**Soft Homoemorphism in Soft**

**Topological space**

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**ABSTRACT**

The study of Soft homeomorphism and Soft Strongly homeomorphism in Soft Topological space is the subject of this article. We come up with a few of its properties and talk about a few composition theorems.

**Keywords**—Soft homeomorphism, Soft Strongly homeomorphism, Soft homeomorphism, Soft mappings.

# INTRODUCTION

Molodtov D [6] introduced the Soft set concept in 1999 to deal uncertainty in a parametric fashion. Naim Cagman et al. [2] in 2011 created a soft topological space and established a soft topology on soft set. In 2002, P. K. Maji et al. [5] defined certain fundamental terminology for the theory, including the equality of two Soft sets, subset and superset of a Soft set, complement of a Soft set, null Soft set, and absolute soft set. C. G. Aras and H. Cakalli [1] presented soft mappings in soft topological spaces in 2013. Soft -homeomorphism in Soft topological space was presented by C. Janaki and D. Sreeja [4] in 2014. We previously discussed the soft closed set, soft open set, soft continuous and soft open map. In this work, we look into soft homeomorphism using an example. Theorems relating to their properties and composition were also examined. Additionally, we discover soft strongly homeomorphisms and their characteristics. Throughout this paper Soft set has been represented as S-set.

# PRELIMINARIES

**Deﬁnition 2.1[6]** A **S-set** on the universe is deﬁned by the set of ordered pairs , where such that for all . Hence is called an approximate function of the S-set . The value of may be arbitrary, some of them may be empty, some may have non empty intersection.

**Definition 2.2[6]**

1. A S-set over is said to be **Null S-Set** denoted by *Fϕ* or if for all , .
2. A S-set over is said to be an **Absolute S-Set** denoted by or if for all *,*

**Definition 2.3[6]** Let . Then denotes the **S-point** over , for which . Also the **S-Singleton Set** corresponding to is denoted by .

**Definition 2.4[2]** Let be a collection of S-sets over with a fixed set of parameters. Then is called a **S-topology** on if

1. belongs to .
2. The union of any number of S-sets in belongs to .
3. The intersection of any two S-sets in belongs to .

The triplet is called **S-topological Space** over . The members of are called **S-open** sets in and complements of them are called **S-closed** sets in .

**Deﬁnition 2.5[2]** A S-map is said to be **S-open (closed)**, if the image of every S-open(closed) set in is S-open (closed) in .

**Deﬁnition 2.6[3]** A S-Subset of a S-Topological space is known as **S-closed set** if whenever and is S-semopen set. represents the collection of all S-closed sets. The complement of S-closed set is **S- open set** and noted by .

**Deﬁnition 2.7[3]** A S-map is said to be **S-continuous map** if is s-closed set in for every s-closed set in .

**Definition 2.8[3]** A S-map is said to be **S-closed (open)** if is s-closed (open) set in for every s-closed (open) set in .

**Definition 2.9[1]** A S-bijective map is called **S-homeomorphism** if is both S-continuous map and S-open map.

**Definition 2.10[3]** A S-topological space is said to be **S- space** if every S-closed set is S-closed.

III. **S-HOMEOMORPHISM**

**Definition 3.1.** A S-bijective map from to is called **S-homeomorphism** if is both S-continuous map and S-open map.

**Example 3.2.** Consider Define and as and . Consider the S-topologies where and where . Therefore the s-mapping is both S-continuous and S-open map. Hence, is S-homeomorphism.

**Theorem 3.3.** A S-bijective map is S-homeomorphism. Then is S-homeomorphism.

**Proof.** Consider is a S-homeomorphism, then is S-continuous and S-open map. Also consider is a S-open set in . Therefore, is S-open set in . We know that all S-open sets are S-open set, is S-open set in . Thus is S-continuous. Again consider is a S-open in . Since is a S-open map, is S-open set in . We know that all S-open sets are S-open set, is S-open set in . Then is a S-open map. Hence is S-homeomorphism.

**Remark 3.4.** S-homeomorphism need not be S-homeomorphism.

**Example 3.5.** Consider Define and as and . Consider the S-topologies where and where . Therefore the S-mapping is both S-continuous and S-open map, but is not S- continuous and S-open map. Since, is not S-open set in , also is not S-closed set in . Hence, is not S-homeomorphism.

**Theorem 3.6.** A S-bijective map is S-continuous, then the following are equivalent

1. is S-open map
2. is S-homeomorphism
3. is S-closed map

**Proof.** (i) ⇒ (ii) Since a S-bijective map is S-continuous, also is S-open map. Then is S-homeomorphism.

(ii) ⇒ (iii) Consider is s-closed set in . Then is s-open set in . By hypothesis, is S-open set in . Therefore, is S- closed set in . Hence, is S-closed map.

(iii) ⇒ (i) Consider is s-open set in . Then is s-closed set in . By hypothesis, is S-closed set in . Therefore, is S- open set in. Hence, is S-open map.

