Parametric study of phenomenological Nuclear Potential

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Abstract: We have constructed an optical potential by considering a family of potentials developed by J. N. Ginocchio. This phenomenological potential is used for analysing experimental data of nuclear scattering and fusion. The potential enables us to reproduce angular distributions of experimental scattering cross-sections in wide angular ranges. Our proposed nuclear potential deals in ten parameters. The study of change in potential shape with variation of parameters educates us to theoretically reproduce experimental results and explain scattering cross-sections and fusion cross-sections of heavy-ion collision systems.

Key Words: heavy-ion collision, optical potential, potential parameters

I INTRODUCTION

The nucleus-nucleus collision or nucleon-nucleus collision is a many-body system of complex nature. It is not easy to handle such many-body systems. That is why it should be reduced to one body problem for easy handling. Therefore we describe the nucleus-nucleus collision by means of a one-body complex potential. The complex potential is known as *optical potential* [1, 2]. The many-body nuclear interaction is thus replaced with a complex nuclear potential. The Schrodinger equation can be easily solved for the effective potential of two colliding (target-projectile) nuclei in optical potential, but it is difficult to solve Schrodinger equation of a many-body system of complex nature such as nucleus-nucleus or nucleon-nucleus collisions.

There are three broad regimes of energy in which people perform experimental works, namely, (a) low and medium energy regime, (b) intermediate energy regime and (c) high energy regime. Most of the nuclear properties are explored with the help of nucleus-nucleus scattering at low energy regime. Experimental results of scattering systems are analysed by various models. The models may be macroscopic and microscopic in nature. The optical model potential has been a well-accepted method to analyse experimental results of nucleus-nucleus or nucleon-nucleus scatterings for exploring nuclear properties. Various phenomenological nuclear potentials are used in the analysis of optical models. An optical model can also be extended to analyse many complicated nuclear phenomena [3, 4].

II THEORETICAL FORMULATION

If the effective potential for the two colliding (target-projectile) nuclei in optical potential is denoted by $V_{eff}(r)$ and the two body system is reduced to one-body problem with reduced mass μ, then we can write the Schrodinger equation for the reduced system as described below.

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_{eff}(r)\right]\Psi(\vec{r}) = E\,\Psi(\vec{r}) \qquad \dots \quad \dots \qquad (1)$$

The effective potential $V_{eff}(r)$ is an optical potential describing nuclear interactions. Here, $\psi(\vec{r})$ represents the wave function in the presence of potential. The optical potential essentially contains *Coulomb potential* and *nuclear potential*. When target-projectile system is reduced to one-body problem, we get angular and radial parts. In the rotating frame of reference, an outward centrifugal force acts on the reduced mass. In order to keep the mass in its orbit, work must be done against the centrifugal force for which the mass gains potential energy at the expense of its kinetic energy. The *centrifugal potential* is nothing but the angular part of the kinetic energy of the reduced mass. Taking contribution of centrifugal force into consideration we have the effective potential as

$$V_{eff}(r) = V_C(r) + V_N(r) + V_{CF.} \qquad \dots \quad \dots \qquad \textbf{(2)}$$

Here, $V_C(r)$ represents the Coulomb potential, which obeys inverse square law, i.e., $V_C(r) \sim 1/r^2$. The other term $V_N(r)$ represents the nuclear potential. This potential arises due to strong, attractive and short range nuclear force. The last term V_{CF} is kept to take care of the centrifugal potential. This is may be represented as $l(l+1)\hbar^2/2\mu r^2$. Here the reduced mass μ can be expressed as $\mu = \frac{m_T m_P}{m_T + m_P}$, where m_P and m_T are masses of projectile and target.

