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Non-stop integral flow in lateral gap distance lattice model

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Abstract The role of non-stop integral flow is studied in a lattice model by assuming the lateral gap among the lattice sites. The proposed model is investigated theoretically as well as numerically. In theoretical evaluation, we derived the stableness criterion and provided the relationship among sensitivity and other parameters. It's far located that similarly to attention of the gap space, the non-stop time of flow reduces the congestion and the unstable region more reinforced via increasing the driver's memory time step.

Keywords Lattice · Non-lane-based · Flow integral · Traffic

1 Introduction

To expose the traffic problems including intrinsic mechanism of traffic congestion, commuting delay, traffic accidents and energy consumption, the modeling of traffic flow has attracted a widespread interest of researchers in latest years. Most of the traffic techniques especially recognition at the reproducing the flow-density-velocity relationship and the phase transition of traffic flow from congested region to free flow region with involving various factors of traffic [1–7]. Also, in order to reveals the actual traffic conditions, a few research have been added to suppress the traffic congestion. These days, the lattice hydrodynamic model which was firstly proposed by Nagatani [8], stimulates a huge interest of many researchers.

As we understand, road condition performs an important function in traffic flow which include narrow lane, curves, and bad road surface make drivers pay more attention on road conditions and decrease their velocity. One of the primary reason behind traffic bottlenecks are due to bad road conditions. In this direction, car-following traffic flow models were proposed through[9–11] with the aid of assuming that vehicles travel in the center of the lane which can be stimulated directly by the only in front

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or behind and no passing is permitted on a single lane highway. By inspiring from Refs.[9–11], the impact of lateral gap has been also studied in lattice model [12] and it is determined that lateral separation performs a critical role in stabilizing the traffic flows in lattice fashions

In real traffic, driver usually observe at the following in addition to the previous vehicles at some point of driving on road. To deal with this phenomena, many lattice models [13–22] had been found within the literature. Currently, to show the impact of historical traffic information, Wang and Ge [23] proposed a lattice model via accounting the backward looking and flow integral effect and it is observed that the stable region enhances efficiently with consideration of these factors. Motivated from this, Peng et al. [24] studied the flux difference memory integral effect in two-lane lattice version and it's far encountered that lane changing performs a vital function in stabilizing the traffic congestion. The continuous memory of vehicles plays an important role in traffic flow modeling and this effect becomes more prominent in non-lane-based lattice model. However, the lateral separation distance of consecutive automobiles has not been studied in driver's continuous memory integral lattice model.

The paper is prepared as follows: we study the lateral separations gap between two consecutive automobiles and presented a lateral-gap-distance lattice model via thinking about the effect of flow memory integral in section 2. In section 3, the model's stability condition is derived the usage of the linear stability theory. Then, numerical simulation is completed out to validate the analytic outcomes and subsequently conclusion is given in final section.

2 Proposed Model

The lattice version of continuum model through considering the idea of car-following model is

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0, \tag{1}$$

with the following flow evolution equation at site j

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j]. \tag{2}$$

where $a = \frac{1}{\tau}$ is the sensitivity; ρ_0 is the average density; $V(.)$ is the optimal velocity function; ρ_j and v_j represent the density and velocity at site j at time t, respectively. Furthermore, to include the lateral separation distance, Peng et al. [12] proposed a lattice hydrodynamic model as follows

$$\rho_j(t+\tau) - \rho_j(t) + \tau\rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0 \tag{3}$$

$$\rho_j(t+\tau) v_j(t+\tau) = \rho_0 V(\rho_{j+1}, \rho_{j+2}) + \kappa G(\triangle Q_{j,j+1}, \triangle Q_{j,j+2}) \tag{4}$$

where κ is the reactive coefficient to the function $G(.)$, $\triangle Q_{j,j+1} = \rho_{j+1} v_{j+1} - \rho_j v_j$, and $\triangle Q_{j,j+2} = \rho_{j+2} v_{j+2} - \rho_j v_j$ are the relative flows among site j & $j+1$ and j & $j+2$, respectively. It's far observed that the free region enhances with an increasing the lateral separation distance of lane width and consequently, this element plays an important role in stabilizing the traffic flow.

