Study of Blast Wave Problem in a Non-ideal Dusty Gas

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ABSTRACT

In this paper an analytical approach is used to derive the exact solution of Euler equations governing the propagation of blast wave for one dimensional adiabatic flow with generalized geometries. Here it is assumed that the density ahead of the shock front varies according to a power of the distance from the source of the explosion. The effect of dust particles and the parameter of non-ideal ness on the radius of blast wave are analyzed. An analytical expression for the total energy carried by blast wave in non-ideal dusty gas is derived. The effect of spherically small solid particles present in dusty gas on the total energy carried by blast wave is also discussed.

Keywords: Blast Wave; Non-ideal Gas; Dusty Gas; Analytical Solution; Total Energy.

1. Introduction

Dusty gas is a mixture of gas and spherically small solid particles. The study of the effect of solid particles on the growth and decay process of strong shock wave yields interesting results applicable to problems arising in the area of astrophysical fluid. In cases when velocity of mixture is very high, the dust particles presented in the mixture behaves like a pseudo fluid. When a strong shock wave is propagated in a non-ideal gas with dust particles, the radius of strong shock wave, the pressure, the density, the speed of shock wave and the energy carried by a strong shock wave change across the shock, and have a significant difference from those which arise when the strong shock wave passes through an ideal and dust-free gas. A variety of phenomenon occurs in astrophysical fluid where mass fraction of solid particles is very small in comparison to the gas particles such as supernova explosions, photo ionized gas, volcanic jets, solid particle motion in rocket exhaust etc. Also, a valid guess regarding the propagation of strong shock waves in a non-ideal dusty gas is very important for the design of warfare and operation of space vehicles.

When a large amount of energy is instantaneously released from a core, a disturbance in the medium is propagated headed by a compressive wave, called a blast wave. Such type of problem is formulated mathematically as a system of quasilinear hyperbolic system of partial differential

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equations. To find the exact solution of the system of equations governing the blast wave problem is almost impossible. In past many attempts have been made to find the analytical/approximate analytical solution of governing system of the blast wave problem using physically relevant assumptions. Taylor (1950a, 1950b) estimated the relationship between the energy input of an extremely powerful explosion and the growth of the resulting fireball and a detailed analysis of Taylor's work is presented by Sedov (1959). Singh *et al.* (1984) studies the flow behind an attached shock wave in a radiating gas. Sachdev *et al.* (2005) obtained the exact solutions of compressible flow equations in spherically symmetric coordinate system for ordinary gas. Murata (2006) have derived the exact solution for the one dimensional blast wave problem for ideal gas dynamics with generalized geometry. Singh *et al.* (2011) studies the imploding of shocks in non-ideal magnetogasdynamics using similarity method. Singh *et al.* (2012) studies the evolution and decay of acceleration waves in radiative magnetogasdynamics Singh *et al.* (2012) obtained the quasi-similar solution of the strong shock wave problem in non-ideal gas

Due to various important applications of blast wave theory, a continuous improvement in the field is desirable. Since blast wave caries gas and small dust particles, so the study of blast wave problem for non-ideal dusty gas is more realistic than the ordinary gas dynamic system. In the present paper, an attempt has been made to find the closed form solution of the system of equations governing the propagation of a blast wave in non-ideal dusty gas with generalized geometry. Here, it is assumed that the density ahead of the shock front varies according to power of the distance from the source of explosion. An expression for the total energy is also determined.

