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Non-stop integral flow in lateral gap distance lattice model

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Abstract The role of non-stop integral flow is studied in a lattice model by assuming the lateral gap among the lattice sites. The proposed model is investigated theoretically as well as numerically. In theoretical evaluation, we derived the stableness criterion and provided the relationship among sensitivity and other parameters. It's far located that similarly to attention of the gap space, the non-stop time of flow reduces the congestion and the unstable region more reinforced via increasing the driver's memory time step.

Keywords Lattice · Non-lane-based · Flow integral · Traffic

1 Introduction

To expose the traffic problems including intrinsic mechanism of traffic congestion, commuting delay, traffic accidents and energy consumption, the modeling of traffic flow has attracted a widespread interest of researchers in latest years. Most of the traffic techniques especially recognition at the reproducing the flow-density-velocity relationship and the phase transition of traffic flow from congested region to free flow region with involving various factors of traffic [1–7]. Also, in order to reveals the actual traffic conditions, a few research have been added to suppress the traffic congestion. These days, the lattice hydrodynamic model which was firstly proposed by Nagatani [8], stimulates a huge interest of many researchers.

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As we know, road conditions have a crucial role in traffic flow. For example, restricted lanes, curves, and poor road surface cause drivers to pay greater attention to road conditions and slow down. Poor road conditions are a major cause of traffic jams. In this direction, car-following traffic flow models were proposed through[9–11] with the aid of assuming that vehicles travel in the center of the lane which can be stimulated directly by the only in front or behind and no passing is permitted on a single lane highway. By inspiring from Refs.[9–11], the impact of lateral gap has been also studied in lattice model [12] and it is determined that lateral separation performs a critical role in stabilizing the traffic flows in lattice fashions

In real traffic, driver usually observe at the following in addition to the previous vehicles at some point of driving on road. To deal with this phenomena, many lattice models [13–22] had been found within the literature. Currently, to show the impact of historical traffic information, Wang and Ge [23] proposed a lattice model via accounting the backward looking and flow integral effect and it is observed that the stable region enhances efficiently with consideration of these factors. Motivated from this, Peng et al. [24] studied the flux difference memory integral effect in two-lane lattice version and it's far encountered that lane changing performs a vital function in stabilizing the traffic congestion. Vehicle continuous memory is useful in traffic modelling, and the effect is amplified in non-lane-based lattice models. However, the lateral separation distance of consecutive autos has not been examined in the driver's continuous memory integral lattice model.

In section 2, we explore the lateral separations gap between two consecutive autos and offer a lateral-gap-distance lattice model while considering the effect of flow memory integral. In Section 3, the model's stability condition is established using linear stability theory. The numerical simulation is then completed in order to verify the analytic results, and the conclusion is stated in the concluding part.

2 Proposed Model

The lattice version of continuum model through considering the concept of car-following model is

$$\partial_t s_j + s_0(s_j v_j - s_{j-1} v_{j-1}) = 0, \tag{1}$$

with the given flow evolution equation at site j

$$\partial_t (s_j v_j) = a[s_0 V(s_{j+1}) - s_j v_j]. \tag{2}$$

where $a = \frac{1}{\tau}$ is the sensitivity; s_0 is the average density; $V(.)$ is the optimal velocity function; s_j and v_j denote the density and velocity at site j at time t, respectively. Furthermore, to include the lateral separation distance, Peng et al. [12] proposed a lattice hydrodynamic model as follows

$$s_j(t+\tau) - s_j(t) + \tau s_0(s_j v_j - s_{j-1} v_{j-1}) = 0 \tag{3}$$

$$s_j(t+\tau) v_j(t+\tau) = s_0 V(s_{j+1}, s_{j+2}) + \kappa G(\triangle Q_{j,j+1}, \triangle Q_{j,j+2}) \tag{4}$$

where κ is the reactive coefficient to the function $G(.)$, $\triangle Q_{j,j+1} = s_{j+1} v_{j+1} - s_j v_j$, and $\triangle Q_{j,j+2} = s_{j+2} v_{j+2} - s_j v_j$ are the relative flows among site j & $j+1$ and j &

$j+2$, respectively. It's far observed that the free region enhances with an increasing the lateral separation distance of lane width and consequently, this element plays an important role in stabilizing the traffic flow.