As we recoinage, driver usually senses the traffic relative information at time t and makes a decision to adjusts speed of his vehicle at later time, in taking this movement there can be delay which influences the traffic. In this course, Gupta and Redhu [26] proposed a hydrodynamics model by thinking about the driver's anticipation effect in sensing relative flux for two-lane system for a fixed delay and studied the impact of driver's expectation on traffic flow. But, it is obvious that the effect of continuous memory has more prominent affects on traffic flow in comparison of fixed delay time and it also investigated in many traffic flow models [23,27]. In literature, we studied that road width performs an essential role in stabilizing the traffic congestion and it will becomes more effective if driver could have the relative records of continuous memory. But, the effect of non-stop memory integral has not been studied untill now.

Here, we are offering a lattice model by considering the continuous historical flux information in term of integration between the time $[t-\tau_0,t]$ and the new evolution is

$$\rho_j(t+\tau)v_j(t+\tau)=\rho_0[V(\rho_{j+1},\rho_{j+2})]+\kappa\int_{t-\tau_0}^{t}G(\triangle Q_{j,j+1}(s),\triangle Q_{j,j+2}(s))ds \quad (5)$$

where τ_0 represents the historical integral time, κ is the corresponding coefficient, $G(.)$ is given by

$$G(\triangle Q_{j,j+1}(s),\triangle Q_{j,j+2}(s)=(1-p_j)\triangle Q_{j,j+1}(s)+p_j\triangle Q_{j,j+2}(s) \quad (6)$$

and

$$V(\rho_{j+1},\rho_{j+2})=V[(1-p_j)\rho_{j+1}+p_j\rho_{j+2}] \quad (7)$$

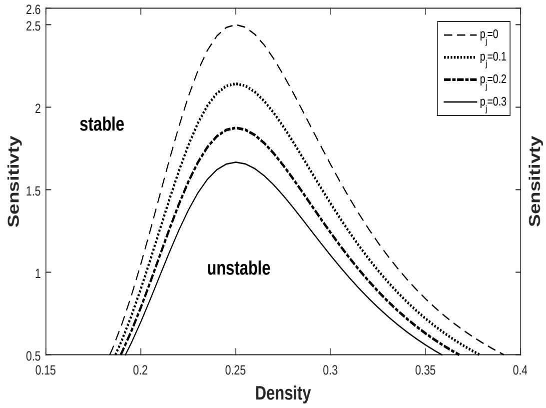
where $p_j=\frac{LS_j}{LS_{max}}$ is the parameter of lateral separation distance, LS_j is the lateral separation distance of sites j and $j+1$ and LS_{max} is the maximum lateral separation distance. The term $\int_{t-\tau_0}^{t}G(\triangle Q_{j,j+1}(s),\triangle Q_{j,j+2}(s))ds$ represents the continuous flux difference information. The modified velocity function for non-lane-based model is

$$V(\rho_{j+1},\rho_{j+2})=\frac{v_{max}}{2}\Big\{tanh\Big[\frac{1}{(1-p_j)\rho_{j+1}+p_j\rho_{j+2}}-\frac{1}{\rho_c}\Big]+tanh\Big(\frac{1}{\rho_c}\Big)\Big\} \quad (8)$$

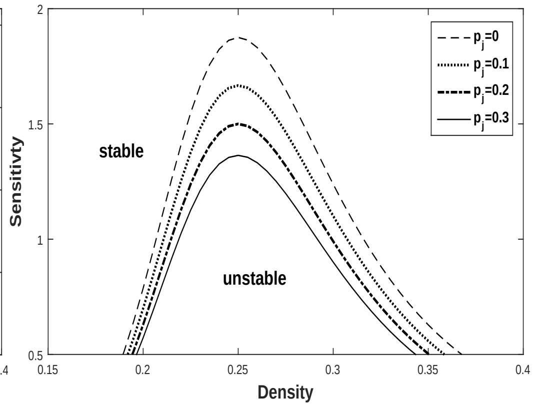
By taking the difference form of Eqs. (1) and (5) and eliminating speed v_j, the density equation is obtained as

$$\rho_j(t+2\tau)-\rho_j(t+\tau)+\tau\rho_0^2[V(\rho_{j+1},\rho_{j+2})-V(\rho_j,\rho_{j+1})]+\tau\kappa[(1-p_j)(-\rho_{j+1}(t)+\rho_{j+1}(t-\tau_0)+\rho_j(t)-\rho_j(t-\tau_0))+p_j(\rho_{j+2}(t)+\rho_{j+2}(t-\tau_0)+\rho_{j+1}(t)-\rho_{j+1}(t-\tau_0))]=0 \quad (9)$$

where $\tau_0=k\tau$, where τ_0 and $k=1,2,3\cdots$ represent the difference time step and integer for the historical time considered.