The nuclear potential $V_N(r)$ is complex in nature. The complex potential can be described by

$$V_N(r) = V_n(r) + i\,W_n(r) \qquad \dots \quad \dots \qquad \textbf{(3)}$$

Where $V_n(r)$ is real the part and $W_n(r)$ is the imaginary part of the complex potential. Real part $V_n(r)$ of the nuclear potential is obtained from one-dimensional attractive Ginocchio potential [5] developed by Joseph N. Ginocchio, in which potential function v(r) is described by Ginocchio as

$$v(r) = -\lambda^2 \upsilon(\upsilon+1)(1-y^2) + \frac{1-\lambda^2}{4}\left[5(1-\lambda^2)y^4 - (7-\lambda^2)y^2 + 2\right](1-y^2) \;\dots \qquad \textbf{(4)}$$

Here parameters υ and λ are dimensionless. Parameter υ measures *depth* and λ is related to shape (*flatness*) of potential. The function y(r) can be related to variable r as follows.

$$r = \frac{1}{\lambda^2}\left[\operatorname{arctanh} y - \sqrt{1-y^2}\,\operatorname{arctanh}\sqrt{1-y^2}\,y\right] \qquad \dots \quad \dots \qquad \textbf{(5)}$$

The potential function in fact represents a family of potentials. The constituent potentials have different depths and shapes. The nuclear part $V_N(r)$ of the effective potential can be expressed [5, 6] by :

$$V_N(r) = \begin{cases} -V_{01}\{\lambda_1{}^2[(B_0 - iW_0) + (B_1 - B_0 + iW_0)(1 - y_1{}^2)] + \xi_1\} & \text{if } 0 < r \le R_0 \\ -V_{02}[\lambda_2{}^2 B_2(1 - y_2{}^2) + \xi_2], & \text{if } r > R_0 \end{cases}$$

The real part can be separated from the above relation to get the expression for $V_n(r)$ as,

$$V_n(r) = \begin{cases} -V_{01}\{\lambda_1{}^2[B_0 + (B_1 - B_0)(1 - y_1{}^2)] + \xi_1\} & \text{for } 0 < r \le R_0 \\ -V_{02}[\lambda_2{}^2 B_2(1 - y_2{}^2) + \xi_2] & \text{for } r > R_0 \end{cases} \quad \cdots \quad \mathbf{(6)}$$

where $$\xi_1 = \frac{1-\lambda_1{}^2}{4}\left[5(1-\lambda_1{}^2)y_1{}^4 - (7-\lambda_1{}^2)y_1{}^2 + 2\right](1-y_1{}^2)$$

and $$\xi_2 = \frac{1-\lambda_2{}^2}{4}\left[5(1-\lambda_2{}^2)y_2{}^4 - (7-\lambda_2{}^2)y_2{}^2 + 2\right](1-y_2{}^2)$$

The parameters V_{01} and V_{02} represent potential strength in MeV. Again the parameters V_{01}, B_0, B_1 and W_0 specify the potential for region $r \le R_0$. Here, V_{01} specifies the strength of potential in that region. On the other hand, parameters V_{02}, B_2 and W_2 specify the potential for region $r > R_0$. The parameter λ basically gives information regarding flatness of the potential. Now, y_1 and y_2 are the function of radial variable 'r'. These can be obtained as follows.

$$\frac{1}{\lambda_n{}^2}\left[tanh^{-1}y_n - \sqrt{1-\lambda_n{}^2}\, tanh^{-1}\left(y_n\sqrt{1-\lambda_n{}^2}\right)\right] = (r - R_0)\sqrt{\frac{2\mu V_{0n}}{\hbar^2}}$$

or, $$\frac{1}{\lambda_n{}^2}\left[tanh^{-1}y_n - \sqrt{1-\lambda_n{}^2}\, tanh^{-1}\left(y_n\sqrt{1-\lambda_n{}^2}\right)\right] = (r - R_0)b_n = \rho_n$$

The parameter b_n is given as, $b_n = \sqrt{\frac{2\mu V_{0n}}{\hbar^2}}$. Therefore, $b_1 = \sqrt{\frac{2\mu V_{01}}{\hbar^2}}$, $\quad b_2 = \sqrt{\frac{2\mu V_{02}}{\hbar^2}}$

In the potential expression, the parameter $\mathbf{b_n}$ (n = 1, 2) is the slope parameter. The ranges of $\boldsymbol{\rho_n}$ and $\mathbf{y_n}$ can be clearly mentioned with variable **r** as follows.

For $r \in (0, R_0]$, $\rho_1 \in (-b_1R_0, 0]$ and $y_1 \in (-1, 0]$.