2. Basic equations

The governing equations describing a non-planar adiabatic non-ideal dusty gas flow are given as [6, 3]

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+\rho\left(\frac{\partial u}{\partial x}+\frac{m}{x}u\right)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}=0, \tag{2}$$

$$\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial x}+c^2\rho\left(\frac{\partial u}{\partial x}+\frac{m}{x}u\right)=0, \tag{3}$$

where $\rho>0$, u and p are density, flow velocity and pressure of the non-ideal dusty gas respectively and $t>0, x\in \mathbb{R}$. $c=\left(\left(\left(\Gamma-\alpha_2\rho^2\right)p\right)/\left(\left(1-\alpha_1\rho+\alpha_2\rho^2\right)\rho\right)\right)^{1/2}$ denotes the speed of sound in the non-ideal dusty gas with $\alpha_1=\theta+\bar{b}$, $\alpha_2=\theta\bar{b}$, $\bar{b}=b\left(1-k_p\right)$ where b is the Van der Waals excluded volume and $k_p=m_{sp}/m_{gd}$ is the mass fraction of solid particles in non-ideal dusty gas, where m_{sp} and m_{gd} are the masses of solid particles and non-ideal dusty gas respectively. $m=0$, 1 and 2 corresponds to the planar, cylindrically symmetric and spherically symmetric flows respectively. The specific heat of non-ideal dusty gas at constant pressure is

given by $c_{pd} = k_p c_{sp} + \left(1-k_p\right) c_p$, where c_p and c_{sp} stands for specific heat of gas and solid particle respectively. If c_{vd} denotes the specific heat of non-ideal dusty gas at constant volume, the ratio of specific heats for dusty gas is given by S. I. Pai. (1977)
$\Gamma = \dfrac{c_{pd}}{c_{vd}} = \dfrac{\gamma\left(1+\delta\beta\right)}{1+\delta\beta}$, where $\delta = k_p/(1-k_p)$, $\beta = c_{sp}/c_p$, $\gamma = c_p/c_v$, c_v specific heat of gas at constant volume. Equation of state for adiabatic non-ideal dusty gas flow is given by Chadha and Jena (2014), $p = \dfrac{\left(1-k_p\right)}{(1-Z)\left(1-\bar{b}\rho\right)}\rho RT$, where $Z = \left(Z_0\rho/\rho_0\right)$ denotes the volume fraction of small solid particle in the mixture and the subscript $'0'$ denotes the value of physical entities in undisturbed region. The relation between entities Z and k_p is given by Pai et al. (1980) as $k_p = Z\rho_{sp}/\rho$, where ρ_{sp} stands for density of solid particles in the mixture. Since mass fraction of solid particle must be constant in the equilibrium flow therefore Z/ρ = constant (say θ), i.e. $Z = \theta\rho$.

3. Boundary conditions

Let R be the position of the shock front from the centre of explosion and is a function of time t, then the propagation velocity of shock front, s, is given by

$$s = \frac{dR}{dt}. \tag{4}$$

If ahead of shock front the undisturbed volume fraction of solid particles and density of the mixture are Z_0, ρ_0 and pressure, density, velocity just behind the shock are p, ρ, u. Then we have the following Rankine-Hugoniot conditions across the shock front

$$\rho = \frac{\Gamma+1}{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}\rho_0, \tag{5}$$

$$u = \frac{2(1-\bar{b}\rho_0 - Z_0)}{\Gamma+1}s, \tag{6}$$

$$p = \frac{2\left(1-\bar{b}\rho_0 - Z_0\right)}{\Gamma+1}\rho_0 s^2. \tag{7}$$

In the present problem, the undisturbed density ρ_0 is taken to vary according to the power law of the radius of the shock front R after the explosion and is given as

$$\rho_0 = \rho_a R^{\alpha}, \tag{8}$$

where ρ_a and α are constants. The constant α is to be determined later.