As we know, a driver often observes traffic relative information at time t and decides to modify the speed of his vehicle at a later time; nevertheless, this movement can cause a delay that effects the traffic. In this course, Gupta and Redhu [26] presented a hydrodynamics model for detecting relative flux for a two-lane system with a fixed delay and explored the effects of driver expectation on traffic flow. However, it is clear that the effect of continual memory has a greater impact on traffic flow than fixed delay time, and this has been examined in many traffic flow models [23,27]. In literature, we studied that road width performs an essential role in stabilizing the traffic congestion and it will becomes more effective if driver could have the relative records of continuous memory. But, the effect of non-stop memory integral has not been studied untill now.

Here, we are offering a lattice model by considering the continuous historical flux information in term of integration between the time $[t-\tau_0, t]$ and the new evolution is

$$s_j(t+\tau)v_j(t+\tau) = s_0[V(s_{j+1}, s_{j+2})] + \kappa \int_{t-\tau_0}^{t} G(\triangle Q_{j,j+1}(s), \triangle Q_{j,j+2}(s))ds \quad (5)$$

where τ_0 represents the historical integral time, κ is the corresponding coefficient, $G(.)$ is given by

$$G(\triangle Q_{j,j+1}(s), \triangle Q_{j,j+2}(s) = (1-p_j) \triangle Q_{j,j+1}(s) + p_j \triangle Q_{j,j+2}(s) \quad (6)$$

and

$$V(s_{j+1}, s_{j+2}) = V[(1-p_j)s_{j+1} + p_j s_{j+2}] \quad (7)$$

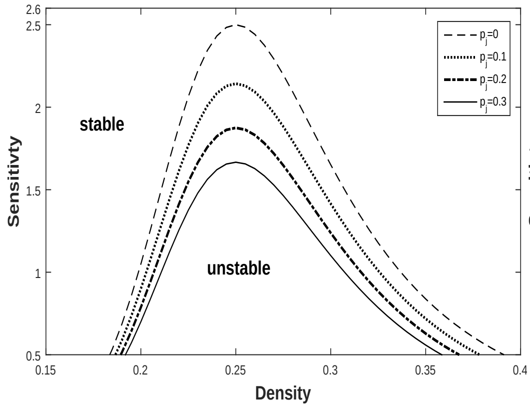
where $p_j = \frac{LS_j}{LS_{max}}$ is the parameter of lateral separation distance, LS_j is the lateral separation distance of sites j and $j+1$ and LS_{max} is the maximum lateral separation distance. The term $\int_{t-\tau_0}^{t} G(\triangle Q_{j,j+1}(s), \triangle Q_{j,j+2}(s))ds$ represents the continuous flux difference information. The modified velocity function for non-lane-based model is

$$V(s_{j+1}, s_{j+2}) = \frac{v_{max}}{2}\Big\{tanh\Big[\frac{1}{(1-p_j)s_{j+1} + p_j s_{j+2}} - \frac{1}{s_c}\Big] + tanh\Big(\frac{1}{s_c}\Big)\Big\} \quad (8)$$