(a)



(b)

Fig. 1 Phase diagram in parameter space (ρ, a), for (a) $\kappa = 0.1$ and (b) $\kappa = 0.3$, respectively.

3 Linear stability analysis

To look the effect of memory flow integral in the proposed model, we assume the steady-state solution of the homogeneous traffic flow as

$$\rho_j(t) = \rho_0, \quad v_j(t) = V(\rho_0). \tag{10}$$

where ρ_0 and $V(\rho_0)$ represent the state of uniform traffic flow. Let $y_j(t)$ be a small perturbation to the steady-state density on site j. Then,

$$\rho_j(t) = \rho_0 + y_j(t). \tag{11}$$

Putting this perturbed density profile into Eq. (9) and linearizing it, we get

$$\begin{aligned} & y_j(t+2\tau) - y_j(t+\tau) + \tau\rho_0^2 V'(\rho_0)[(1-p_j)(y_{j+1} - y_j(t)) + p_j(y_{j+2} - y_{j+1})] + \\ & \tau\kappa[(1-p_j)(-y_{j+1}(t) + y_{j+1}(t-\tau_0) + y_j(t) - y_j(t-\tau_0)) + p_j(y_{j+2}(t) + y_{j+2}(t-\tau_0) \\ & + y_{j+1}(t) - y_{j+1}(t-\tau_0))] = 0 \end{aligned} \tag{12}$$

Substituting $y_j(t) = exp(ikj + zt)$ in Eq. (12), we obtain

$$\begin{aligned} & e^{2\tau z} - e^{\tau z} + \tau\rho_0^2 V'(\rho_0)[(1-p_j)(e^{ik} - 1) + p_j(e^{ik-\tau_0 z} - e^{-\tau_0 z})] + \tau\kappa[(1-p_j) \\ & (-e^{ik} + e^{ik-\tau_0 z} + 1 - e^{\tau_0 z}) + p_j(e^{2ik} + e^{2ik-\tau_0 z} + e^{ik} - e^{ik-\tau_0 z})] = 0. \end{aligned} \tag{13}$$

Inserting $z = z_1(ik) + z_2(ik)^2 \ldots$ into Eq. (13), we will obtain the first-order and second-order terms of the coefficient ik and $(ik)^2$, respectively, we get

$$z_1 = -\rho_0^2 V'(\rho_0), \tag{14}$$

$$z_2 = -\frac{3\tau z_1^2}{2} - \frac{\rho_0^2 V'(\rho_0)}{2}(1 + 2p_j) + \kappa\tau_0. \tag{15}$$