4. Exact solution of the blast wave problem

The expression for the pressure behind the shock front satisfying the RH conditions given by equations (5)–(7) is given as

$$p = \frac{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2\left(1 - \bar{b}\rho_0 - Z_0\right)} \rho u^2 . \tag{9}$$

By equation (8), equations (2) and (3), can be rewritten as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \kappa_1 \left(\frac{u^2}{\rho}\frac{\partial \rho}{\partial x} + 2u\frac{\partial u}{\partial x} \right) = 0 , \tag{10}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \kappa_2 \left(\frac{\partial u}{\partial x} + \frac{m}{x} u \right) u = 0 , \tag{11}$$

where κ_1 and κ_2 are given as

$$\kappa_1 = \frac{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2\left(1 - \bar{b}\rho_0 - Z_0\right)},$$

$$\kappa_2 = \frac{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)\left[(\Gamma - 1)\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right) + \alpha_1(\Gamma + 1)\rho_0\right] - 2\alpha_2(\Gamma + 1)^2 \rho_0{}^2}{2\left[\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)\left\{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right) - \alpha_1(\Gamma + 1)\rho_0\right\} + \alpha_2(\Gamma + 1)^2 \rho_0{}^2\right]} .$$

Combining (10) and (11) and after integration we have the resulting equation as,

$$f(t) = \rho u^{\xi} x^{-m\eta} , \tag{12}$$

where $f(t)$ is function of time only and ξ and η are given as

$\xi = \left(2\kappa_1 - \kappa_2\right)/\kappa_1,$

$\eta = \kappa_2/\kappa_1 .$

By equation (12) and (1), we have

$$\frac{\xi}{u}\frac{\partial u}{\partial t} - (1 - \xi)\frac{\partial u}{\partial x} - \frac{m(\eta + 1)}{x} u - \frac{1}{f}\frac{df}{dt} = 0 . \tag{13}$$

Solving equations (11) and (13), we have

$$u = -\chi \frac{x}{f}\frac{df}{dt} , \tag{14}$$

where χ is given as

$$\chi = \frac{1}{\left(\xi\kappa_2 + \eta + 1\right)m + \left(1 + \kappa_2\xi\right)} . \tag{15}$$

Also $f(t) = f_0 t^{-\tau}$, (16)

where, f_0 is arbitrary constant and τ is given as

$$\tau = \frac{\xi}{\left\{ (\xi - 1)\chi - m(\eta + 1)\chi + 1 \right\}} .$$

Rankine- Hugoniot condition (5) yields the analytical expression for the radius of the shock front as

$$R(t) = t^{\frac{\Gamma+1}{2(1-\bar{b}\rho_0 - Z_0)}\chi\tau} . \tag{17}$$

Rankine- Hugoniot condition (5) yields the following value of α which is given as

$$\alpha = \frac{(m-1)(\Gamma+1) - 2\left(1 - \bar{b}\rho_0 - Z_0\right)\left\{(\eta+1)m - \xi + 1\right\}}{(\Gamma+1)} .$$

Therefore, the solution of strong shock wave problem is given as

$$\rho = \frac{f_0 t^{-\tau+\xi} x^{m\eta-\xi}}{\chi^{\xi}\tau^{\xi}} , u = \chi\tau\, x/t , \; p = \frac{\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2\left(1 - \bar{b}\rho_0 - Z_0\right)} \frac{1}{(\chi\tau)^{\xi-2}} f_0\, x^{(m\eta-\xi+2)} t^{(\xi-\tau-2)} . \tag{18}$$

After determining the physical quantities density, velocity and pressure behind the shock front, we can also calculate the total energy E (sum of kinetic and heat energy) within the blast wave in non-ideal dusty gas at any time t as [5]

$$E = 4\pi \int_0^R \left\{ \frac{1}{2}\rho u^2 + \frac{(1-Z)\left(1-\bar{b}\rho\right)}{\Gamma - 1} p \right\} x^m dx , \tag{19}$$