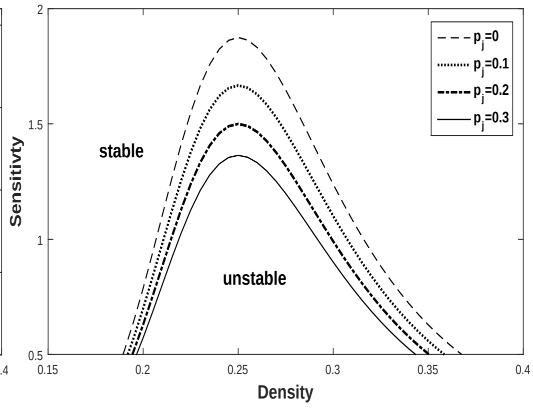
By taking the difference form of Eqs. (1) and (5) and eliminating speed v_j, the density equation is obtained as

$$s_j(t+2\tau) - s_j(t+\tau) + \tau s_0^2[V(s_{j+1}, s_{j+2}) - V(s_j, s_{j+1})] + \tau\kappa[(1-p_j)(-s_{j+1}(t) + s_{j+1}(t-\tau_0) + s_j(t) - s_j(t-\tau_0)) + p_j(s_{j+2}(t) + s_{j+2}(t-\tau_0) + s_{j+1}(t) - s_{j+1}(t-\tau_0))] = 0 \quad (9)$$

where $\tau_0 = k\tau$, where τ_0 and $k = 1, 2, 3\cdots$ represent the difference time step and integer for the historical time considered.



(a)



(b)

Fig. 1 Phase diagram in parameter space (s,a), for (a) $\kappa = 0.1$ and (b) $\kappa = 0.3$, respectively.

3 Linear stability analysis

To look the effect of memory flow integral in the proposed model, we assume the steady-state solution of the homogeneous traffic flow as

$$s_j(t) = s_0, \quad v_j(t) = V(s_0). \tag{10}$$

where s_0 and $V(s_0)$ represent the state of uniform traffic flow. Let $y_j(t)$ be a small perturbation to the steady-state density on site j. Then,

$$s_j(t) = s_0 + y_j(t). \tag{11}$$

Putting this perturbed density profile into Eq. (9) and linearizing it, we get

$$\begin{aligned} &y_j(t+2\tau) - y_j(t+\tau) + \tau s_0^2 V'(s_0)[(1-p_j)(y_{j+1} - y_j(t)) + p_j(y_{j+2} - y_{j+1})] + \\ &\quad \tau\kappa[(1-p_j)(-y_{j+1}(t) + y_{j+1}(t-\tau_0) + y_j(t) - y_j(t-\tau_0)) + p_j(y_{j+2}(t) + y_{j+2}(t-\tau_0) \\ &\quad + y_{j+1}(t) - y_{j+1}(t-\tau_0))] = 0 \end{aligned} \tag{12}$$

Substituting $y_j(t) = exp(ikj + zt)$ in Eq. (12), we obtain

$$\begin{aligned} &e^{2\tau z} - e^{\tau z} + \tau s_0^2 V'(s_0)[(1-p_j)(e^{ik} - 1) + p_j(e^{ik-\tau_0 z} - e^{-\tau_0 z})] + \tau\kappa[(1-p_j) \\ &\qquad (-e^{ik} + e^{ik-\tau_0 z} + 1 - e^{\tau_0 z}) + p_j(e^{2ik} + e^{2ik-\tau_0 z} + e^{ik} - e^{ik-\tau_0 z})] = 0. \end{aligned} \tag{13}$$

Inserting $z = z_1(ik) + z_2(ik)^2 \ldots$ into Eq. (13), we will obtain the first-order and second-order terms of the coefficient ik and $(ik)^2$, respectively, we get

$$z_1 = -s_0^2 V'(s_0), \tag{14}$$

$$z_2 = -\frac{3\tau z_1^2}{2} - \frac{s_0^2 V'(s_0)}{2}(1 + 2p_j) + \kappa\tau_0. \tag{15}$$

