the maximum operator.

**Proposition 3.2.2:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2. If is a FNS in and , then and .

Proof:

1. Since the value of the cosine function is within [0,1], the similarity measure based on the cosine function is also within [0,1]. Thus, there is .
2. For any two FNSs and in if , then and for Thus,

 So,

If this implies for Since cos(0) =1. Then, there are

 and for Hence

.

1. Proof is straightforward.
2. If then there are , and for Then, , and

Thus, we have

,

,

,

,

 and

.

Hence, and for as the cosine function is a decreasing function with the interval

Thus, the proofs of these properties are completed.

 In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cosine similarity measure between FNSs and is proposed as follows:

 --------------(21)

 --------------(22)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is .

Obviously, the weighted cosine similarity measures also satisfy the axiomatic requirements of similarity measures in Proposition 2.

**Proposition 3.2.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2.
3. If is a FNS in and , then W and .

By using similar proof in Proposition 1, we can give the proofs of these properties (1) – (4).

 **3.3 Similarity Measures of FNSs based on the Cotangent Function:**

In this section, we shall propose two cotangent similarity measures between FNSs.

**Definition 3.3.1**:

Suppose that there are two FNSs and in the universe of discourse , we further propose the cotangent similarity measures between FNSs as follows:

 --------------(23)

 --------------(24)

Where the symbol is the maximum operator.

**Proposition 3.3.2:**

For any two FNSs and in the cotangent similarity measures should satisfy the following properties (1) – (4):

1. If is a FNS in and , then and .

**Proof:**

1. Since,

 ,

It is obvious that the cotangent function are within 0 and 1.

1. It is obvious that the proposition is true.
2. When , then obviously . On the other hand if then,

 and for

This implies

1. If then we can write , and for Then, , and

The cotangent function is decreasing function within the interval .

Hence we can write and .

Thus, the proofs of these properties are completed.

 In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cotangent similarity measure between FNSs and is proposed as follows:

 --------------(25)

 ----------------(26)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if

, then the weighted cotangent similarity measure reduces to cotangent similarity measure. That is to say, if we take then there is .

**Proposition 3.3.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2.
3. If is a FNS in and , then W and .

By using similar proof in Proposition 3, we can give the proofs of these properties (1) – (4)

4.**APPLICATIONS**

In this section, the cosine and cotangent similarity measures for FNSs are applied to pattern recognition and medical diagnosis to illustrate the feasibility of the proposed methods and deliver a comparative analysis

* 1. **Example 1: Pattern Recognition**

Let us consider, a three known patterns which are represented by the FNSs: in the feature space as

Consider an unknown pattern that will be recognized, where

 The purpose of this problem is classify the pattern in one classes and . For it, the proposed similarities degrees have been computed from to and are given in Table 1.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8704 | 0.8320 | **0.8992** |
|  | 0.8747 | 0.8360 | **0.9041** |
|  | 0.9104 | 0.8863 | **0.9328** |
|  | 0.5967 | 0.5533 | **0.654** |
|  | 0.7695 | 0.7327 | **0.7898** |

Table 1: The similarity measures between and

 From the numerical results presented in Table 1, we know that the degree of similarity between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

 If we consider the weight of are 0.5, 0.3 and 0.2 respectively. Then we use the proposed weighted similarities measures have been computed from to and are given in Table 2.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8244 | 0.8145 | **0.8692** |
|  | 0.8631 | 0.8622 | **0.8808** |
|  | 0.8938 | 0.8975 | **0.9221** |
|  | 0.5818 | 0.5877 | **0.6161** |
|  | 0.7503 | 0.7426 | **0.7753** |

Table 2: The weighted similarity measures between and

 From the numerical results presented in Table 2, we know that the weighted similarity measures between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

* 1. **Example 2: Medical Diagnosis**

Let us consider a set of diagnosis

 and a set of symptoms

 Suppose that a patient, with respect to all symptoms, can be depicted by the following FNS:

 ( (

And then each diagnoses can be viewed as FNSs with respect to all the symptoms as follows:

 (

The purpose of this problem is classify the pattern in one classes . For this, the proposed similarities measures have been computed from to and are given in Table 3.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5811 | 0.8863 | 0.9288 | 0.9420 | **0.9469** |
|  | 0.7440 | 0.8752 | 0.8911 | **0.9208** | 0.8685 |
|  | 0.7644 | 0.9236 | 0.9200 | **0.9355** | 0.9224 |
|  | 0.5083 | 0.6002 | 0.6328 | **0.7018** | 0.6005 |
|  | 0.6628 | 0.7741 | 0.7700 | **0.8155** | 0.7717 |

Table 3: The similarity measures between and

From the numerical results presented in Table 3, expect for the , we know that the similarity measures between and is the largest one as derived by five similarity measures. That is, the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficients method [10] we can get same result that the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs expect for the

If we consider the weight of is respectively. Then we apply the proposed weighted similarities measures, which have been computed from to and are given in Table 4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5608 | 0.7994 | 0.8251 | 0.8235 | **0.8517** |
|  | 0.6889 | 0.7881 | 0.8020 | **0.8204** | 0.7699 |
|  | 0.7083 | 0.8280 | 0.8201 | **0.8382** | 0.8206 |
|  | 0.4830 | 0.5465 | 0.5780 | **0.6245** | 0.5304 |
|  | 0.6149 | 0.6986 | 0.6892 | **0.7327** | 0.6865 |

Table 4: The weighted similarity measures between and

From the numerical results presented in table 4, we get the following results:

1. For similarity measures , the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs.
2. For similarity measures the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs. At the same time, for this case compared with Garg’s correlation coeeficients method [10], we can get the same result that the pattern P should belong to the class of the known diagnoses according to the principle of the maximum degree of similarity between FNSs.
3. **ADVANTAGES OF THE PROPOSED SIMILARITY MEASURES:**
* Although, Atannasov’s IFSs theory has been successfully applied in different areas, but there are situations in real life which cannot be represented by Atannasov’s IFSs. FNSs are extension of Atannasov’s IFSs. The FNS is characterized by the membership degree, the non-membership degree and the degree of hesitancy whose sum of squares is less than or equal to 1, the FNS is more general than the IFS. All the intuitionistic fuzzy degrees are a part of Spherical Fuzzy degrees, which indicates that the FNS is more powerful to handle the uncertain problems. Therefore, the MADM with FNSs is more suitable for real scientific and engineering applications.
* Also it has been observed from the existing studies [29, 22, 27, 20, 30]that the various researchers proposed algorithms by using similarity measures for Atannasov’s IFSs. As mentioned above, there are some situations that cannot be depicted by Atannasov’s IFSs, so their corresponding algorithm may not give appropriate results.
* The similarity measures Atannasov’s IFSs are special case of the similarity measures of FNSs. Therefore, the proposed similarity measures are more generalized and suitable to solve the real- life problem more accurately than the existing ones.
1. **CONCLUSION**

 In this paper, we presented another form of five similarity measures between FNSs based on the cosine function between FNSs by considering the degree of membership, degree of non-membership and the degree of hesitation in FNSs. Then, we applied these similarity measures and weighted similarity measures between FNSs to pattern recognition and medical diagnosis. Finally, two illustrative examples are given to demonstrate the efficiency of the similarity measures for pattern recognition and medical diagnosis.

**REFERENCES:**

[1] I.Antonov,On a new geometrical interpretation of the intuitionistic fuzzy sets, Notes on

 Intuitionistic Fuzzy Sets 1(1) 564 (1995), 29–31.

[2] C. Antony Crispin Sweety, R. Jansi, Fermatean Neutrosophic Sets, International Journal of

 Advanced Research in Computer and Communication Engineering Vol.10, Issue 6, June

 2021.

[3] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems 20(1) (1986), 87-96.

[4] K. T. Atanassov, More on Intuitionistic fuzzy sets, Fuzzy sets and Systems. 1989; 33:37-

 46.

[5] Bhattacharya A. On a measure of divergence of two multinomial populations. Sankhya.

 1946; 7:401-406.

[6] Bustince H, Barrenechea E, Pagola M. Restricted equilance functions, Fuzzy Sets Syst.

 2006; 157: 2333-2346.

[7] Bustince H, Barrenechea E, Pagola M. Image thresholding using restricted equilance

 functions and maximizing the measures of similarity. Fuzzy Sets Syst. 2007; 158:496-

 516.

[8] Bustince H, Barrenechea E, Pagola M. Relationship between restricted dissimilarity

 functions, restricted equivalence functions and normal E-N functions: image thresholding

 invariant. Pattern Recognit Lett. 2008; 29: 525-536.