For $r \in (R_0, \infty]$, $\rho_2 \in (0, \infty]$ and $y_2 \in (0, 1]$.

Near the surface at $r = R_0$, we have $y_1 = (r - R_0)\, b_1 = 0$. Similarly, at $r = R_0$, we have $y_2 = (r - R_0)\, b_2 = 0$. Putting value of $y_1 = 0$ in **Eq.6**, we get the height of barrier at $r = R_0$ as,

$$V_n(r = R_0) = -V_{01}[\lambda_1{}^2 B_1 + \xi_1] = -V_{01}\left[\lambda_1{}^2 B_1 + \frac{1-\lambda_1{}^2}{2}\right] = -V_{B1}$$

Putting the value of $y_2 = 0$ in **Eq.6**, we get the height of barrier at $r = R_0$ as,

$$V_n(r = R_0) = -V_{02}[\lambda_2{}^2 B_2 + \xi_2] = -V_{02}\left[\lambda_2{}^2 B_2 + \frac{1-\lambda_2{}^2}{2}\right] = -V_{B2}$$

We have to match both the results at $r = R_0$ to create single barrier of height V_B (in MeV) at position $r = R_0$. Thus, we can have the Barrier height V_B as,

$$V_B = V_{B1} = V_{B2}$$

i.e., $V_{01}\left[\lambda_1{}^2 B_1 + \frac{1-\lambda_1{}^2}{2}\right] = V_{B1} = V_B$ and $V_{02}\left[\lambda_2{}^2 B_2 + \frac{1-\lambda_2{}^2}{2}\right] = V_{B2} = V_B$

Then, $V_{01} = \frac{V_B}{\left[\lambda_1{}^2 B_1 + \frac{1-\lambda_1{}^2}{2}\right]}$ and $V_{02} = \frac{V_B}{\left[\lambda_2{}^2 B_2 + \frac{1-\lambda_2{}^2}{2}\right]}$

The parameters λ_n and B_n are important, because λ_n controls flatness of the potential, when B_n decides the range of the potential. If we put the condition, $\lambda_1=1$ and $\lambda_2=1$, we will get, $\xi_1 = 0$ and $\xi_2 = 0$. The zero values of ξ_1 and ξ_2 will give,

$V_{01} = \frac{V_B}{B_1}$ and $V_{02} = \frac{V_B}{B_2}$

Thus, real part of nuclear potential $V_n(r)$ assumes the expression as described below.

$$V_n(r) = \begin{cases} -\frac{V_B}{B_1}\{B_0 + (B_1 - B_0)(1 - y_1{}^2)\} & \text{for } 0 < r \le R_0 \\ -V_B(1 - y_2{}^2) & \text{for } r > R_0 \end{cases} \quad \ldots \quad \textbf{(7)}$$

Putting $\lambda_n = 1$ in the expression of ρ_n, i.e.,

$\rho_n = \frac{1}{\lambda_n{}^2}\left[tanh^{-1} y_n - \sqrt{1-\lambda_n{}^2}\, tanh^{-1}\left(y_n \sqrt{1-\lambda_n{}^2}\right)\right] = (r - R_0) b_n$, we get,

$\tanh^{-1} y_n = (r - R_0) = \rho_n$ or, $y_n = \tanh(\rho_n)$

Using the relations $y_1 = \tanh(\rho_1)$ and $y_2 = \tanh(\rho_2)$ in **Eq.7**, we obtain

$$V_n(r) = \begin{cases} -\frac{V_B}{B_1}\{B_0 + (B_1 - B_0)(1 - \tanh^2\rho_1)\} & \text{for } 0 < r \le R_0 \\ -V_B(1 - \tanh^2\rho_2) & \text{for } r > R_0 \end{cases}$$

$$= \begin{cases} -\frac{V_B}{B_1}\{B_0 + (B_1 - B_0)\,\mathrm{sech}^2\rho_1\} & \text{for } 0 < r \le R_0 \\ -V_B\,\mathrm{sech}^2\rho_2 & \text{for } r > R_0 \end{cases}$$

or, $$V_n(r) = \begin{cases} -\frac{V_B}{B_1}\left[B_0 + \frac{B_1 - B_0}{\cosh^2\rho_1}\right], & \text{for } 0 < r \le R_0 \\ -\frac{V_B}{B_2}\left[\frac{B_2}{\cosh^2\rho_2}\right], & \text{for } r > R_0 \end{cases} \quad \ldots \quad \textbf{(8)}$$