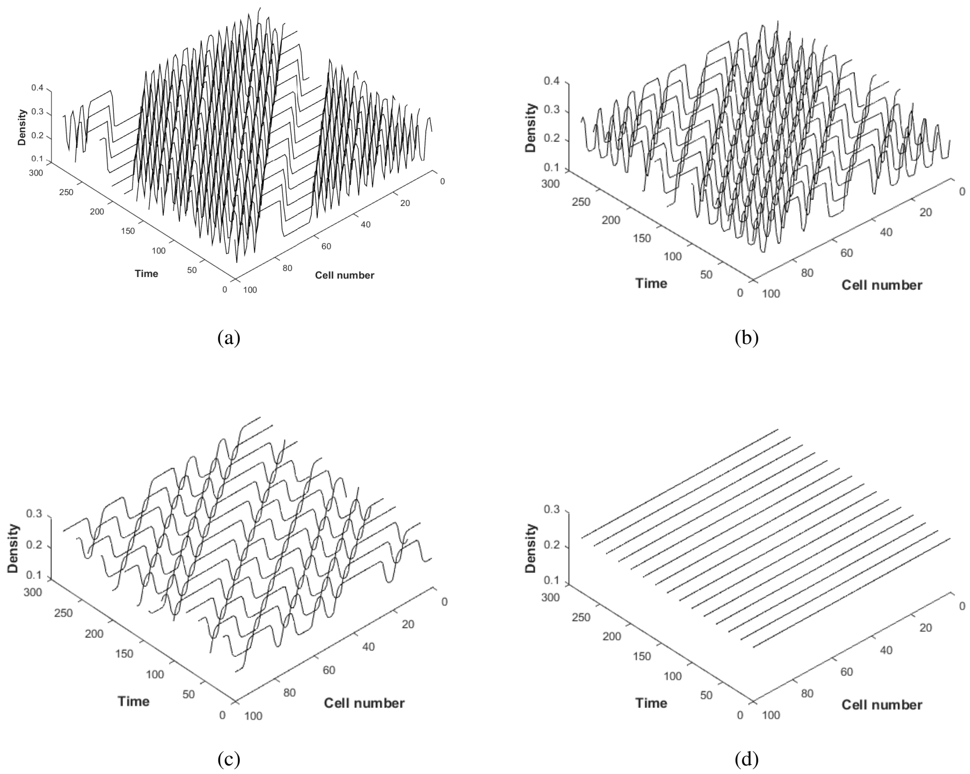


Fig. 2 Space time evolution after time $t = 20000$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, for $\kappa = 0.1$.

When $z_2 < 0$, the uniform steady-state flow becomes unstable for long-wavelength waves. For $z_2 > 0$ the uniform flow becomes stable. Thus, the stability condition for the steady-state is

$$\tau = -\frac{1 + 2p_j + 2\kappa\tau_0}{3s_0^2 V'(s_0)}. \tag{16}$$

The instability condition for the homogeneous traffic flow can be described as

$$\tau > -\frac{1 + 2p_j + 2\kappa\tau_0}{3s_0^2 V'(s_0)}. \tag{17}$$

For $\kappa = 0$, and $p_j = 0$, the above unstability criteria (Eq. 17) will becomes same as that of Nagatani's [8] model.

Figure 1 shows the phase digram in the parameter space (s, a) for different values of p_j. It is clear form Fig. 1(a) that the amplitude of the neutral stability curves decreases with an increases in the value of p_j when $\kappa = 0.1$. Further increase in the value of κ, stable region enhances with an increase in the value of p_j. On comparing

