Some Fixed Point Theorems for Occasionally WeaklyCompatible Mappings in Intuitionistic Fuzzy Metric Spaces.

Neerja Namdeo[1], Dr. U.K.Shrivastava[2]

[1]Assistant Professor, Govt. Dau Kalyan Arts and Commerce Postgraduate College, BalodaBazar
Distt - BalodaBazar(C.G.), India(neerjanamdeo1982@gmail.com)

[2]Professor, Govt. E.R.P.G.College, Bilaspur
Distt - Bilaspur(C.G.), India(profumesh18@yahoo.co.in)

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Abstract

Common fixed point, occasionally weakly Compatible mapping, Implicit relations, Complete intuitionistic fuzzy metric spaces.

Introduction

Fuzzy set theory was introduced by Zadeh in 1965 [16].Many authors have introduced and discussed several notions of fuzzy metric space in different ways [10], [4], [5] and also proved fixed point theorems with interesting consequent results in the fuzzy metric spaces [6]. The important result in the theory of fixed point of compatible mappings obtained by G. Jungck[12, 13] and the concept of intuitionistic fuzzy metric space was given by Park [13] and the subsequent fixed point results in the intuitionistic fuzzy metric spaces are investigated by Alaca and et al. [2] and Mohamad [10]. Al-Thagafi and N.Shahzad[3] introduced the concept of occasionally weakly compatible maps.

In this paper, as an application of occasionally weakly compatible mappings, we prove common fixed point theorems for intuitionistic fuzzy metric spaces.

Preliminaries

Definition 1.1[14] A binary operation $*$: $[0,1]\times[0,1] \rightarrow [0,1]$ is *continuous t-norm* if $*$ is satisfying the following conditions :

(i) $*$ is commutative and associative,

(ii) $*$ is continuous,

(iii) $a * 1 = a$ for all $a \in [0,1]$,

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d $\in [0,1]$.

Definition 1.2 A binary operation $\Diamond$: $[0,1]\times[0,1] \rightarrow [0,1]$ is *continuous t-conorm* if $\Diamond$ is satisfying the following conditions :

(i) $\Diamond$ is commutative and associative,

(ii) $\Diamond$ is continuous,

(iii) $a \Diamond 0 = a$ for all $a \in [0,1]$,

(iv) $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ for all a, b, c, d $\in [0,1]$.

Definition 1.3[2] A 5- tuple $(X, M, N, *, \Diamond)$ is called a *intuitionistic fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: For all x, y, z $\in$ X and s, t > 0

(IFM-1) $M(x, y, t) + N(x, y, t) \leq 1$,
(IFM-2) $M(x, y, 0) = 0$,
(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$,
(IFM-4) $M(x, y, t) = M(y, x, t)$,
(IFM-5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(IFM-6) $M(x, y, .) : (0, \infty) \to (0,1]$ is left continuous,
(IFM-7) $\lim_{t\to\infty} M(x, y, t) = 1$,
(IFM-8) $N(x, y, 0) = 1$,
(IFM-9) $N(x, y, t) = 0$ if and only if $x = y$,
(IFM-10) $N(x, y, t) = N(y, x, t)$,
(IFM-11) $N(x, y, t) \lozenge N(y, z, s) \leq N(x, z, t + s)$,
(IFM-12) $N(x, y, .) : (0, \infty) \to (0,1]$ is right continuous,
(IFM-13) $\lim_{t\to\infty} N(x, y, t) = 0$,

Then (M, N) is called an *intuitionistic fuzzy metric on X*. The function M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 1.4 Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1\text{-}M, * ,\lozenge)$ such that t-norm $*$ and t-conorm $\lozenge$ are associated i.e. $x \lozenge y = 1- ((1-x) * (1-y))$ for all $x, y \in X$.

Example 1.5(Induced intuitionistic fuzzy metric space) Let (X, d) be a metric space. Define $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$ for all $a, b \in [0, 1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t+d(x,y)}, \quad N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}.$$

Then $(X, M_d, N_d, * ,\lozenge)$ is an intuitionistic fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric space.