$$E = At^{\frac{(\Gamma+1)\chi\tau\{m(\eta+1)-\xi+3\}}{2(1-\bar{b}\rho_0 - Z_0)}+\xi-\tau-2} + Bt^{\frac{(\Gamma+1)\chi\tau\{m(\eta+1)-\xi+3\}}{2(1-\bar{b}\rho_0 - Z_0)}+\xi-\tau-2} + Ct^{\frac{(\Gamma+1)\chi\tau\{m(2\eta+1)-2\xi+3\}}{2(1-\bar{b}\rho_0 - Z_0)}+2\xi-2\tau-2} + Dt^{\frac{(\Gamma+1)\chi\tau\{m(2\eta+1)-2\xi+3\}}{2(1-\bar{b}\rho_0 - Z_0)}+2\xi-2\tau-2} + Et^{\frac{(\Gamma+1)\chi\tau\{m(3\eta+1)-3\xi+3\}}{2(1-\bar{b}\rho_0 - Z_0)}+3\xi-3\tau-2} , \tag{20}$$

where $A = \dfrac{f_0}{2(\chi\tau)^{\xi-2}\left\{m(\eta+1) - \xi + 3\right\}}$, $\quad B = \dfrac{f_0\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2(\Gamma-1)\left(1 - \bar{b}\rho_0 - Z_0\right)(\chi\tau)^{\xi-2}\left\{m(\eta+1) - \xi + 3\right\}}$,

$$C = \frac{-bf_0^{\,2}\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2(\Gamma-1)\left(1 - \bar{b}\rho_0 - Z_0\right)(\chi\tau)^{2\xi-2}\left\{m(2\eta+1) - 2\xi + 3\right\}} ,$$

$$D = \frac{-\theta f_0^{\,2}\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2(\Gamma-1)\left(1 - \bar{b}\rho_0 - Z_0\right)(\chi\tau)^{2\xi-2}\left\{m(2\eta+1) - 2\xi + 3\right\}} ,$$

$$E = \frac{b\theta f_0^{\,3}\left(\Gamma - 1 + 2\bar{b}\rho_0 + 2Z_0\right)}{2(\Gamma-1)\left(1 - \bar{b}\rho_0 - Z_0\right)(\chi\tau)^{3\xi-2}\left\{m(3\eta+1) - 3\xi + 3\right\}} .$$

6. Result and Discussion

The effects of dust particles and non-idealness parameters on the radius of blast wave are shown in figures1- 5. The effect of volume fraction of the dust particles present in the gas on the radius of the blast wave in planer, cylindrical symmetric and spherically symmetric flows are shown in Figs.1-3. The values of the constants appearing in the computations are taken as: $\beta = 1.0$ and $K_p = 0.1$, and volume fraction $Z_0 = 0.0,\ 0.01,\ 0.02,\ 0.04$.

Here, $b = 0.0, Z_0 = 0.0$ corresponds to the ordinary gas dynamics case and $b = 0.02, Z_0 = 0.0$ corresponds to the dust free non-ideal gas. It is observed that an increase in the volume fraction of dust particles and non-ideal ness parameter causes to increase the radius of the blast wave for very short time and then decreases continuously. From Fig.2 and Fig.3 it is clear that the variation of radius of blast wave in planar and cylindrically symmetric flows have similar trend as in case of spherically symmetric flow but rate of variation is slowed down in planar case as compared to cylindrically symmetric and spherical symmetric flow. The rate of variation of radius in cylindrically symmetric flow is slowed down as compared to spherical symmetric flow.

The effect of mass fraction and specific heat of the dust particles present in the gas on the radius of the blast wave at constants $Z_0 = 0.01$, $\beta = 1.0$ and $Z_0 = 0.01$, $K_p = 1.0$ in spherically symmetric flows are shown in Figs.4-5 respectively. Due to the similar behavior of solid particles in planar and cylindrically symmetric flow details are omitted. Since increment in the mass fraction of solid particles have very less effect on the value of volume fraction of solid particles but have sufficient effect on the value of Γ. So, an increment in mass fraction of solid particles causes to decrease the radius of blast wave for very short time and then increases continuously. Small change in the value of specific heat of dust particle does not have a visible effect on the radius of blast wave as shown in Fig.5

The effects of volume fraction of the dust particles present in the gas and non-idealness parameter of gas on the energy carried by blast wave in spherically symmetric flows are shown in Fig.6. The values of the constants appearing in the computations are taken as: $\beta = 1.0$, $K_p = 0.1$, $b = 0.02$ and Volume fraction $Z_0 = 0.0,\ 0.01,\ 0.02,\ 0.03, 0.04$ *i.e.* $\theta = 0.01$ and $\rho_0 = 0,\ 1,\ 2,\ 3,\ 4$.