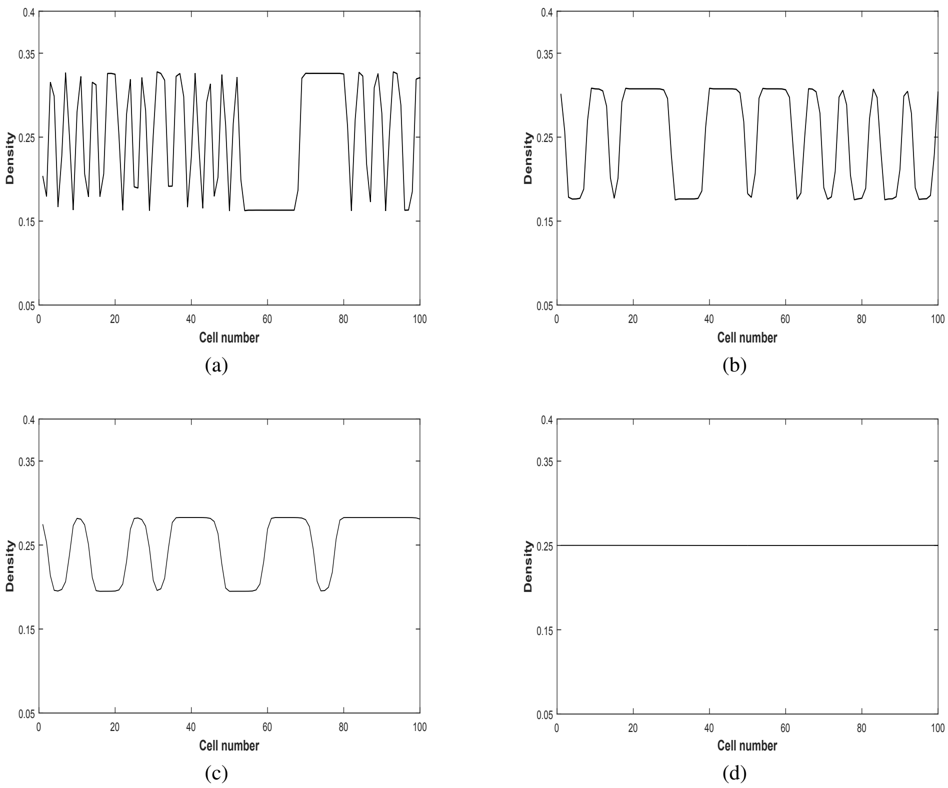


Fig. 3 Density profile at time $t = 20300$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, respectively for $\kappa = 0.1$.

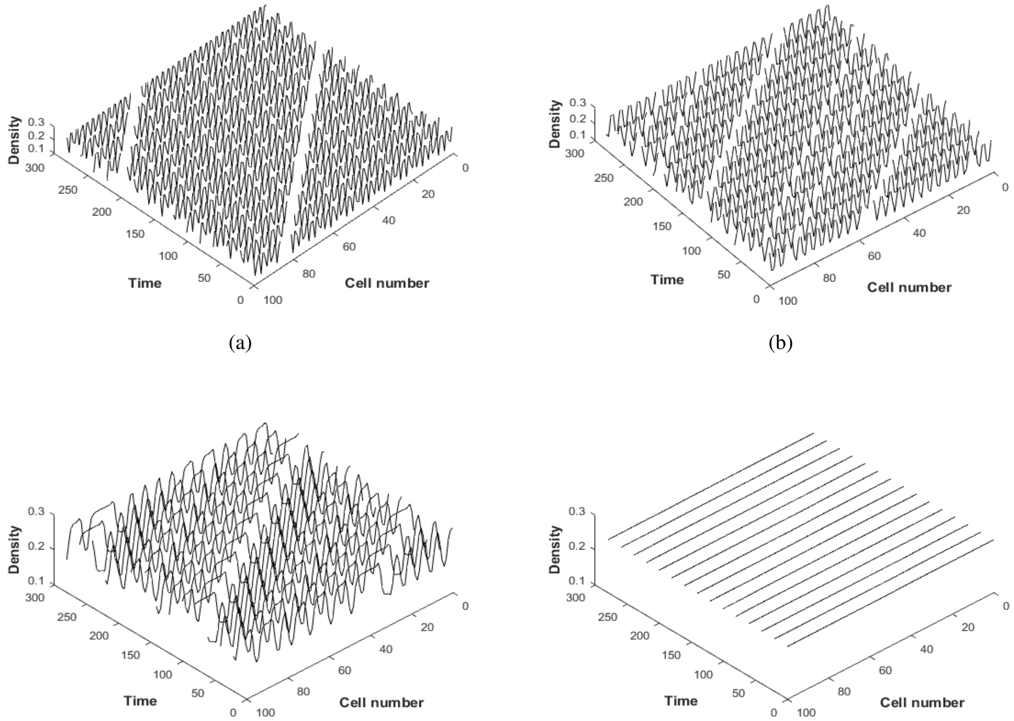
the results for $\kappa = 01.$ and $\kappa = 0.3$, it is concluded that the stable region expands with increase in the value of κ which further enhances with the increment in the value of p_j. If we compare our result with the Peng et al. model [25] for $\kappa = 0.1$ it is concluded the the stable region is more in proposed model which shows that the continuous delayed of flow integral plays a effective role in stabilizing the traffic flow.