The parameter λ is related to shape of potential. The barrier flatness is controlled by lambda (λ). Although 0 < λ < ∞. We choose λ = 1 in order to get smooth shape which can serve our purpose to analyse various results. Sahu et al. [7] have discussed in detail about the variation of this parameter of the potential

We have mentioned a slope parameter b_n earlier. This slope parameter is defined as $b_n = \sqrt{\frac{2\mu V_{0n}}{\hbar^2}} = \sqrt{\frac{2\mu V_B}{\hbar^2 B_n}}$ and V_B is the barrier height in MeV at r = R_0, where R_0 is the radial distance. Parameter B_0 controls potential depth at the origin r = 0, whereas, parameter V_B is related to the potential depth at R_0 position. V_B controls parameter b_n as well. The slope parameter is also affected by parameter B_n on either side of R_0.

Our potential possesses an inbuilt *non-trivial neck-formation*. For a given energy if we vary the amplitude of wave function with radial distance, then it does not possess any discontinuity at position r = R_0. The potential consists of two analytically solvable regions. One is inner (volume) region and the other is outer (surface) region. The slopes of volume region and surface region are taken to be b_1 and b_2 respectively. The two regions are smoothly joined near r = R_0. The location where the volume and surface regions meet is analytically solvable. Schrodinger equation can also be solved in this region [8, 9, 10]. Both the regions smoothly coincide here. The derivatives respect to 'r' have the same value (zero) at the point [11]. The smooth continuation nature of wave function justifies the physical nature of adopted nuclear potential for analysing heavy ion collisions.

As stated earlier, the nuclear potential has real and imaginary parts. Imaginary part has similar structure. The strength of real part differs from that of imaginary part. Following the structure of real section in **Eq.7** and **Eq.8**, the imaginary part is given as,

$$W_n(r) = \begin{cases} -\frac{V_{BW}}{W_1}\{W_0 + (W_1 - W_0)(1 - y_1{}^2)\} & \text{for} \quad 0 < r \leq R_0 \\ -\frac{V_{BW}}{W_2}[W_2(1 - y_2{}^2)] & \text{for} \quad r > R_0 \end{cases} \qquad \ldots \qquad \textbf{(9)}$$

With the substitution of $y_n = \tanh(\rho_n)$, $V_{01W} = V_{BW}/W_1$ and $V_{02W} = V_{BW}/W_2$ the imaginary part of the nuclear potential may be expressed as,

$$W_n(r) = \begin{cases} -V_{01W}\left[W_0 + \frac{W_1 - W_0}{\cosh^2\rho_1}\right], & \text{if } 0 < r \leq R_{0W} \\ -V_{02W}\left[\frac{W_2}{\cosh^2\rho_2}\right], & \text{if } r > R_{0W} \end{cases} \qquad \ldots \qquad \ldots \qquad \textbf{(10)}$$

Two nuclei, namely, projectile (P) and target (T) interact in the process. They behave as a uniformly charged sphere. The radius (R_C) of that sphere is known as *reduced radius* and is represented as $R_C = r_C (A_P^{1/3} + A_T^{1/3})$. The Coulomb potential $V_C(r)$ arises due to these two interacting nuclei involved in the scattering system. The potential can be given by

$$V_C(r) = \begin{cases} \frac{Z_P Z_T e^2}{2\,R_C^3}\left(3{R_C}^2 - r^2\right), & \text{if } r < R_C \\ \frac{Z_P Z_T e^2}{r}, & \text{if } r \geq R_C \end{cases} \quad \dots \quad \dots \quad \textbf{(11)}$$

The atomic numbers in the above expression are denoted by $\mathbf{Z_P}$ and $\mathbf{Z_T}$ respectively for the projectile nucleus and the target nucleus. In the expression of reduced radius, $\mathbf{r_C}$ is the Coulomb radius parameter, $\mathbf{A_P}$ is the mass number of projectile nucleus and $\mathbf{A_T}$ is the mass number of target nucleus. The value of the Coulomb radius parameter (r_C) is taken within the range from 1.2 fm to 1.4 fm while analyzing different collision systems.