Definition 1.6[2] Let $(X, M, N, * ,\lozenge)$ be an intuitionistic fuzzy metric space. Then
(a)A sequence $\{x_n\}$ in X is said to be *convergent to a point* x in X if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each $t > 0$.
(b)A sequence $\{x_n\}$ in X is called *Cauchy sequence* if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$ for each $p > 0$ and $t > 0$.
(c)An intuitionistic fuzzy metric space $(X, M, N, * ,\lozenge)$ is said to be *complete* if and only if every Cauchy sequence in X is convergent in X.

Lemma 1.7[13] Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, * ,\lozenge)$ with $t * t \geq t$ and $(1-t) \lozenge (1-t) \leq (1-t)$ for all $t \in [0, 1]$. If $\exists$ a number $q \in (0, 1)$ such that $M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, qt) \leq N(x_{n+1}, x_n, t)$, for all $t > 0$ and $n \in \mathbf{N}$, then $\{x_n\}$ is a Cauchy sequence in X.
Proof :– For $t > 0$ and $q \in (0, 1)$ we have,

$$M(x_2, x_3, qt) \geq M(x_1, x_2, t) \geq M(x_0, x_1, t/q)$$

or

$$M(x_2, x_3, t) \geq M(x_0, x_1, t/q^2)$$

By simple induction, we have for all $t > 0$ and $n \in \mathbf{N}$

$$M(x_{n+1}, x_{n+2}, t) \geq M(x_1, x_2, t/q^n)$$

Thus for any positive number p and real number $t > 0$, we have

$$M(x_n, x_{n+p}, t) \geq M(x_n, x_{n+1}, t/p) * \ldots\ldots * M(x_{n+p-1}, x_{n+p}, t/p) \quad \text{[By IFM – 5]}$$
$$\geq M(x_1, x_2, t/pq^{n-1}) * \ldots\ldots * M(x_1, x_2, t/pq^{n+p-2})$$

Therefore by IFM – 7, we have

$$M(x_n, x_{n+p}, t) \geq 1 * \ldots\ldots * 1 \geq 1,$$

Similarly, for $t > 0$ and $q \in (0, 1)$ we have,

$N(x_2, x_3, qt) \leq N(x_1, x_2, t) \leq N(x_0, x_1, t/q)$

or

$N(x_2, x_3, t) \leq N(x_0, x_1, t/q^2)$

By simple induction, we have for all $t > 0$ and $n \in \mathbf{N}$

$N(x_{n+1}, x_{n+2}, t) \leq N(x_1, x_2, t/q^n)$

Thus for any positive number p and real number $t > 0$, we have

$N(x_n, x_{n+p}, t) \leq N(x_n, x_{n+1}, t/p) \lozenge \ldots\ldots \lozenge N(x_{n+p-1}, x_{n+p}, t/p)$ [By IFM – 11]

$\leq N(x_1, x_2, t/pq^{n-1}) \lozenge \ldots\ldots \lozenge N(x_1, x_2, t/pq^{n+p-2})$

Therefore by IFM – 13, we have

$N(x_n, x_{n+p}, t) \leq 0 \lozenge \ldots\ldots \lozenge 0 \leq 0,$

$\Rightarrow \{x_n\}$is a Cauchy sequence in X. This completes the proof ♦

Lemma 1.8[13] Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. If $\forall$ x, y $\in$ X and $t > 0$ with positive number $q \in (0, 1)$ and $M(x, y, qt) \geq M(x, y, t)$ and $N(x, y, qt) \leq N(x, y, t)$, then x = y.

Proof :– If for all $t > 0$ and some constant $q \in (0, 1)$, then we have

$M(x, y, t) \geq M(x, y, t/q) \geq M(x, y, t/q^2) \geq \ldots\ldots \geq M(x, y, t/q^n) \geq \ldots,$

and $N(x, y, t) \leq N(x, y, t/q) \leq N(x, y, t/q^2) \leq \ldots\ldots \leq N(x, y, t/q^n) \leq \ldots,$

$n \in \mathbf{N}$ and for all $t > 0$ and x, y $\in$ X. Let $n \to \infty$, we have $M(x, y, t) = 1$ and $N(x, y, t) = 0$ and thus x = y ♦

Definition 1.9[7] Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are called *compatible* if $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$, $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$ whenever $\{x_n\}$ is a sequence in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$, for some $x \in X$.

Definition 1.10[8]Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are called *weakly compatible* if they commute at their coincidence points. i.e. if Au = Su for some $u \in X$, then ASu = SAu.