Here, it is observed that the increment in the volume fraction of dust particles and non-idealness parameter causes the decrement of the energy carried by blast wave in non-ideal gas. The effects of mass fraction of the dust particles present in the gas on the energy carried by blast wave in spherically symmetric flows are shown in Fig.7. The values of the constants appearing in the computations are taken as: $\beta = 1.0$, $Z_0 = 0.01$, $b = 0.02$ and mass fraction $k_p = 0.1,\ 0.2,\ 0.3,\ 0.3, 0.4$.

It is observed that an increment in the mass fraction of dust particles causes to increase the energy carried by blast wave in non-ideal gas. Due to the similar effect of dust particles on the energy carried by blast wave details are omitted.

5. Conclusion:

In the present article the exact analytical solution for the problem of blast wave in a non-ideal dusty gas has been derived. The solution of Euler's equation in a non-ideal dusty gas obtained here is a new one. The behavior of variations of the radius of blast wave in the mixture of non-ideal gas and small solid particles are similar to that as in an ideal and non-ideal gas whereas the behavior of energy in a non-ideal dusty gas are quite different to that of an ideal gas and are similar to that of a non-ideal gas. Here, it is observed that the solution of blast wave problem for adiabatic non-ideal dusty gas given by equation (18-20) reduces to the solution presented by Murata for $\theta = b = 0$.

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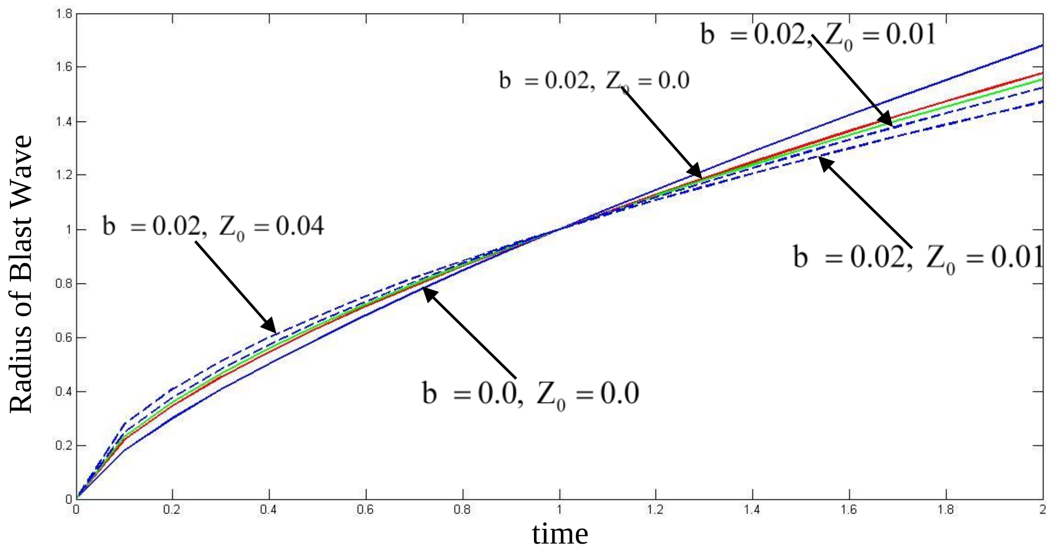

Fig 1: Behavior of the radius of the Blast Wave for $m = 2$.