III STUDY OF PARAMETERS OF THE OPTICAL POTENTIAL

The proposed nuclear potential deals in ten parameters. The parameters are R_0, V_B, B_0, B_1, B_2, R_{0W}, V_{BW}, W_0, W_1 and W_2. Five parameters, namely, R_0, V_B, B_0, B_1, and B_2 are used for the real part, whereas, the other five parameters R_{0W}, V_{BW}, W_0, W_1, and W_2 are used for the imaginary part of the complex optical potential. All these parameters simultaneously help us theoretically reproduce different experimental results in different energy ranges. They also explain angular distribution in scattering cross-sections as well as fusion cross-sections of different scattering systems.

(a) Parameters of real part

Real part $V_n(r)$ of the potential $V_N(r)$ is represented in **Fig.1**. It has two parameters B_0 and V_B to specify the depths of potential. Parameter B_0 gives potential depth at origin, $r = 0$. Parameter V_B describes the potential depth at $r = R_0$. Therefore, the depth of attractive potential well can be increased by increasing the B_0 value. Similarly, the depth of neck from the zero-line can be altered with the variation of V_B. Greater the magnitude of V_B, the bigger will be the depth of neck-structure from the zero-line. The non-trivial structure (neck formation) occurs at the surface region around $r = R_0$. The distance from the origin to the location of this non-trivial structure is R_0. The location of the neck-structure is dependent upon R_0 value. The position of the neck-structure is shifted towards right when the value of R_0 increases, and shifted towards left when the value of parameter R_0 decreases.

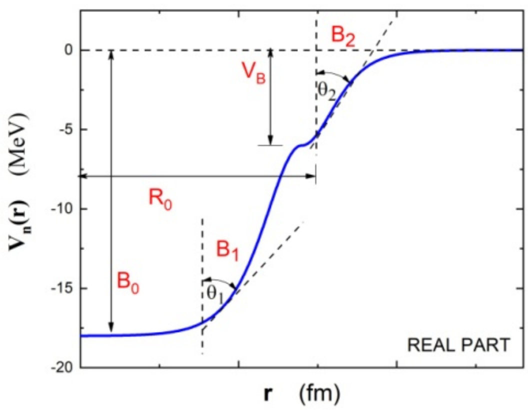

Fig.1 : real part

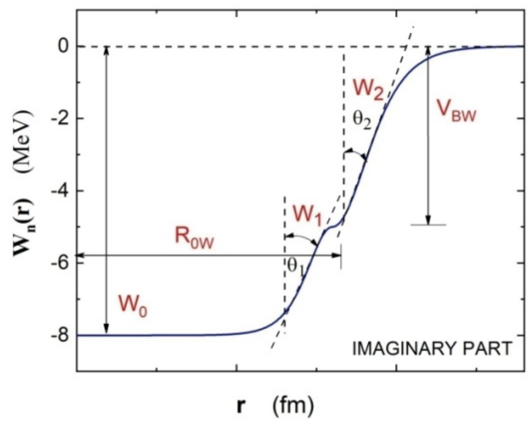

Fig.2 : imaginary part

Fig.1 *: Representation of real part of potential (V_N). Real part has five parameters, i.e., V_B, B_0, R_0, B_1 and B_2. Parameters V_B and B_0 represent depths of the potential. B_1 and B_2 are the slope parameters. The parameter R_0 represents nuclear radius.*

Fig.2 *: Representation of imaginary part of potential (V_N). It has five parameters, i.e., V_{BW}, W_0, R_{0W}, W_1 and W_2. Parameters V_{BW} and W_0 specify depths of the potential. W_1 and W_2 are the slope parameters. The parameter R_{0W} represents nuclear radius.*

As far as B_1 and B_2 are considered, these are connected with slope parameter. Parameter B_1 controls slope of potential curve in region $r < R_0$, whereas, parameter B_2 controls slope of potential curve in region $r > R_0$. If we increase B_1, then the value of θ_1 will increase, and the slope of the lower part of the curve will decrease. If we decrease B_1, then the value of θ_1 will decrease, and the lower part of the curve will be steeper. The upper part of the curve also behaves in similar fashion. When B_2 increases, the value of θ_2 increases, and hence the slope of the upper portion decreases. When B_2 decreases, the value of θ_2 decreases, and the upper portion becomes more steep. Thus, B_n is directly proportional to θ_n.