Definition 1.11[1]Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ are called *occasionally weakly compatible (owc)* if and only if a point x in X which is coincidence point of A and S at which A and S commute.

Lemma 1.12[1] Let A and S are two owc self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$. If A and S have unique point of coincidence, w = Ax = Sx, then w is unique common fixed point of A and S.

Proof: Since A and S are owc, there exists a point *x* in *X* such that w = Ax = Sx and ASx = SAx.Thus, AAx = ASx = SAx , which says that AAx is also a point of coincidence of A and S. Since the point of coincidence w = Ax is unique by hypothesis, SAx = AAx = Ax, and w = Ax is a common fixed point of A and S.

Moreover, if z is any common fixed point of A and S then z = Az = Sz = w by the uniqueness of the point of coincidence♦

A. Al-Thagafi and Naseer Shahzad [3] shown that occasionally weakly is weakly compatible but converse is not true.

Example 1.13[3] Let R be the usual metric space. Define S, T: $R \to R$ by Sx = 2x and $Tx = x^2$ for all $x \in R$. Then Sx = Tx for x = 0,2 but ST0 =TS0, and ST2 $\neq$ TS2. S and T are occasionally weakly compatible self maps but not weakly compatible.

Main Results

Following theorem is given by [12]

Theorem Let (X, M, $*$) be a complete fuzzy 2-metric space and Let A, B, S and Tbe self mappings of X. Let the pairs {A, S} and {B, T} be owc. If there exist $q \in (0,1)$ such that

$$M(Ax, By, a, qt) \geq \alpha_1 M(Sx, Ty, a, t) + \alpha_2 M(Ax, Ty, a, t) + \alpha_3 M(By, Sx, a, t) \quad (3.1)$$

For all x, y $\in$ X, where $\alpha_1, \alpha_2, \alpha_3 > 0$, $\alpha_1 + \alpha_2 + \alpha_3 > 1$ then there exist a unique point w $\in$ X such that Aw = Sw = w and a unique point z $\in$ X such that Bz = Tz = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Here we generalized this theorem in intuitionistic fuzzy metric spaces as follows:

Theorem 2.1 Let the pairs (A, S) and (B, T) are occasionally weakly compatible self mappings on complete intuitionistic fuzzy metric space (X, M, N, *, ◊) for any x, y $\in$ X and t > 0 with positive number q $\in$ (0, 1) such that

$$M(Ax, By, qt) \geq \alpha_1 M(Sx, Ty, t) + \alpha_2 M(Ax, Ty, t) + \alpha_3 M(By, Sx, t) \quad \ldots(i)$$

and

$$N(Ax, By, qt) \leq \beta_1 N(Sx, Ty, t) + \beta_2 N(Ax, Ty, t) + \beta_3 N(By, Sx, t), \quad \ldots(ii)$$

For all x, y $\in$ X, where $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3 > 0$, $\alpha_1 + \alpha_2 + \alpha_3 > 1$ and $\beta_1 + \beta_2 + \beta_3 < 1$ then there exist a unique point w $\in$ X such that Aw = Sw = w and a unique point z $\in$ X such that Bz = Tz = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Proof: Let the pairs {A, S} and {B, T} be owc, so there are points $x, y \in X$ such that

Ax = Sx and By = Ty . We claim that, Ax = By. If not, by inequality (i)

$$\begin{aligned} M(Ax, By, qt) &\geq \alpha_1 M(Sx, Ty, t) + \alpha_2 M(Ax, Ty, t) + \alpha_3 M(By, Sx, t) \\ &= \alpha_1 M(Ax, By, t) + \alpha_2 M(Ax, By, t) + \alpha_3 M(By, Ax, t) \\ &= (\alpha_1 + \alpha_2 + \alpha_3) M(Ax, By, t) \end{aligned}$$

Which gives contradiction because $\alpha_1 + \alpha_2 + \alpha_3 > 1$.