4 Numerical Simulation

In this portion, we applied periodic boundary conditions to run a numerical simulation to validate the theoretical conclusions. The initial conditions are adopted as follows:

$$s_j(1) = s_j(0) = \begin{cases} s_0; & j \neq \frac{L}{2}, \frac{L}{2}+1 \\ s_0 - \sigma; & j = \frac{L}{2} \\ s_0 + \sigma; & j = \frac{L}{2}+1 \end{cases}$$

where, σ is the initial disturbance, L is the total number of sites taken as 100 and other parameters are set as follows: $\sigma = 0.1, \tau = \frac{1}{a}$.



(a) (b) (c) (d)

Fig. 4 Space time evolution after time t = 20000 for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, when $\kappa = 0.3$.

The dynamical changes of density waves at time $t = 20000s - 20300s$ for numerous values of p_j when $a = 1.7$ and $\kappa = 0.1$ are shown in Fig. 2. The traffic congestion develops in the unstable zone in the form of kink-antikink types of density waves that arise at each site and propagate in the backward direction over time, as seen in Figs. 2(a)-(c). We enter the stable region when $p_j = 0.3$, and the density waves dissipate and the traffic flow turns uniform. The density profile after a suitably long time $t = 20300$ is shown in Fig.3, which corresponds to the panel in Fig.2. The intensity of the kink-antikink density wave diminishes as the value of p_j grows, and the flow becomes uniform at $p_j = 0.3$.

Figure 4 depicts the spatiotemporal evolutions of density waves at time $t = 20000s - 20300s$ for various values of p_j when $a = 1.52$ and $\kappa = 0.3$, and Figure 5 depicts the density profile at a sufficiently long period $t = 20300$ corresponding to panel of Fig. 4. The initial disturbance turns into density waves in the unstable region, as shown in Fig.4(a)-(c), and these density waves fade out in the free flow region, as seen in Fig.4(d). The deviation occurs around the critical density in the crowded zone, as illustrated in Figs.5(a)-(c), and this deviation disappears in Fig. 5(d). As a result, we