(b) Parameters of Imaginary Part

Imaginary part $W_n(r)$ of the potential $V_N(r)$ is represented in **Fig.2**. It has two parameters W_0 and V_{BW} to control the depths of the potential. The parameter W_0 describes potential depth at origin, $r = 0$. Other parameter V_{BW} describes the potential depth at $r = R_0$. Therefore, the depth of attractive potential well can be increased by increasing the W_0 value and the depth decreases with fall of W_0. Similarly, the depth of analytic junction from the zero-line can be altered with the variation of V_{BW}. If the magnitude of V_{BW} increases, then the depth of neck-structure from the zero-line increases. The distance from the origin to the location of this non-trivial neck-structure is given by R_{0W}. The location of the neck-structure shifts towards right when the value of R_{0W} increases, and shifts towards left when the value of R_{0W} decreases.

The other two parameters W_1 and W_2 behave in similar fashion as that of real potential parameters B_1 and B_2. The parameters W_1 and W_2 describe the inclinations of the lower portion and the upper portion of the potential curve. So these are connected to the slope parameter. Parameter W_1 controls slope of the potential curve in region $r < R_0$, whereas, parameter W_2 controls the slope in region $r > R_0$. If we increase W_1, then the value of θ_1 will increase, and the lower portion will move away from vertical line and incline more towards the horizontal line. Thus, the slope of the lower part of the curve will decrease. If we decrease W_1, then the value of angle θ_1 will decrease, and simultaneously the lower part of the curve will be steeper. The upper part of the curve also behaves in similar fashion. When W_2 increases, the value of angle θ_2 increases, and hence the slope of the upper portion decreases. In the similar fashion, when W_2 decreases, the value of θ_2 decreases, and the upper portion becomes more steep. Thus, the slope parameter W_n is directly proportional to θ_n.

(c) Parameters of Effective Potential

The effective potential explicitly depends on five parameters, namely, R_0, V_B, B_0, B_1 and B_2. The other parameters R_{0W}, V_{BW}, W_0, W_1 and W_2 have no effect on the effective potential. The effects of those five parameters, namely, R_0, V_B, B_0, B_1 and B_2 on the effective potential are discussed in detail in this chapter. The effective potential $V_{eff}(r)$ is represented in **Fig.3**. The physical description of potential parameters is depicted in the figure. The attractive potential-well is shown with a depth B_0, when the depth of neck-formation from the maximum of surface part is determined by the parameter V_B. The slope parameters B_1 and B_2 are also shown separately. R_0 locates the non-trivial structure from the origin.

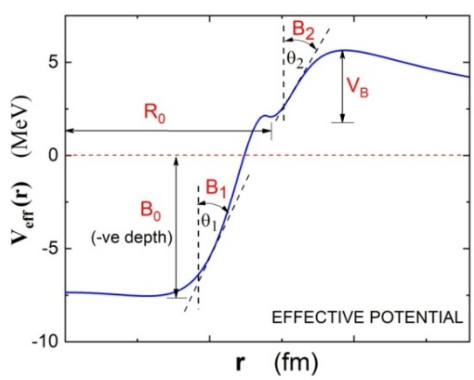

***Fig.3** : The figure represents the effective potential. Parameters, i.e., V_B, B_0, R_0, B_1 and B_2 are physically shown in the figure. The parameters V_B and B_0 are related to the depths in the potential, whereas, B_1 and B_2 are the slope parameters. The parameter R_0 decides the location of non-trivial structure from the origin.*

(i) Effect of R_0 on effective potential

The non-trivial structure (neck formation) occurs at the surface region around r = R_0. The distance from the origin to the location of this non-trivial structure is given by R_0. The location of the neck-structure shifts towards right when the value of R_0 increases, and shifts towards left when the value of R_0 decreases. This is shown in **Fig.4** with three different values of parameter R_0, i.e., R_0 = 6 (blue line), 7 (red line), and 8 (black dash). The depths of potential at the origin remain the same for all values of R_0. The three curves are almost parallel, which indicates that the corresponding slopes remain unchanged.