Similarly, by inequality (ii)

$$\begin{aligned} N(Ax, By, qt) &\leq \beta_1 N(Sx, Ty, t) + \beta_2 N(Ax, Ty, t) + \beta_3 N(By, Sx, t) \\ &= \beta_1 N(Ax, By, t) + \beta_2 N(Ax, By, t) + \beta_3 N(By, Ax, t) \\ &= (\beta_1 + \beta_2 + \beta_3) N(Ax, By, t) \end{aligned}$$

a contradiction, since $(\beta_1 + \beta_2 + \beta_3) < 1$. And by Lemma 1.8 Ax = By, i.e. Ax = Sx = By = Ty. Suppose that there is another point z such that Az = Sz, then by (i) and (ii), we have Az = Sz = By = Ty. So, Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 1.12 w is the only common fixed point of A and S, i. e. w = Aw = Sw. Similarly there is a unique point z $\in$ X such that z = Bz = Tz.

Assume that w $\neq$ z. We have,

$$\begin{aligned} M(w, z, qt) &= M(Aw, Bz, qt) \\ &\geq \alpha_1 M(Sw, Tz, t) + \alpha_2 M(Aw, Tz, t) + \alpha_3 M(Bz, Sw, t) \\ &= \alpha_1 M(w, z, t) + \alpha_2 M(w, z, t) + \alpha_3 M(z, w, t) \\ &= (\alpha_1 + \alpha_2 + \alpha_3) M(w, z, t) \end{aligned}$$

Which gives contradiction because $(\alpha_1 + \alpha_2 + \alpha_3) > 1$.

Similarly,

$$\begin{aligned} N(w, z, qt) &= N(Aw, Bz, qt) \\ &\leq \beta_1 N(Sw, Tz, t) + \beta_1 N(Aw, Tz, t) + \beta_1 N(Bz, Sw, t) \end{aligned}$$

$$= \beta_1 N(w, z, t) + \beta_2 N(w, z, t) + \beta_3 N(z, w, t)$$

$$= (\alpha_1 + \alpha_2 + \alpha_3) N(w, z, t)$$

a contradiction, since $(\beta_1 + \beta_2 + \beta_3) < 1$. And by Lemma 1.8 z = w. Also by Lemma 1.12, z is the common fixed point of A, B, S and T. The uniqueness of the fixedpoint holds from (i) and (ii) ♦

Theorem 2.2 Let the pairs (A, S) and (B, T) are occasionally weakly compatible self mappings on complete intuitionistic fuzzy metric space (X, M, N, *, ◊) for any x, y ∈ X and t > 0 with positive number q ∈ (0, 1) such that

$$M (Ax, By, qt) \geq \alpha_1 \min\{ M (Sx, Ty, t), M (Sx, Ax, t)\} + \beta_1 \min\{ M(By, Ty, t), M (Ax, Ty, t)\} + \gamma_1 M (By, Sx, t) \quad \ldots(iii)$$

and

$$N (Ax, By, qt) \leq \alpha_2 \min\{ N(Sx, Ty, t), N(Sx, Ax, t)\} + \beta_2 \min\{ N(By, Ty, t), N(Ax, Ty, t)\} + \gamma_2 N(By, Sx, t) \quad \ldots(iv)$$

For all x, y ∈ X, where $\alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2 > 0$, $\alpha_1 + \beta_1 + \gamma_1 > 1$ and $\alpha_2 + \beta_2 + \gamma_2 < 1$then there exist a unique point w ∈ X such that Aw = Sw = w and a unique point z ∈ X such that Bz = Tz = z. Moreover, z = w, so that there is a unique common fixed point of A, B, S and T.

Let the pairs {A, S} and {B, T} be owc, so there are points x, y ∈ X such that

Ax = Sx and By = Ty . We claim that, Ax = By. If not, by inequality (iii)

$$M (Ax, By, qt) \geq \alpha_1 \min\{M(Sx, Ty, t), M(Sx, Ax, t)\} + \beta_1 \min\{M(By, Ty, t), M(Ax, Ty, t)\} + \gamma_1 M (By, Sx, t)$$

$$= \alpha_1 \min\{M(Ax, By, t), M(Ax, Ax, t)\} + \beta_1 \min\{M(By, By, t), M(Ax, By, t)\} + \gamma_1 M(By, Ax, t)$$

$$= \alpha_1 \min\{M(Ax, By, t), 1\} + \beta_1 \min\{1, M (Ax, By, t)\} + \gamma_1 M (By, Ax, t)$$