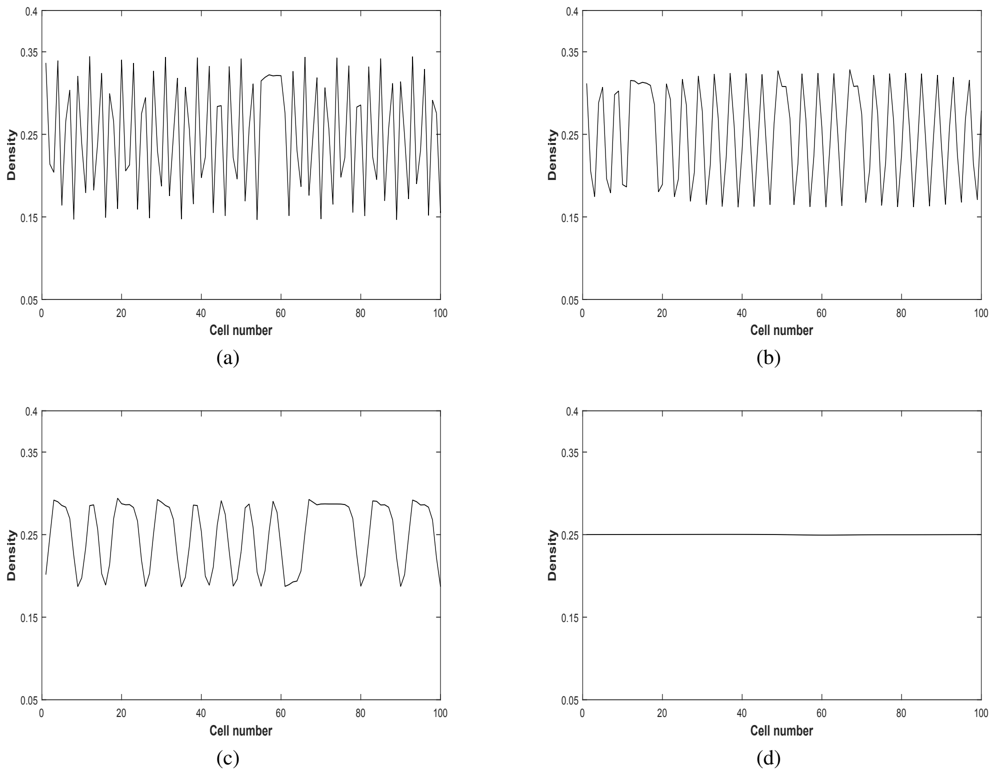


Fig. 5 Density profile at time $t = 20300$ for (a) $p_j = 0$, (b) $p_j = 0.1$, (c) $p_j = 0.2$, and (d) $p_j = 0.3$, respectively for $\kappa = 0.3$.

may deduce that the lateral separation distance plays an important role in traffic flow stabilisation.

When the results for $kappa = 0.1$ and $kappa = 0.3$ are compared, it is concluded that the information of continuous memory integral plays a vital role in traffic flow theory, and its influence is more pronounced in non-lane-based lattice hydrodynamic models.

5 Conclusion

A non-lane-based lattice traffic flow model is proposed with consideration of continuous flow integral effect. Through linear analysis, the condition of stability is derived to analyze the traffic congestion region. To validate the theoretical results, simulation is carried out with periodic boundary conditions. We investigated the effect of lateral separation distance on traffic flow for fixed values of *kappa* and concluded that the coefficient of flow integral effect stabilises the traffic flow and that this factor should be addressed in traffic flow modelling..

Conflicts of Interest

The authors state that they have no competing interests in the publication of this research.