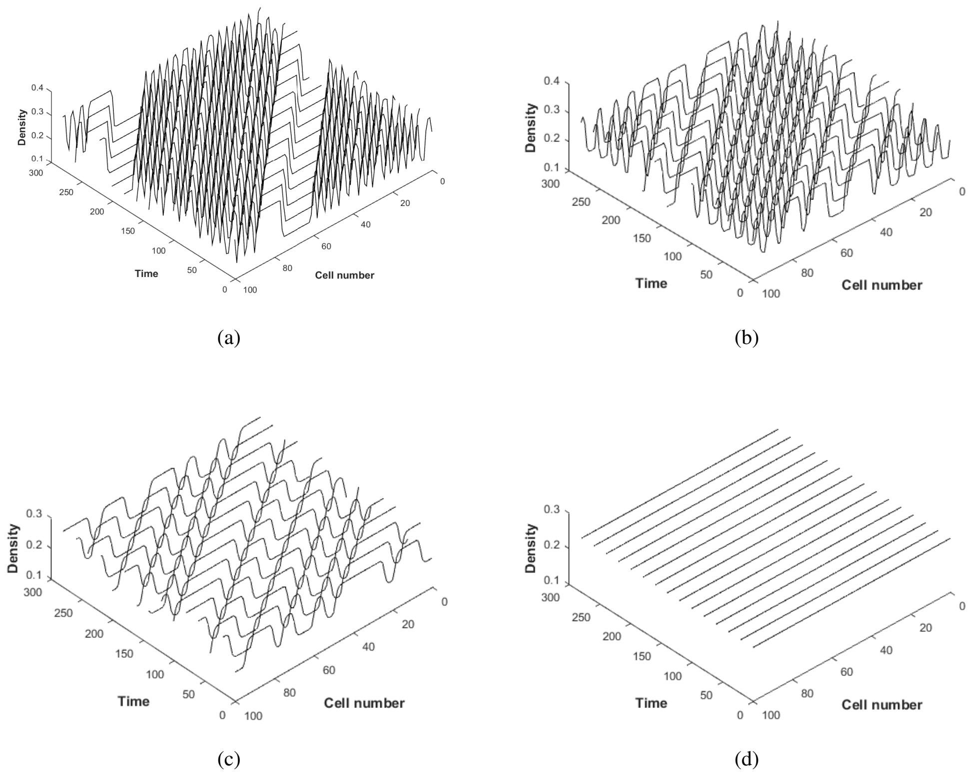


Fig. 2 Space time evolution after time $t = 20000$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, for $\kappa = 0.1$.

When $z_2 < 0$, the uniform steady-state flow becomes unstable for long-wavelength waves. For $z_2 > 0$ the uniform flow becomes stable. Thus, the stability condition for the steady-state is

$$\tau = -\frac{1+2p_j+2\kappa\tau_0}{3\rho_0^2 V'(\rho_0)}. \tag{16}$$

The instability condition for the homogeneous traffic flow can be described as

$$\tau > -\frac{1+2p_j+2\kappa\tau_0}{3\rho_0^2 V'(\rho_0)}. \tag{17}$$

For $\kappa = 0$, and $p_j = 0$, the above unstability criteria (Eq. 17) will becomes same as that of Nagatani's [8] model.

Figure 1 shows the phase digram in the parameter space (ρ, a) for different values of p_j. It is clear form Fig. 1(a) that the amplitude of the neutral stability curves decreases with an increases in the value of p_j when $\kappa = 0.1$. Further increase in the value of κ, stable region enhances with an increase in the value of p_j. On comparing

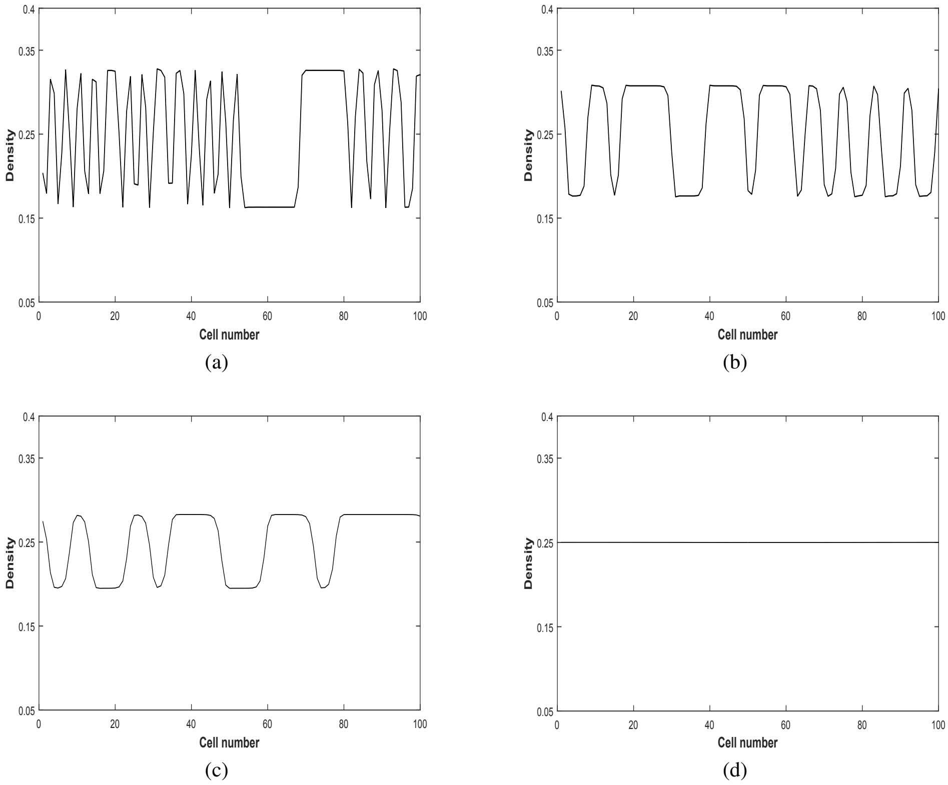


Fig. 3 Density profile at time $t = 20300$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, respectively for $\kappa = 0.1$.

the results for $\kappa = 01.$ and $\kappa = 0.3$, it is concluded that the stable region expands with increase in the value of κ which further enhances with the increment in the value of p_j. If we compare our result with the Peng et al. model [25] for $\kappa = 0.1$ it is concluded the the stable region is more in proposed model which shows that the continuous delayed of flow integral plays a effective role in stabilizing the traffic flow.

