# **MUCUS FLOW IN HUMAN LUNG AIRWAYS BY USUNG MATHEMATICAL methods**

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**Abstract:** Mucus flow in human lung airways can be modelled using mathematical equations that describe the transport and properties of the mucus. There are various approaches to modelling mucus flow, but a common one is to use the Navier-Stokes equations, which describe the motion of fluids. The Navier-Stokes equations can be applied to the mucus layer in the airways, considering it as a non-Newtonian fluid. Non-Newtonian fluids have complex rheological properties that are dependent on the applied stress and strain rate. The mucus layer in the airways is a viscoelastic fluid, which means it has both viscous and elastic properties.

***Keywords:*** *Mucus flow, human lung airways, mathematical modelling, Navier stokes equation.*

# **Introduction**

Mucus flow in the human lung airways has been studied by several investigators [Lucas and Douglas (1934),Wilkision (1960), Weibel (1963), Barton and Raynor (1967), Schroter and Sudlow (1969), Pedley et al.(1970), Blake (1971a, 1971b, 1973, 1974, 1975), Clarke et al. (1970),Clarke (1973), Ross and Corrsin (1974), Sleigh (1977,1990), Scherer and Burtz (1978), Blake and Winet (1980), Winet and Blake (1980), Yeats et al.(1980,1981), Wanner (1981), Blake and Fulford (1984), Puchelle et al.(1980), Sleigh et al. (1988), Zahm et al. (1989,1991), Agarwal et al. (1989), King et al. (1982,1985,1989,1993,1995), Bennet et al. (1990), Mogami et al. (1992), Tomkiwicz et al. (1994), Agarwal and Verma (1997,1998), Kim (1997), Rubin (2002), Verma (2007,2009,2010,2011,2012), Satpathi (2007), Smith et al.(2008), Polak (2008), Benjamin (2011)].

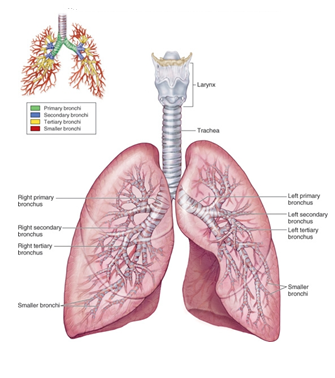


Figure 1: A general structure of Human Lung

Barton and Raynor (1967) presented an analytical model for mucus transport by considering cilium as an oscillating cylinder with a greater height during the effective stroke and smaller height during the recovery stroke. Blake (1975) considered a two-layer Newtonian fluid model, one serous layer fluid and the other mucus and pointed out the importance of gravity and effect of air flow on mucus transport. Another mathematical analysis of two-layer fluid model is given by Blake and Winet (1980). They suggested that if cilia just penetrate the upper, much more viscous layer , then the mucus transport rates would be substantially enhanced.

Though the air flow resistance in bronchial airways has been studied by Schroter and Sudlow (1969) and Pedley et al.(1970), the role of mucus interaction with mucus in bronchial clearance has been emphasized by Clarke (1973), Clarke et al. (1970) and many others, including Puchelle et al. (1983), Zahm et al. (1989), King et al. (1982,1985,1989) in their experimental studies. Scherer and Burtz (1978) conducted experiments relevant to coughing and showed the importance of viscosity of the fluid. King et al. (1982) also studied the interaction of airflow with the mucus gels in a simulated cough machine under study state and oscillatory airflow conditions and pointed out the importance of viscosity of mucus gel on transport. Agarwal et al. (1989) have studied the mucus transport by airflow interaction in a miniaturized simulated cough machine and found that mucus transport increases as the viscosity of the serous layer simulant decreases or as the mucus filance (spinnability) decreases.

**MATHEMATICAL MODELLING**

It may be noted here that very little attention has been paid to explain these experimental observations using mathematical models. Verma (2007) presented a two layer planar steady state mathematical model to study mucus transport in the respiratory tracts due to airflow and formation of cilia porous bed in serous sub- layer in contact with epithelium. The effect of airflow was considered by prescribing air velocity at the mucus air interface. It had been shown that mucus transport increases as the pressure drop or force due to gravity, air velocity and porosity parameter increase. It was also observed that mucus transport increases as the viscosity of mucus or serous layer fluid decreases.