References

1. Tang, T.Q., Li, C.Y., Wu, Y.H., Huang, H.J., Impact of the honk effect on the stability of traffic flow, Phys. A, 390, 3362-3368 (2011)
2. Tang, T.Q., Li, C.Y., Huang, H.J., A new car-following model with the consideration of the driver's forecast effect, Phys. Lett. A, 374, 3951-3956 (2010)
3. Tang, T.Q., Wang, Y.P., Yang, X.B., Wu, Y.H., A new car-following model accounting for varying road condition, Nonlinear Dyn., 70, 1397-1405 (2012)
4. Daganzo, C.F., Requiem for second-order fluid approximations of traffic flow, Trans. Res. Part B Methodol., 29, 277-286 (1995)
5. Berg, P., Mason, A., Woods, A., Continuum approach to car-following models, Phys. Rev. E, 61, 1056 (2000)
6. Jiang, R., Wu, Q.S., Zhu, Z.J., A new continuum model for traffic flow and numerical tests, Trans. Res. Part B Methodol., 36, 405-419 (2002)
7. Gupta, A.K., Redhu, P., Analyses of the driver's anticipation effect in a new lattice hydrodynamic traffic flow model with passing, Nonlinear Dyn., 76, 1001-1011 (2014)
8. Nagatani, T., Modified kdv equation for jamming transition in the continuum models of traffic, Phys. A, 261, 599-607 (1998)
9. Gunay, B., Car following theory with lateral discomfort, Transp. Res. B 41, 722,(2007)
10. Jin, S., Wang, D.H., Tao, P.F., Li, P.F., Non-lane-based full velocity difference car following model, Phys. A, 389, 4654-4662 (2010)
11. Li, Y., Zhang, L., Peeta, S., Pan, H., Zheng, T., Li, Y. and He, X., Non-lane-discipline-based car-following model considering the effects of two-sided lateral gaps, Nonlinear Dyn., 80, 227-238 (2015)
12. Peng, G.H., Cai, X.H., Cao, B.F., Liu, C.Q., Non-lane-based lattice hydrodynamic model of traffic flow considering the lateral effects of the lane width, Phys. Lett. A, 375, 2823-2827 (2011)
13. Nagatani, T., Chaotic jam and phase transition in traffic flow with passing, Phys. Rev. E, 60, 1535 (1999)
14. Ge, H.X., Cheng, R.J., The backward looking effect in the lattice hydrodynamic model, Phys. A, 387, 6952-6958 (2008)
15. Peng, G.H., Cai, X.H., Liu, C.Q. and Cao, B.F., A new lattice model of traffic flow with the consideration of the driver's forecast effects. Phys. Lett. A, 375, 2153-2157 (2011)
16. Peng, G., Nie, F., Cao, B. and Liu, C., A driver's memory lattice model of traffic flow and its numerical simulation. Nonlinear Dyn., 67(3), 1811-1815 (2012)
17. Tian, J.F., Yuan, Z.Z., Jia, B., Li, M.H. and Jiang, G.J., The stabilization effect of the density difference in the modified lattice hydrodynamic model of traffic flow. Phys. A, 391(19), 4476-4482 (2012)
18. Peng, G., A new lattice model of two-lane traffic flow with the consideration of optimal current difference. Commun. Nonlinear Sci. Numer. Simul., 18(3), 559-566 (2013)
19. Ge, H.X., Zheng, P.J., Lo, S.M. and Cheng, R.J., TDGL equation in lattice hydrodynamic model considering driver's physical delay. Nonlinear Dyn., 76(1), 441-445 (2014)
20. Redhu, P., Gupta, A.K., Jamming transitions and the effect of interruption probability in a lattice traffic flow model with passing, Phys. A, 421, 249-260 (2015)
21. Redhu, P. and Gupta, A.K., Effect of forward looking sites on a multi-phase lattice hydrodynamic model. Phys. A, 445, 150-160 (2016)
22. Redhu, P. and Siwach, V., An extended lattice model accounting for traffic jerk. Phys. A, 492, 1473-1480 (2018)
23. Wang, Q. and Ge, H., An improved lattice hydrodynamic model accounting for the effect of "backward looking" and flow integral. Phys. A, 513, 438-446 (2019)
24. Peng, G., Yang, S., Zhao, H. and Qing, L., The flux difference memory integral effect on two-lane stability in the lattice hydrodynamic model, Int. J. Mod. Phys. C, 29, 1850083 (2018)
25. Peng, G.H., Cai, X.H., Cao, B.F. and Liu, C.Q., Non-lane-based lattice hydrodynamic model of traffic flow considering the lateral effects of the lane width, Phys. Let. A, 375, 2823-2827 (2011)

26. Gupta, A.K. and Redhu, P., Analyses of driver's anticipation effect in sensing relative flux in a new lattice model for two-lane traffic system, Phys. A, 392(22), 5622-5632 (2013)
27. Cao, B.G., A new car-following model considering driver's sensory memory, Phys. A, 427, 218-225 (2015)