**Darcy-Forchheimer flow of Casson nanofluid past a stretching cylinder in the presence of magnetic field**

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**Abstract**

Heat transfer as well as flow of fluid can be effectively controlled by the magnetic field. Also it can also be used to increase thermodynamic efficiency. The use of porous media and nanofluids are numerous techniques for enhancing heat transfer. Porous media can increase pressure drop in addition to enhancing heat transfer. This article is a numerical study of magnetohydrodynamic (MHD) Darcy-Forchheimer flow and heat transfer of Casson nanofluid past a stretching cylinder embedded in a porous medium in presence of magnetic field. The governing partial differential equations (PDEs) are converted into non-linear ordinary differential equations (ODEs) by using suitable similarity transformations which are solved numerically by effective shooting technique by the help of MATLAB software. The influences of effective parameters on velocity, temperature profiles, skin friction coefficient, and Nusselt number are discussed through graphs and tables.

**Keywords:** MHD,Darcy-Forchheimer flow, Stretching cylinder, Casson fluid.

**1. Introduction**

Casson fluid is a non-Newtonian fluid with yield stress which describes blood flow in narrow arteries. When blood flows from larger diameter arteries at high shear rates it shows Newtonian behaviour. But it exhibits non-Newtonian behaviour when it flows through small diameter arteries at low shear rates. Casson [1] considered the Casson fluid model relating to the flow of blood and perceived that at low shear rates the yield stress for blood is nonzero. Misra and Pandey [2] studied the peristaltic transport of blood in small vessels. Animasaun [3] examined the Casson fluid flow with higher order chemical reaction. Senapati et al. [4] have numerically investigated the Casson nanofluid flow over a stretching sheet: a three dimensional analysis.

The flow and heat transfer past a stretching sheet has many industrial and engineering applications such as cooling of metallic plates, polymer industries, and paper production etc. Bhukta et al. [5] examined the mixed convection flow over a stretching sheet. Swain et al. [6-8] investigated the nanofluid flow, heat and mass transfer over a stretching sheet in presence of chemical reaction. Hayat et al. [9] studied the flow and heat transfer over a stretching cylinder with thermal radiation. Hussain et al. [10] examined the effects of Joule heating and viscous dissipation on Sisko nanofluid over a stretching cylinder. Tamoor et al. [11] studied the flow of Casson fluid over a stretching cylinder. Mahdy and Chamkha [12] and Hayat et al. [13] investigated the flow of Maxwell fluid in non-Darcy porous medium with chemical reaction. Rasool et al. [14] observed the Darcy-Forchheimer nanofluid flow over nonlinear stretching sheet.

The main objective of the present study is to analyse the flow and heat transfer of Casson fluid past a stretching cylinder using Darcy-Forchheimer model. The governing PDEs are converted into coupled non-linear ODEs by using suitable similarity solutions. These ODEs are solved numerically by effective shooting technique by the help of MATLAB software. Numerical calculations are carried out of operating parameters up to desired level of accuracy. The effects of different parameters are discussed through graphs and tables.

**2. Mathematical Formulation**

Consider a steady two dimensional electrically conducting Darcy-Forchheimer flow of Casson fluid past an elongating cylinder in a saturated porous medium. The flow is caused by the linear stretching of cylinder with velocity . The flow is along axial (*x*) direction whereas radial direction is perpendicular to *x*. A uniform transverse magnetic field of strength is applied normally to the surface of cylinder (along *r* direction). The flow is subjected to an additional body force due to the presence of porous matrix. It is assumed that

* the magnetic Reynolds number of the fluid is very small so that the effect of induced magnetic field is neglected.
* there is no slip between fluid and the bounding surface.
* the heat transfer takes care of viscous dissipation due to internal resistance, Joule heating due to a resistance to passage of electric current.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid (Senapati et al. [4]) is expressed as



where is the rate of strain tensor,  is the component of stress tensor,  is the Casson coefficient of viscosity,  is the product of the rate of strain tensor with itself,  is the critical value of the product of the rate of strain tensor with itself,  is the yield stress of the fluid and is the Casson fluid parameter.

Under the boundary layer approximation and above assumptions, the governing equations of continuity, momentum and energy are given by

 (1)

 (2)

 (3)

where are velocity components in *x* and *y* directions respectively, is the magnetic field strength, is the kinematic viscosity, is the electrical conductivity, is the density, is the thermal conductivity, is the temperature of the fluid, is the specific heat, and is the quadratic drag coefficient.

The boundary conditions are

 (4)

where is the reference velocity,  is the characteristic length,  is the heat transfer coefficient, and is the ambient temperature.