In recent decades, the mucus flow in human lung airways has been studied by several investigators. Agarwal and Verma (1997) and Verma (2007, 2010) have studied the mucus flow by taking the effect of porosity due to formation of porous matrix by immotile cilia.

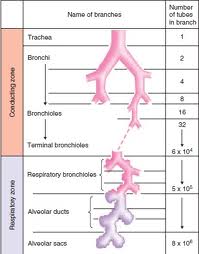


Figure 2: A schematic diagram of human lung

The flow of viscous fluids over and through porous media has been the subject of intensive studies during recent decades because of its natural occurrences and importance in many problems.

The flow of mucus is governed by the mechanical forces of cilia beating and airflow, which are counter-acted by frictional and inertial forces of the mucus. The bronchial secretions form at least two-layers possibly three. The cilia reside in the periciliary layer (serous layer) and the mucus layer lies at the top of the cilia. It is hypothesized that the serous and mucus layers might be separated by a layer of surfactant. Probably only the mucus layer fluid is transported, but serous layer is essential for mucus transport because it provides the conditions necessary for the cilia to beat effectively. The flowing surface area at a given level in the bronchial tree is determined by the number and diameter of the airways. From the central to the peripheral airways, the airway diameter decreases and the number of airways increases exponentially, so the total airways diameter and the flowing surface decrease from the peripheral to the central airways and somewhat reduced at bifurcations.

To investigate the flow of serous fluid and mucus in the human respiratory tracts due to beating activity of cilia of length L, we suppose that one of the tubes in the lung under consideration is cylindrical having its radius R. The total depth of fluids (serous fluid and mucus) at the top of the cilia is H as shown in Fig 3:

In the normal state of the lung, the ratio is very small i.e. of the order of 10-2, being the same order of magnitude as. But in diseased state, the ratio may become as large as, or even greater than one half. In all airways of the lung, the ratio is always very small i.e. of the order of 10-2, therefore, we can approximate the cilia sub-layer and mucus layer as shown in Fig.4.

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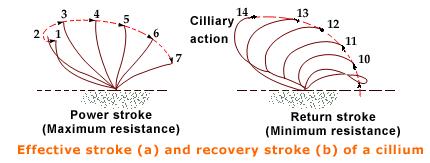
**Fig.3. Circular model of mucus flow in the human Lung airways.**



**Fig.4 Planar model of mucus flow in the human lung airways**

Cilia beating consist of an effective stroke (power stroke), in which the extended cilia sweep through a large volume of fluid and a recovery stroke (return stroke), in which the cilia bent and perform an unrolling movement across the cell surface. The Fig.5 shows the ciliary beat activity:

In the effective stroke, the cilium moves in a plane nearly perpendicular to the cell surface, but in most cases it moves one side in the recovery stroke, keeping near the cell surface, increasing the difference in height between the two strokes and maximizing the net flow of fluid by reducing the back-flow produced by the recovery (return) stroke.

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**Fig 5 Ciliary beat activity**

( <http://www.tutorvista.com/content/biology/biology-iii/cell-organization/nonmembranous-cell-organelles.php> )

Most serous layer fluid propelling cilia perform three-dimensional cycle, usually cilia move counter clockwise in the recovery stroke.

Serous layer fluid propelling cilia seldom stop moving the effective and recovery strokes merging together where the transitions occur between them, so that instantaneous pressure show all cilia in some phase of active movement. By contrast, mucus propelling cilia normally rest between beats and this rest takes place after the effective stroke has been completed, so that the tip of the resting cilium points in the direction of the mucus flow from this rest position, the curve of cilia movement moves through a clockwise recovery stroke, which leads directly into the effective stroke.