4 Numerical Simulation

In this section by using the periodic boundary conditions, we carried out the numerical simulation to check the theoretical results. The initial conditions are adopted as follows:

$$\rho_j(1) = \rho_j(0) = \begin{cases} \rho_0; & j \neq \frac{L}{2}, \frac{L}{2}+1 \\ \rho_0 - \sigma; & j = \frac{L}{2} \\ \rho_0 + \sigma; & j = \frac{L}{2}+1 \end{cases}$$

where, σ is the initial disturbance, L is the total number of sites taken as 100 and other parameters are set as follows: $\sigma = 0.1, \tau = \frac{1}{a}$.

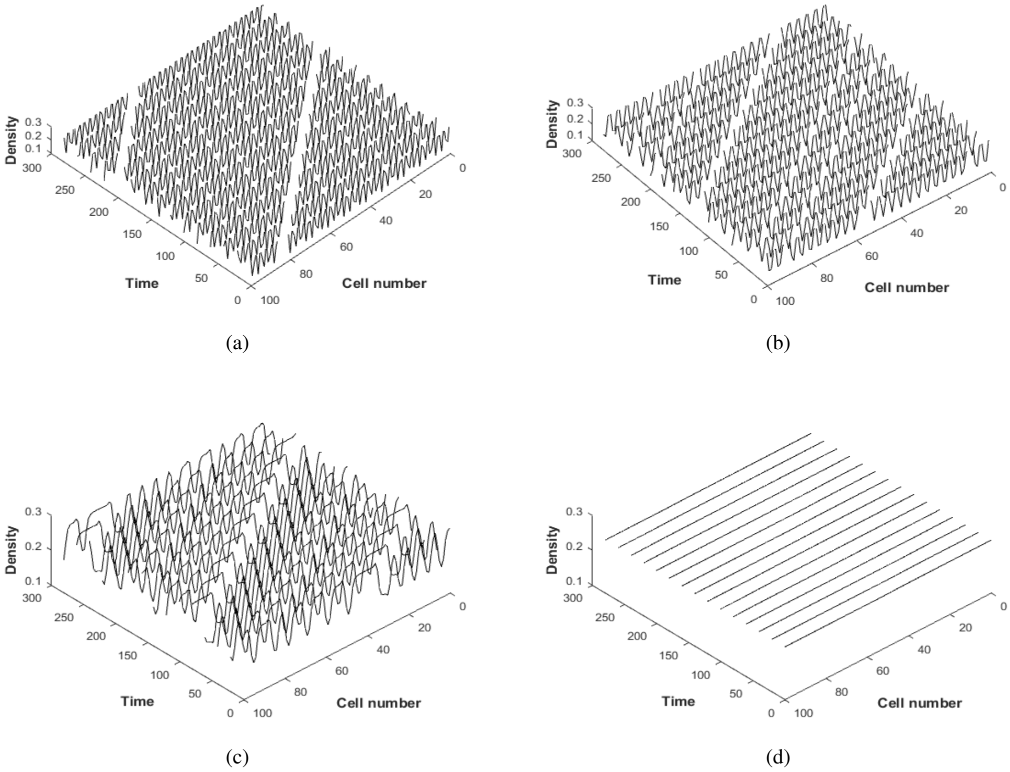


Fig. 4 Space time evolution after time $t = 20000$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, when $\kappa = 0.3$.

Fig. 2 represents the spatiotemporal evolutions of density waves at time $t = 20000s - 20300s$ for different values of p_j when $a = 1.7$ and $\kappa = 0.1$. In the unstable region, the traffic jam appears in term of kink-antikink types of density wave which arise at each site and propagates in the backward direction with time as shown in the Figs. 2(a)-(c). For $p_j = 0.3$, we enter into the stable region and the density waves disappear and traffic flow becomes uniform. Fig. 3 shows the density profile after a sufficiently long time $t = 20300$ corresponding to panel of Fig. 2. It is clear form the Figs. 3(a)-(c) that the amplitude of the kink-antikink density wave decreases with increasing the value of p_j and the flow becomes uniform for $p_j = 0.3$.