[9] Fatma Kutlu Gundogu and Cengiz Kahraman, Spherical Fuzzy Sets and Spherical Fuzzy

 Topsis method, Journal of Intelligent & Fuzzy Systems.

[10] Garg H. A novel correlation coefficients between Pythagorean fuzzy sets and its applications to

 decision-making processed. Int J Intell Syst. 2016; 31(12): 1234-1252.

[11] Guiwu Wei and Yu Wei, Similarity measures of Pythagorean fuzzy sets based on the cosine

 function and their applications, Int J Intell Syst. 2018; 33: 634-652.

[12] Hung WL, Yang MS. Similarity measures of intuitionistic fuzzy sets based on Hausdorff

 distance. Pattern Recognit Lett. 2004; 25: 1603-1611.

[13] Hung WL, Yang MS. Similarity measures of intuitionistic fuzzy sets based on Lp metric. Int J

 Approxiamate Reasoning. 2007; 46:120-136.

[14] Hung KC. Applications of medical information: using an enhanced likelihood measured

 approach based on intuitionistic fuzzy sets. IIE Trans Healthcare Syst Eng. 2012; 2(3):224-231.

[15] Lee SH, Pedrycz W, Sohn G. Design of similarity and dissimilarity measures for fuzzy sets on

 the basis of diatance measure. INt J Fuzzy Syst. 2009; 11(2): 67-72.

[16] Li D, Cheng C. New similarity measures of intuitionistic fuzzy sets and application to pattern

 recognition. Pattern Recognit Lett. 2002; 23: 221-225.

[17] Li YH, Olson DL, Zheng Q. Similarity measures between intuitionistic fuzzy (vague) sets:a

 comparative analysis. Pattern Recognit Lett. 2007; 28: 278-285.

[18] Liu HW. New similarity measures between intuitionistic fuzzy sets and between elements. Math

 Comput Modell. 2005;42: 61-70.

[19] Mitchell HB. On the Dengfeng- Chuntian similarity measures and its application to pattern

 recognition. Pattern Recognit Lett. 2003; 24: 3103-3104.

[20] Rajarajeswari P, Uma N. Intuitionistic fuzzy multi similarity measure based on cotangent

 function . Int J Eng Res Technol. 2013; 2(11):1323-1329.

[21] Salton G, McGill MJ. Introduction to Modern Information Retrieval. New York, NY: McGraw-

 Hill; 1983.

[22] Shi LL, Ye J. Study on fault diagnosis on turbine using an improved cosine similarity measure

 for vague sets. J Appl Sci. 2013; 13(10): 1781-1786.

[23] Son LH, Phong PH. On the performance evaluation of intuitionistic vector similarity measures

 for medical diagnosis. J Intell Fuzzy Syst. 2016; 31(3): 15 97-1608.

[24] Szmidt E, Kacprzyk J. Distances between intuitionistic fuzzy sets. Fuzzy Sets Syst. 2000;

 114:505-518.

[25] Szmidt E, Kacprzyk J. A concept of similarity measure for intuitionistic fuzzy sets and its use in

 group decision-making. Lect Notes Comp Sci(Subseries LNAI) 2005; 3558:272-282.

[26] Szmidt E. Distances and similarities in intuitionistic fuzzy sets. Studies in Fuzziness and Soft

 Computing. Berlin, Germany: Springer; 2014.

[27] Tian MY. A new fuzzy similarity based on cotangent function for medical diagnosis. Adv Model

 Optim. 2013;15(2):151-156.

[28] Xu ZS, Xia MM. Some new similarity measures for intuitionistic fuzzy values and their

 application in group decision making. J Syst Sci Eng. 2010; 19: 430-452.

 [29] J. Ye, “Cosine similarity measures for intuitionistic fuzzy sets and their applications”,

 Mathematical and Computer Modelling, Vol. 53, No. 1, pp. 91-97, 2011.

[30] J. Ye. Similarity measures for intuitionistic fuzzy sets based on cosine function for the

 decision making of mechanical design schemes. J Intell Fuzzy Syst. 2016; 30(1): 1597-

 1608.

[31] Zadeh LA. Fuzzy sets. Inform Control. 1965; 8:338-356.