Cilia that propel serous layer fluid and those that propel mucus commonly beat at frequencies of between 10 to 20 Hz, although frequencies above 50 Hz have been recorded in some water propelling examples. The main propulsive thrust of the effective stroke is imparted to serous layer fluid or mucus by the distal part of the cilium. At the same frequency, the tip velocity of a serous layer fluid propelling cilium is higher than that of a mucus propelling cilium because the former is longer, although the proportion of the cycle time occupied by the effective stroke tends to be smaller for a mucus propelling cilium because of the rest phase. Because of its visco-elastic nature, the transport rate of mucus by cilia beating is similar to the tip velocity of cilia concerned, where as serous fluid is less viscous and is transported at only about one fourth of the tip velocity of the cilia. The flow rate of mucus to be relatively independent of the load but under some conditions the effective strokes of the cilia may be slowed and transport rates fall.

The closely packed cilia on a mucus-flowing epithelium rest in each cycle and each beat begins with a recovery stroke. Before any movement occur the cilia of an area lie still with the cilia bent over in the direction of mucus flow [Sleigh et al. (1972)].

The relevant equations of motion for the flow of fluid under consideration are dependent on the Reynolds number which gives an indication of the relative importance of inertial and viscous effects. While investigating cilia induced flow, the Reynolds number of basic importance is the Cilium Reynolds number which is defined as follows:

(1)

where is the angular frequency, is the length, is the characteristic radius of cilium, while is the kinematic viscosity. In the respiratory tract, the Cilium Reynolds number is always very small i.e. of the order of 10-4 indicating that the viscous effects are more important. The equations of motion, called the Navier Stokes flow equations at low Reynolds number are defined as:

and **(2)**

where is the pressure, is the velocity vector and the dynamic viscosity which we relate with the kinematic viscosity as follows :

, being the density. (3)

In our case when cilia become immotile and form a porous matrix bed (or porous medium), the fluid flows through two regions. Region-I, where there is no porous medium and fluid flows freely, is called free flow region. Region-II, is called as porous region, where fluid flows through the pores of the porous medium. Flows through these two regions are matched by suitable boundary conditions at the interface called as matching conditions. The mathematical theory of the flow of a viscous fluid through a porous medium was established empirically by Darcy and is known by his name as Darcy’s law. Since then, Darcy’s law has been verified experimentally by several workers. In our case, we shall use the boundary condition as proposed by Beavers and Joseph (1967) at the interface under consideration. We are interested to study mucus transport in the respiratory tracts of the lung which will give us a better understanding of hydro-dynamical and some physiological aspects of mucus transport in the human lung airways. We shall use the principles and theories of Fluid Mechanics while discussing the mucus flow problems. It is well known that the flow problems concerning the motion of incompressible viscous fluids are governed by the Navier-Stokes equations

(4)

Along with the continuity equation

(5)

where***q*** is the velocity vector, ***p*** the pressure, F the external body force, the kinematic coefficient of viscosity. The second term in the L.H.S.νdensity and of equation (4) viz. ***q.gradq***, the convective part of the inertia term, is non-linear in q and makes the solution of the equations a difficult task. Therefore, we shall consider some slow motion problems by writing equation (2) in non-dimensional form as given below:

(6)

Where is Reynolds Number L being the Characteristic length and is characteristic velocity. Thus if Reynolds number approaches to zero we get the following equation

(7)

We find the solution of above equations using certain boundary and matching conditions either by applying simple methods of solving partial differential equations or by any other method e.g. the integral transform method. After getting the solution, we shall interpret the results which will be helpful in gaining insight into the pathogenesis of the respiratory diseases by applying the different aspects of mucus clearance altered in health and disease or by physical or pharmacologic intervention. We may gain tools for diagnosis, prognosis and for evaluation of therapy for some airway diseases and thus have much intrinsic scientific interest.

**METHODOLOGY**

The flow of viscous fluids over and through porous media has been the subject of intensive studies during recent decades because of its natural occurrences and importance in many problems. In our case when cilia become immotile and form a porous matrix bed (or porous medium), the fluid flows through two regions. Region-I, where there are no porous medium and fluid flows freely, is called free flow region. Region-II, is called as porous region, where fluid flows through the pores of the porous medium. Flows through these two regions are matched by suitable boundary conditions at the interface called as matching conditions [Fig .4].