Fig. 4 represents the spatiotemporal evolutions of density waves at time $t = 20000s - 20300s$ for different values of p_j when $a = 1.52$ and $\kappa = 0.3$ and Fig. 5 shows the density profile after a sufficiently long time $t = 20300$ corresponding to panel of Fig. 4. It is clear from the Fig. 4(a)-(c) that in the unstable region, the initial disturbance converts into the density waves and these density waves dies out in the free flow region as shown in Fig. 4(d). In the congested region, the deviation occurs around the critical density as shown in Figs. 5(a)-(c) and this deviation disappears

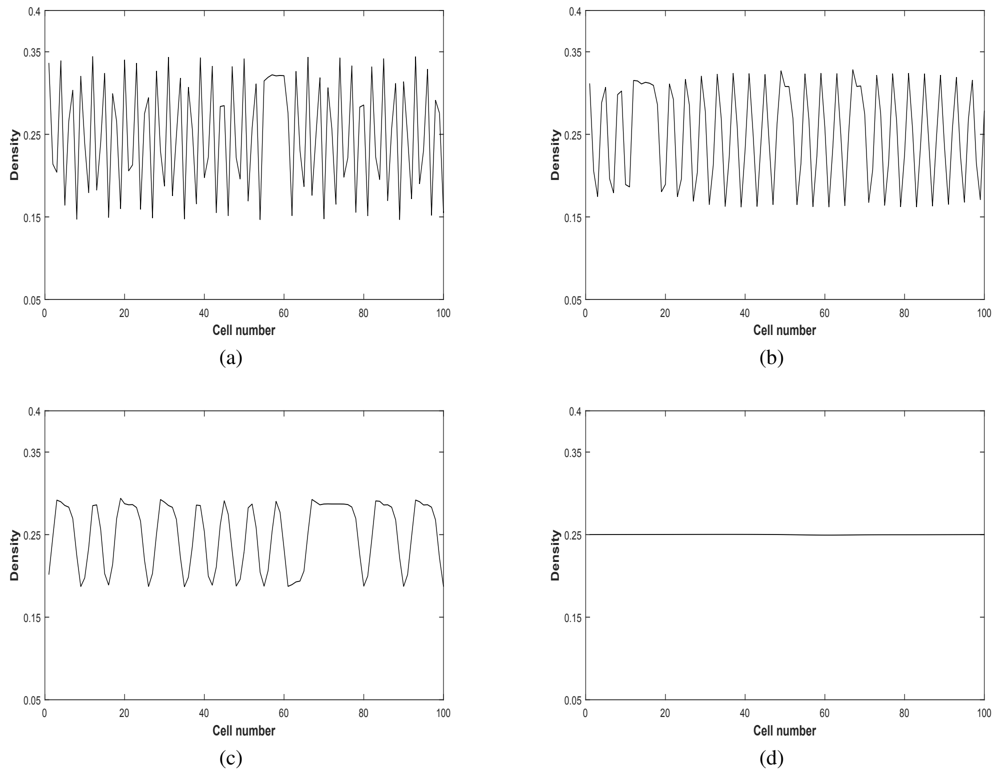


Fig. 5 Density profile at time $t = 20300$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, respectively for $\kappa = 0.3$.

in Fig. 5(d). Therefore, we can conclude that the lateral separation distance plays a prominent role in stabilizing the traffic flow.

On comparing the results for $\kappa = 0.1$ and $\kappa = 0.3$, it is conclude that the information of continuous memory integral plays an important role in traffic flow theory and its affect becomes more impressive in non-lane-based lattice hydrodynamic model.

5 Conclusion

A non-lane-based lattice hydrodynamic traffic flow model is proposed with consideration of continuous flow integral effect. Through linear stability analysis, the stability condition is derived to analyze the traffic congestion region. To validate the theoretical results, numerical simulation is carried out with periodic boundary conditions. For fix values of κ, we studied the affect of lateral separation distance on traffic flow and it is concluded that the coefficient of flow integral effect stabilizes the traffic flow and this factor should be considered in traffic flow modeling.

Conflicts of Interest

Te authors declare that there are no conficts of interest regarding the publication of this paper.

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