$$= \alpha_1 M(Ax, By, t) + \beta_1 M(Ax, By, t) + \gamma_1 M(Ax, By, t)$$

$$= (\alpha_1 + \beta_1 + \gamma_1) M (Ax, By, t)$$

Which gives contradiction because $\alpha_1 + \beta_1 + \gamma_1 > 1$

Similarly, by inequality (iv)

$$N (Ax, By, qt) \leq \alpha_2 \min\{N(Sx, Ty, t), N (Sx, Ax, t)\} + \beta_2 \min\{N(By, Ty, t), N(Ax, Ty, t)\} + \gamma_2 N (By, Sx, t)$$

$$= \alpha_2 \min\{N(Ax, By, t), N (Ax, Ax, t)\} + \beta_2 \min\{N(By, By, t), N(Ax, By, t)\} + \gamma_2 N(By, Ax, t)$$

$$= \alpha_2 \min\{N(Ax, By, t), 1\} + \beta_2 \min\{1, N (Ax, By, t)\} + \gamma_2 N(By, Ax, t)$$

$$= \alpha_2 N(Ax, By, t) + \beta_2 N(Ax, By, t) + \gamma_2 N(Ax, By, t)$$

$$= (\alpha_2 + \beta_2 + \gamma_2) N(Ax, By, t)$$

a contradiction, since $\alpha_2 + \beta_2 + \gamma_2 < 1$. And by Lemma 1.8 Ax = By, i.e.Ax = Sx = By = Ty. Suppose that there is another point z such that Az = Sz, then by (iii) and (iv), we have Az = Sz = By = Ty. So, Ax = Az and w = Ax = Sx is the unique point of coincidence of A and S. By Lemma 1.12 w is the only common fixed point of A and S, i. e. w = Aw = Sw. Similarly there is a unique point z ∈ X such that z = Bz = Tz.

Assume that w ≠ z. We have,

$$M (w, z, qt) = M (Aw, Bz, qt)$$

$$\geq \alpha_1 \min\{M(Sw, Tz, t), M (Sw, Aw, t)\} + \beta_1 \min\{M(Bz, Tz, t), M(Aw, Tz, t)\} + \gamma_1 M (Bz, Sw, t)$$

$$= \alpha_1 \min\{M(w, z, t), M (w, w, t)\} + \beta_1 \min\{M (z, z, t), M(w, z, t)\} + \gamma_1 M (z, w, t)$$

$$= \alpha_1 \min\{M (w, z, t), 1\} + \beta_1 \min\{1, M (w, z, t)\} + \gamma_1 M (w, z, t)$$

$$= \alpha_1 M(w, z, t) + \beta_1 M(w, z, t) + \gamma_1 M(w, z, t)$$

$= (\alpha_1 + \beta_1 + \gamma_1)\, M(w, z, t)$

Which gives contradiction because $\alpha_1 + \beta_1 + \gamma_1 > 1$

Similarly,

$$
\begin{aligned}
N(w, z, qt) &= N(Aw, Bz, qt) \\
&\leq \alpha_2 \min\{N(Sw, Tz, t), N(Sw, Aw, t)\} + \beta_2 \min\{N(Bz, Tz, t), N(Aw, Tz, t)\} + \gamma_2 N(Bz, Sw, t) \\
&= \alpha_2 \min\{N(w, z, t), N(w, w, t)\} + \beta_2 \min\{N(z, z, t), N(w, z, t)\} + \gamma_2 N(z, w, t) \\
&= \alpha_2 \min\{N(w, z, t), 1\} + \beta_2 \min\{1, N(w, z, t)\} + \gamma_2 N(w, z, t) \\
&= \alpha_2 N(w, z, t) + \beta_2 N(w, z, t) + \gamma_2 N(w, z, t) \\
&= (\alpha_2 + \beta_2 + \gamma_2)\, N(w, z, t)
\end{aligned}
$$

a contradiction, since $\alpha_2 + \beta_2 + \gamma_2 < 1$. And by Lemma 1.8 z = w. Also by Lemma1.12, z is the common fixed point of A, B, S and T. The uniqueness of the fixedpoint holds from (i) and (ii) ♦

Conclusion

We prove common fixed point results for occassionally weakly compatible in Intuitionistic Fuzzy Metric Spaces which improve and generalize the result of various authors present in fixed point theory of Fuzzy Metric Spaces.

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