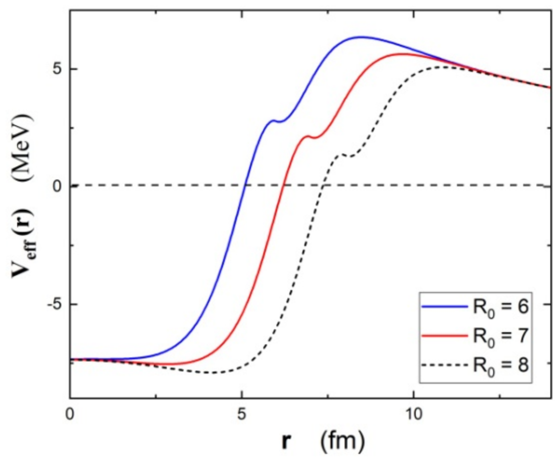

Fig.4 : parameter R_0

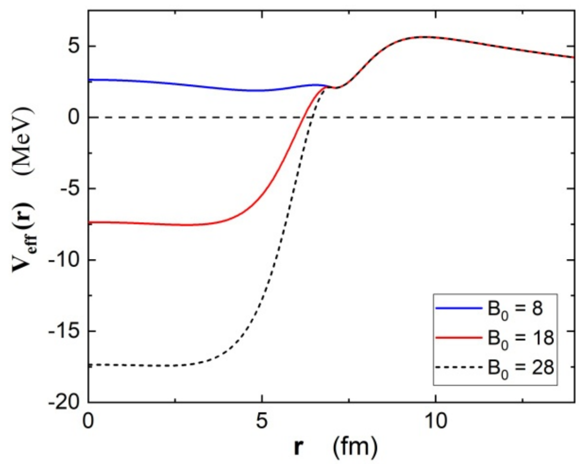

Fig.5 : parameter B_0

Fig.4 *: Plot showing the effect of parameter R_0 on the effective potential. Three curves represent effective potential for three different values of R_0 parameter. They are R_0 = 6 (blue line), R_0 = 7 (red line), and R_0 = 8 (black dash).*

Fig.5 *: Plot showing the effect of parameter B_0 on the effective potential. Three curves of different shapes represent the effective potential for three different values of B_0 parameter, namely, B_0 = 8 (blue line), B_0 = 18 (red line), and B_0 = 28 (black dash).*

(ii) Effect of B_0 on effective potential

The parameter B_0 specifies the depth of the effective potential. Particularly, the potential depth at origin r = 0 is affected by B_0 value. The depth of attractive potential-well can be increased by increasing B_0 value, which is evident from the three different curves obtained for three different values of B_0. The variations in the shape of potential are explicitly presented in **Fig.5**. The dashed curve (black) is shown for B_0 = 28. The depth of red curve drawn for $B_0 = 18$ at the origin is less than that of black curve. Further, the potential level (blue line) for $B_0 = 8$ is above the level of other two potentials at the origin.

(iii) Effect of V_B on effective potential

The parameter V_B specifies the potential depth at $r = R_0$, where the non-trivial neck-formation is located. The depth of non-trivial structure from the maximum of surface part can be altered with V_B value. More the magnitude of V_B, more will be the depth of neck-structure from the maximum. This is evident from the **Fig.6** in which three curves are represented for three different values of V_B. When the curves are compared for parameter values $V_B = 4$, $V_B = 6$ and $V_B = 8$, the depth for the highest V_B value is found maximum, which is represented by black dash. The curve (blue) with lowest V_B is found to have the minimum depth. The curve (red) for intermediate value $V_B = 6$ finds its place between black and blue necks.