**Theorem 3.7.** Every S-homeomorphism from a S- space into another S- space is S-homeomorphism.

**Proof.** Consider a s-map is S-homeomorphism and be a s-open set in . Since is S-open and is a S- space, is s-open set in . Thus, is a s-open map. Since is S-continuous and is a S- space, is s-closed in. Therefore, is s-continuous. Hence, is s-homeomorphism.

**Theorem 3.8.** Consider be a S-topological space, then is a group under the composition of maps.

**Proof.** Let us define a binary operation by for all and is the usual operation of composition of maps. Then . We know that the composition of maps is associative, also the identity map belonging to serves as the identity element. For every , . Therefore the inverse exists for each element of . Hence, forms a group under the operation of composition of maps.

**Theorem 3.9.** If is a S-homeomorphism. Then, induces a S-isomorphism from the group onto the group .

**Proof.** Suppose . We define a function by for every . Therefore is a S-bijection. Now for all , we get .

**Theorem 3.10.** Consider be a S-homeomorphism and be a S-closed subset of . Also, be a s-closed subset of such that . Assume that is closed under any s-intersection. Then the restriction is a S-homeomorphism.

**Proof.**

1. Consider is a S-homeomorphism. Since is S-1-1 and s-onto, is S-1-1 and such that is s-onto. Hence, is s-bijection.
2. Consider be an s-open set of . , for some s-open set in . Since, is S-1-1 then . Since, m is S-open and is s-open set in , then is S-open in . Therefore is a S-open in . Hence, is S-open map.
3. Consider be a s-closed in . Then for some s-closed set in . Since is a s-closed set in , then is a s- closed set in . By hypothesis and assumption, (say) is a S-closed set in . Since , It is adequate to prove that is S-closed in . Let be S--open set in such that . Then by hypothesis and by relativity of S--open set, is S--open set in . Since is a S-closed set in , . Since is S-open, and so is S-closed in .

Therefore, is s-bijection, S-continuous and S-open map. Hence the restriction map is a S-Homeomorphism.

**Theorem 3.11.** If is S-strongly continuous and be a S-homeomorphism then is S- homeomorphism.

**Proof.** Consider be a s-open set in . Since every s-open set is S-open set, is a s-open set in . Then is S-open set in Also, consider be a s-closed in , is a S-closed set in . Since is S-strongly continuous and is S-continuous, is S-closed set in . Thus is S-open map and S-continuous. Hence, is S- homeomorphism.

**IV. S-STRONGLY HOMEOMORPHISM**

**Definition 4.1.** A s-bijective map from to is S-strongly homeomorphism if and are both S-irresolute.

**Example 4.2.** Consider Define and as and . Consider the S-topologies where and where . Therefore the s-mapping is S- irresolute also the s-mapping is S-irresolute. Hence, is S-strongly homeomorphism.

**Theorem 4.3.** If a s-map is S-strongly homeomorphism then is a S-homeomorphism.

**Proof.** Consider be a S-strongly homeomorphism, then and are S-irresolute. Since every S-irresolute map is S-continuous and is S-open map if is S-continuous. Then is S-continuous and S-open map. Hence, is S- homeomorphism.

**Remark 4.4.** The reverse implication of the above theorem is not true.

**Example 4.5.** Let Define and as and . Consider the S-topologies where and where ,, . Therefore the s-mapping is both S-continuous and S-open map, but is S-irresolute and is not S-irresolute. Since, is not S-closed set in . Hence, is not S-strongly homeomorphism.

**Theorem 4.6.** If is a S-strongly homeomorphism, then for every s-subset of .

**Proof.** Consider is a S-strongly homeomorphism, then is S-strongly continuous. Since is S-closed set in , is S-closed in . Now, and so . Also, since is a S-strongly homeomorphism, then is S-irresolute. Since is S-closed set in , is S-closed in . Now, then . Therefore, . Hence, .

**Theorem 4.7:** If is a S-strongly homeomorphism, then for all .

**Proof.** Consider is a S-strongly homeomorphism, then is S-irresolute. Since is a S-closed set in , is S-closed set in . Now, then . Also, since is a S-strongly homeomorphism, is S-irresolute. Since is S- closed set in , is S-closed set in . Now , . Thus and hence the equality holds.

**Theorem 4.8.** The composition of two S-strongly homeomorphism is S-strongly homeomorphism.

**Proof.** Consider the s-mappings and be two S-strongly homeomorphism and be a S-closed set in . Since and are S- irresolute, is S-closed set in . Thus is S- irresolute. Now, consider is S-closed set in . Since and are S- irresolute, is S-closed set in . Therefore, is S-irresolute. Hence, is S-strongly homeomorphism.

**Theorem 4.9.** Consider be a S-strongly homeomorphism and be a S-homeomorphism then is S- homeomorphism.

**Proof.** Consider be a S-closed set in . Since, is S-irresolute and is S- continuous. is S-closed set in . Therefore,  is S-continuous. Now, consider be a s-open set in , is S-open set in . Since, is S-irresolute and is S-open map. Thus  is S-open map. Hence,  is S-homeomorphism.

v. CONCLUSION

We have introduced S-closed (open) maps with S-closed (open) set in S-topological space. We have learned some of its properties and composition theorems. Then we have studied about S-strongly closed maps and S-quasi closed maps with some properties.

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