Consider the stream function  such that dimensionless similarity transformations 

Therefore, the equation (1) is identically satisfied and equations (2) and (3) convert

 (5)

 (6)

The corresponding boundary conditions (4) become

 (7) where is the Hartman number , is the porosity parameter, is the curvature parameter, is the local inertia parameter, is the Prandtl number, and is the conjugate parameter.

The surface conditions of engineering interest such as the skin friction and Nusselt number are given by andrespectively.

Here wall shear stress and wall heat flux and is the local Reynolds number.

**3. Results and Discussion**

The dimensionless coupled non-linear ODEs (5) - (6) with appropriate boundary conditions (7) are solved numerically by shooting technique using MATLAB software with step length and the error tolerance. During calculation we fix the parameters as and unless otherwise the values are mentioned. The values of is calculated to validate our results with previously published results of Tamoor et al. [11] for various values of as shown in Table 1. It is found that our numerical results are in good agreement.

**Table 1** Comparison of  for various values of when and 

|  |  |  |
| --- | --- | --- |
|  |  | |
|  | Tamoor et al. [11] | Present study |
| 0.0 | 1.00000 | 1.0000083 |
| 0.2 | 1.01980 | 1.0198092 |
| 0.5 | 1.11803 | 1.1180343 |
| 0.8 | 1.28063 | 1.2806249 |
| 1.0 | 1.41421 | 1.4142136 |

Fig. 1 displays the influence of Hartman number  on velocity profile in the presence and absence of porous matrix. It is seen that in absence of porous matrix the velocity of the fluid is higher than that of presence of porous matrix. Further,  declines with an increase in. Physically, in presence of magnetic field generates a resistive force called Lorentz force which deceleratesto contribute a thinner momentum boundary layer. The impact of inertia coefficient  onis presented in Fig. 2 for both Casson fluidand Newtonian fluid. It is seen that Casson parameter enhances the velocity profile, whereas the Forchheimer parameter (local inertia parameter), which is responsible for the inertia drag has a reverse effect on the fluid velocity. Fig. 3 portrays the outcome of radius of curvatureon. The velocity profile slightly declines with higher values of near the cylinderbut increases with increase values of. The same observation was made by Hayat et al. [9].

From Fig. 4 it is observed that the temperature profileenhances with increase values ofand . Physically, Lorentz force resists the fluid flow and consequently, causes extra heat producing a thicker thermal boundary layer. Fig. 5 shows the behaviour of on  for both Casson fluidand Newtonian fluid. It is perceived that decreases with an increase in. Physically, higher Prandtl number fluid having lower thermal diffusivity which reduces the conduction and consequently thermal boundary layer decreases. Moreover, Casson fluid has lower velocity than that of Newtonian fluid. Fig. 6 describes the influence of curvature parameter and conjugate parameteron. It is seen that as  increases, more amount of heat is transferred from heated surface of the cylinder to cooled surface of the fluid. Consequently, thickness of thermal boundary layer enhances but the adverse effect is observed on the fluid temperature in case of.



Fig. 1 Effect of andon velocity profile



Fig. 2 Effect of on velocity profile



Fig. 3 Effect of on velocity profile



Fig. 4 Effect of andon temperature profile



Fig. 5 Effect of  on temperature profile



Fig. 6 Impact of and on temperature profile

**Table 2** Computation of and when 

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 0.1 | 0.1 | 0.1 | 2.325000 | 1.497189 |
| 0.3 |  |  | 2.377730 | 1.493493 |
| 0.5 |  |  | 2.480170 | 1.486296 |
|  | 0.5 |  | 2.638612 | 1.477482 |
|  | 1 |  | 2.824700 | 1.467178 |
|  |  | 0.3 | 3.091958 | 1.904015 |
|  |  | 0.5 | 3.354159 | 2.237804 |

Table 2 shows the computed values of skin friction coefficientand local Nusselt numberfor various values of operating parameters such as and. It is perceived that and enhance the skin friction coefficient but the rate of heat transfer at the surface decreases with increasing values ofand  but increases with. Thus there exists Reynolds analogy between skin friction and surface flux [15]. The same observation was made by Hussain et al. [10] and Tamoor et al. [11].

**4. Conclusions**

From the present study the major finding are:

* Higher values of Hartman number declines the velocity profile but enhances the thermal resistance that leads to enhance the temperature profile.
* Casson parameter enhances the velocity profile, whereas the Forchheimer parameter, which is responsible for the inertia drag has a reverse effect on the fluid velocity.
* and enhance the skin friction coefficient but the rate of heat transfer at the surface decreases with increasing values ofand  but increases with.
* The temperature profile decreases with enlarging values of Prandtl number.

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