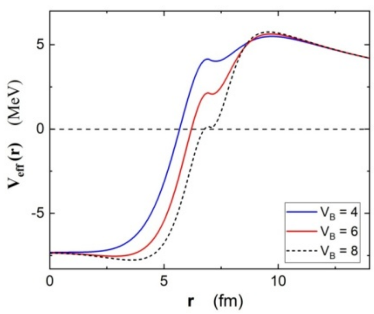

Fig.6 : parameter V_B

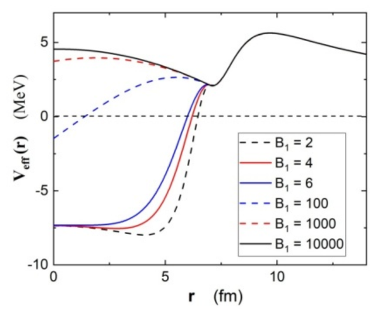

Fig.6 : parameter B_1

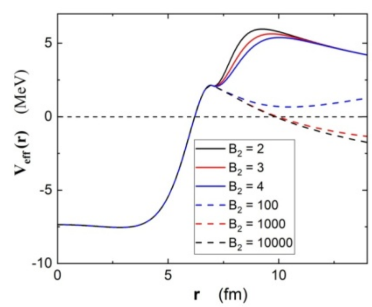

Fig.6 : parameter B_2

Fig.6 *: Plot showing the effect of parameter V_B on the effective potential. Three curves of different colours represent the effective potential for three different values of the parameter, i.e., $V_B = 4$ (blue line), $V_B = 6$ (red line), and $V_B = 8$ (black dash). V_B determines the depth of non-trivial structure from the top of surface part.*

Fig.7 *: Plot showing the effect of parameter B_1 on the shape of effective potential. Six different curves represent the effective potential for six different values of the parameter, i.e., $B_1 = 2, 4, 6, 100, 1000$ and 10000. Parameter B_1 determines slope of the lower part.*

Fig.8 *: Plot showing the effect of parameter B_2 on the shape of effective potential. Six different curves represent the effective potential for six different values of the parameter, i.e., $B_2 = 2, 3, 4, 100, 1000$ and 10000. Parameter B_2 determines slope of the upper part.*

(iv) Effect of B_1 on effective potential

The parameter B_1 is related to the slope of the lower (volume) part, as it controls the slope of potential curve in region $r < R_0$. If we increase the value B_1, then the value of θ_1 (**Fig.3**) will increase in clockwise sense. Thus the slope of lower part decreases with rise of B_1 value. If we decrease B_1, then θ_1 will decrease making the lower part steeper. With $B_1 = 0$, the lower portion of the curve ($r < R_0$) vanishes and the upper portion only appears. The lower portion only changes with B_1 without altering the upper portion ($r > R_0$). Different orientations of the lower portion imply that B_1 is directly proportional to θ_1 for non-zero B_1.

(v) Effect of B_2 on effective potential

The parameter B_2 is related to the slope of the upper (surface) part. It controls the slope of potential curve in region $r > R_0$. If we increase the value B_2, then the value of θ_2 (**Fig.3**) will increase in clockwise sense. Thus the slope of upper part decreases with rise of B_2 value. If we decrease B_2, then θ_2 will decrease making the upper part steeper. With $B_2 = 0$, the upper portion of the curve vanishes and the lower portion only appears. The upper portion only changes with B_2 without altering the lower portion. Different orientations of the upper portion imply that B_2 is directly proportional to θ_2 for non-zero B_2.

IV CONCLUSION

We introduce some basic concepts of the optical model. The idea of optical model was introduced long back in late 40s. Interestingly, this simple theory is still used for many types of common applications. The optical model is a general nuclear reaction model that is proposed to explain heavy-ion collision phenomena. We use an optical potential of complex nature developed from a versatile Ginocchio potential. It has ten different parameters, five each for real and imaginary parts. We study the effect of all those parameters on the shape of the potential. The detail study parameters help us compute scattering and fusion cross-sections theoretically in order to reproduce experimental data. Therefore, we can explain various experimental results.

The potential has non-trivial behaviour at $r = R_0$. This arises due to presence of two different parts, i.e., surface part and volume part. This small disturbance has been helpful for us in explaining different complicated results of scattering experiments. In particular, the region $r = R_0$ has been taken as the sensitive region. Therefore, we check the sensitivity of the optical potential for different systems in our works prior to explaining experimental results.

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