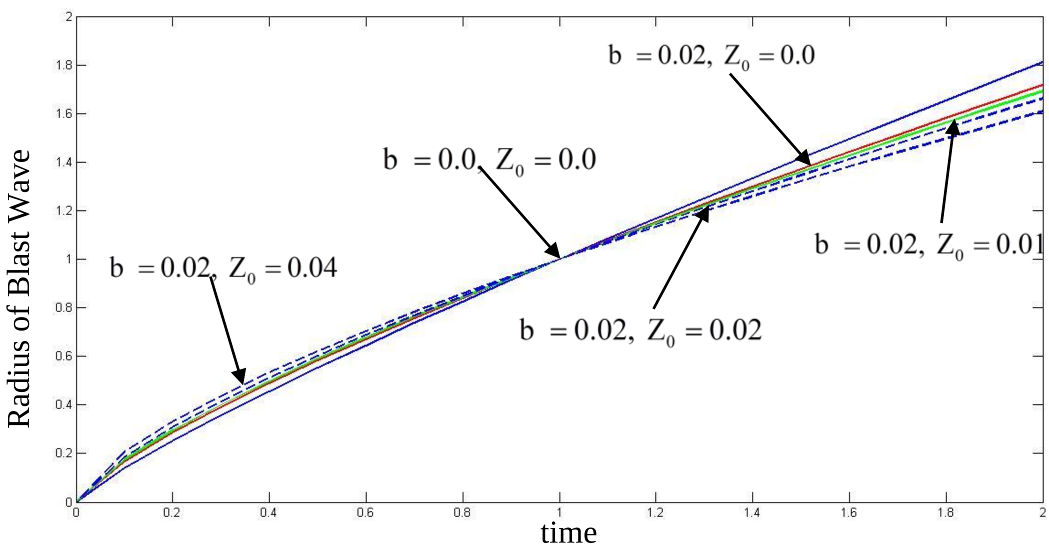

Fig 2: Behavior of the radius of the Blast Wave for $m = 1$.

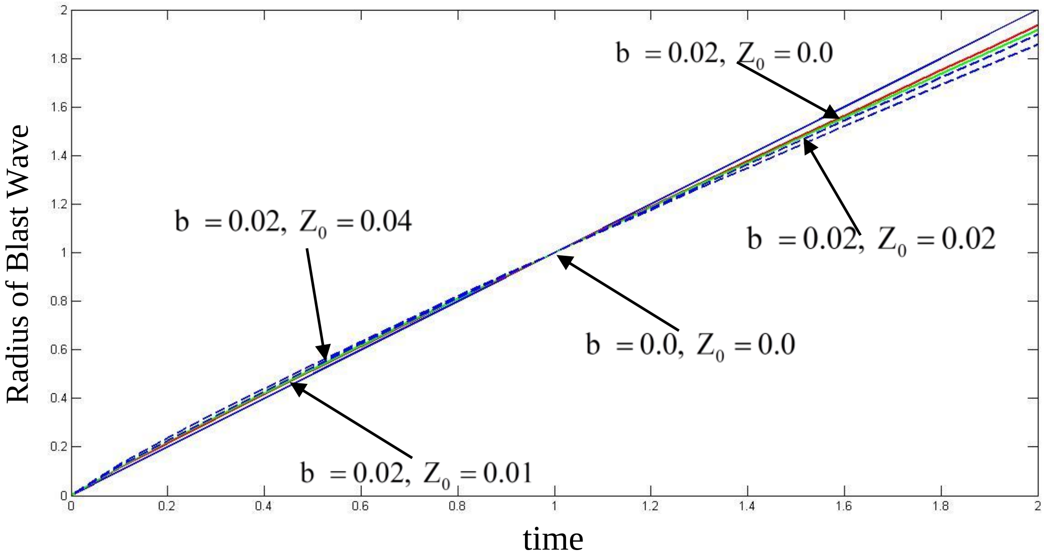

Fig 3: Behavior of the radius of the Blast Wave for $m = 0$.