The mathematical theory of the flow of a viscous fluid through a porous medium was established empirically by Darcy and is known by his name as Darcy’s law. Since then, Darcy’s law has been verified experimentally by several workers.

In our case, we shall use the boundary condition as proposed by Beavers and Joseph (1967) at the interface under consideration.

**(i) Steady State Planar Model**

Let an incompressible viscous fluid be in steady motion bounded by planes and . Let the plane .i.e- axis be at rest while plate has a velocity along - axis. If be the fluid velocity at point **P** (), then,

**(12)**

Therefore, from continuity equation (1) , we can write

**(13)**

so that is independent of . Also, by symmetry is independent of .

Consequently **(14)**

Again, from Navier- Stokes equations of motion (4), in absence of body forces, we have

**(15)**

Since motion is steady state, therefore, we have

Thus we have [by (13) and (14)]

Now, equation (15) becomes

or **(15)**

This is equivalent to the following equations:

**(16)**

**(17)**

**(18)**

From (1.16) and (1.17), it is clear that is a function of only i.e.

Therefore, from equations (1.12) and (1.15) we can write:

**(19)**

From equation (15), it is seen that L.H.S. is a function of only while R.H.S. is a function of only. Hence, each side is a constant. As the fluid is flowing in a positive direction of , the pressure decreases as increases so that

Thus , we may take

**(20)**

This implies that **(21)**

Using conditions (i) and (ii), ,equation (21) may be written as :

**(22)**

This shows that the velocity profile between the two plates is parabolic.

The volumetric transport rate is given by

**(23)**

or **(26)** .

**(ii) Steady State Circular Model**

Let an incompressible viscous fluid be in steady motion in a cylindrical pipe. We take axis of along the axis of cylinder. Also suppose that the direction of flow is parallel to axis so that . In this case, equation of continuity is given by

**(25)**

Navier Stokes equations of motion in absence of body forces become:

**(26)**

Since motion is steady state, therefore, we have **(27)**

Thus, we have

Now, equation (26) becomes

or **(28)**

This is equivalent to the following equations:

, , **(29)**

From equations (26), (28) and (29), we can write:

**(30)**

Inspection of (30) shows that L.H.S. is a function of only while R.H.S. is a function of . Hence, each side is a constant. Further, since fluid is moving along positive direction of axis, the pressure should decrease as increases. Thus, we have

Thus, we may take **(31)**

Changing the above equation into cylindrical co-ordinates using the transformations we obtain

**(32)**

From (32), we can also write or **(33)**

Integrating (33), we have

**(34)**

**CONCLUSION**

The study of mucus flow in human lung airways involves the application of various mathematical concepts and models. One of the primary mathematical tools used in this field is fluid mechanics, which deals with the behavior of fluids in motion. Specifically, researchers use the principles of fluid dynamics to understand the movement of mucus through the airways.

There are several mathematical models used to describe mucus flow in the airways. One of the most common is the two-phase model, which considers mucus as a non-Newtonian fluid that is transported by a layer of air flowing through the airways. This model takes into account the complex rheological properties of mucus, which makes it difficult to move through the narrow airways.

Other mathematical models include the continuum model, which assumes that the air and mucus are continuous fluids, and the discrete particle model, which treats mucus as a collection of discrete particles. Researchers use these models to study different aspects of mucus transport, such as the effects of cilia movement and mucus composition on transport rates.

In addition to fluid mechanics, mathematical tools such as computational fluid dynamics (CFD) and finite element analysis (FEA) are used to simulate the transport of mucus in the airways. These tools allow researchers to visualize and analyze mucus transport under different conditions and to test different hypotheses about the underlying mechanisms.

Overall, the mathematical study of mucus transport in human lung airways is a complex and interdisciplinary field that requires the integration of multiple disciplines, including mathematics, physics, biology, and engineering. However, advances in mathematical modeling and simulation techniques are helping researchers gain a better understanding of this crucial biological process and develop new treatments for respiratory diseases.

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