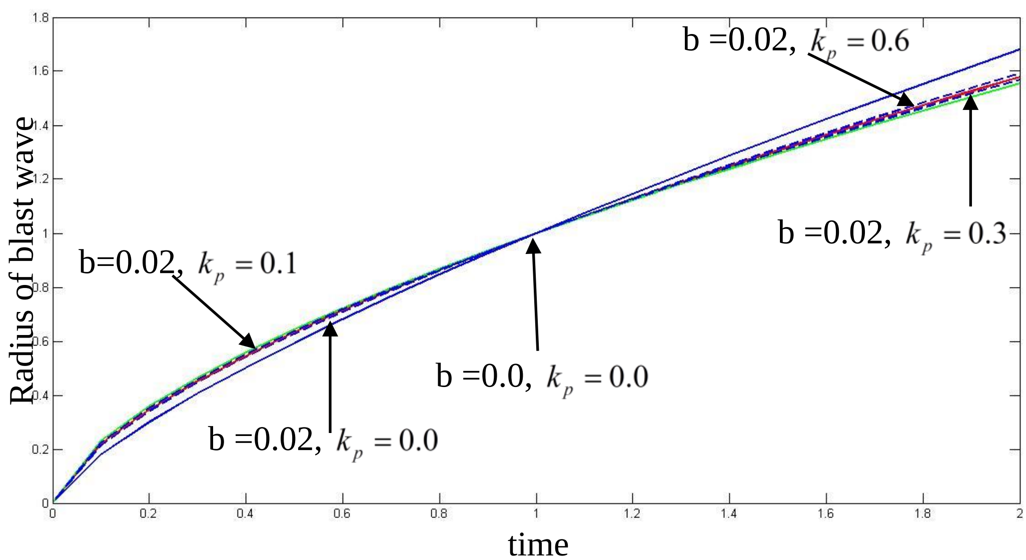

Fig 4: Behavior of the radius of the Blast Wave for $m = 2$

$m = 2.$

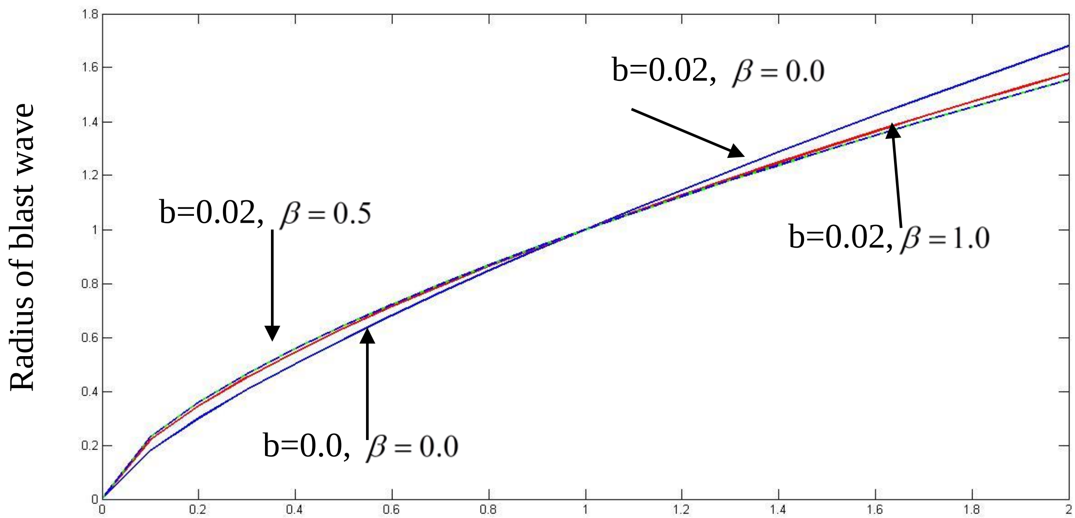

Fig 5: Behavior of the radius of the Blast Wave for $m = 2$.

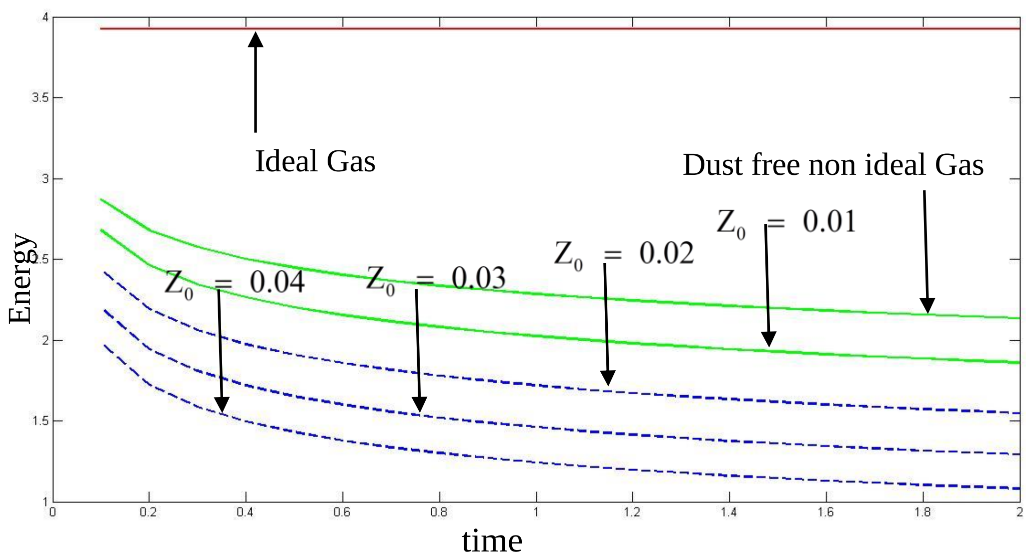

Fig 6: Behavior of energy carried by the blast wave for $m = 2$.

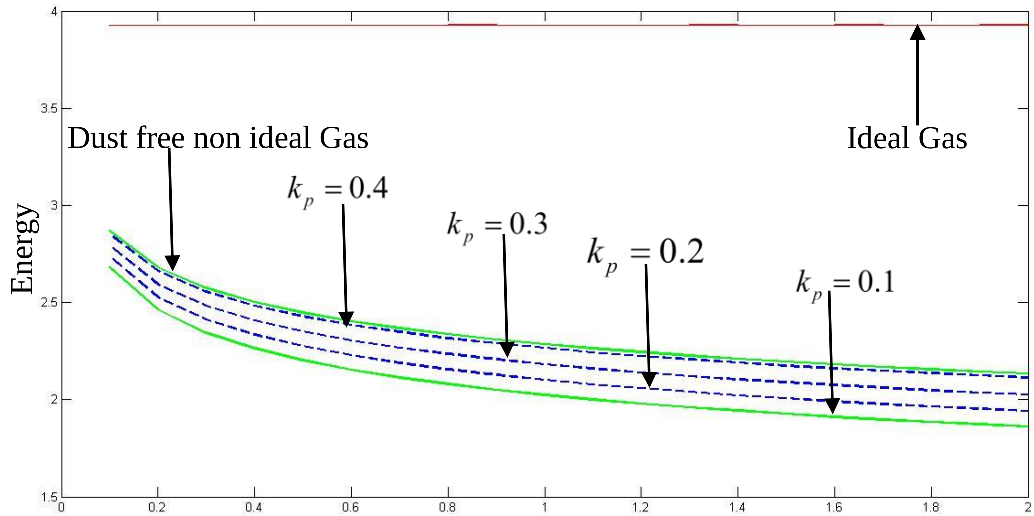

Fig 7: Behavior of energy carried by the blast wave for $m = 2$.