**SIMILARITY MEASURES OF FERMATEAN NEUTROSOPHIC SETS BASED ON THE COSINE FUNCTION AND THEIR APPLICATIONS**

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**ABSTRACT:**

In this study, we analyse the degree of hesitation, non-membership, and membership in Fermatean Neutrosophic sets (FNSs) and give similarity metrics between them based on the cosine function. Next, we utilise these similarity measures along with weighted measures amongst FNSs for diagnosing medical conditions and identifying patterns. Lastly, two instances are provided to show how effective similarity measures are at identifying patterns and making medical diagnoses.

1. **INTRODUCTION**

The similarity measures are valuable instruments for assessing how similar two objects are to one another. Due to their widespread use in a variety of domains, including pattern recognition, machine learning, decision-making, and image processing, measures of similarity between fuzzy sets have drawn the attention of researchers. In recent years, numerous measures of similarity between fuzzy sets have been proposed and investigated[6–8, 15]. Since its introduction by Zadeh[31], fuzzy set theory has been extensively utilised to simulate uncertainty that arises in practical applications. Fuzzy sets were expanded by Atanassov[3,4] to become Atanassov's intuitionistic fuzzy sets (IFSs); numerous similarity measures between IFSs have been studied in the literature[17]. A suitable similarity measure between IFSs was proposed by Li and Cheng [16] and applied to pattern recognition challenges. Liang and Shi [16] explored the connections between several IFSs and proposed a few similarity mereasus to distinguish between them. Additionally, Mitchell [19] changed the measurements made by Li and Cheng. Szmidt and Kacprzyk [24,25] devised a similarity measure between IFSs based on the Hamming distance, based on the extension of Hamming distance on fuzzy sets. Based on the Hausdorff distance, Hung and Yang[12] computed the distance between IFSs and produced a few similarity measures. Liu [18] created a few new similarity measures between elements and IFSs. Hung and Yang [13]proposed a similarity measures between IFSs based on the measures. The geometric distance and similarity measures of IFSs for group decision-making problems were defined by Xu and Xia [28]. The cosine similarity measure between IFSs was proposed by Ye [29]. The likelihood-based measurement of IFSs for medical diagnostic and bacteria classification problems was developed by Hung [14]. Shi and Ye [22] enhanced the IFSs' cosine similarity measures even further. The cotangent similarity measure between IFSs was suggested by Tian [27] for use in medical diagnosis. The cotangent similarity measure, which took into account the degrees of hesitation, non-membership, and membership as stated in IFSs, was further introduced by Rajarajeswari and Uma [20]. Szmidt [26] also covered the distances between IFSs and presented a family of similarity measure that took into account the degrees of hesitation, membership, and non-membership that are expressed in IFSs. Ye [30] presented two new weighted cosine similarity measures and two new cosine similarity measures based on the information conveyed by the membership, non-membership, and hesitation degrees in IFSs as well as the cosine function. The intuitionistic vector similarity measurements for medical diagnostics were provided by Son and Phong [23]. According to Antony Jansi[1], Fermatean Neutrosophic Sets were proposed.

The format of this paper is as follows: We present some fundamental ideas about IFSs and FNSs, as well as some measures of similarity between IFSs, in the following section. Based on the idea of the cosine function, we are going to suggest some similarity measures and weighted similarity measures between FNSs in Section 3. The application of FNSs' similarity measure to pattern recognition and medical diagnosis is discussed in Section 4. The benefits of the suggested similarity measures are covered in Section 5, and some closing thoughts are included in the final section to wrap up the work.

**2. PRELIMINARIES**

In the following, we introduce some basic concepts related to IFSs and some similarity measure between IFSs.

**Definition 2.1 [3,4] :**

An IFS in is given by,

 ------------(1)

Where and where The number and represents, respectively the membership degree and non-membership degree of the element to the set .

**Definition 2.2 [3,4]:**

For each IFS in , if

 ----------(2)

Then is called the degree of indeterminacy of to .

Suppose that there are two IFSs and in the universe of discourse

Ye[30] proposed the cosine similarity measure between IFSs and as following:

 ---------------(3)

Shi and Ye [22] further presented the cosine similarity measure by considering membership degree , non-membership degree, and hesitancy degree in IFSs as the vector space of the three terms:

 ------------(4)

Based on the cosine function, Ye [30] proposed two cosine similarity measures between IFSs A and B.

 -------------(5)

 --------------(6)

On the other hand, Tian [27] proposed a cotangent similarity measure between IFSs A and B as following:

 -------------(7)

Where the symbol is the maximum operation.

 When the three terms such as the membership degree, non-membership degree, and hesitancy degree are considered in IFSs, Rajarajeswari and Uma [20] defined the cotangent similarity measure of IFSs:

 ------------(8)

In the following, we introduce the weighted cosine and cotangent similarity measures between IFSs A and B, respectively [29, 22, 27, 20, 30]**.**

 --------------(9)

 ------------(10)

 -------------(11)

 ----------------(12)

 ---------------(13)

 ---------------(14)

Where is the weight of an element and and the symbol is the maximum operation.

**Definition 2.3[2]:**

Let X be a universe of discourse. A Fermatean Neutrosophic set [FN Set] A on X is an object of the form: 𝐴 = , where and .

Then, for all .

 is the degree of membership function, is the degree of indeterminacy and is the degree of non- membership function. Here and are dependent components and is an independent components.

**Definition 2.4[2]:**

Let X be a non-empty set and I the unit interval [0,1]. A Fermatean Neutrosophic sets M and N of the form and

1. .
2. = .
3. =
4. **SOME SIMILARITY MEASURE BASED ON THE COSINE FUNCTION FOR FERMATEAN NEUTROSOPHIC SETS**

**3.1 Cosine Similarity Measure for FNSs**

Let be a FNS in an universe of discourse , the FNS is characterized by the degree of membership , the degree of non-membership , and the degree of hesitation , , which can be considered as a vector representation with the three elements. Therefore, a cosine similarity measure and a weighted cosine similarity measure for FNSs are proposed in an analogous manner to the cosine similarity measure based on Bhattacharya’s distance [21, 6]and cosine similarity measure for IFS [28]**.**

 Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

 ------------(17)

If we take n=1, then the cosine similarity measure between FNSs and becomes the correlation coefficient between FNSs and . Therefore, the cosine similarity measure between and also satisfies the following properties:

1. .
2. if

Proof:

1. It is obvious that the proposition is true according to the cosine value.
2. It is obvious that the proposition is true.
3. When there are , and So there is .

Therefore, we have finished the proofs.

If we consider the weights of , a weighted cosine similarity measure between FNSs and is proposed as follows:

 --------------(18)

 Where is the weight vector of , with , In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is . Obviously, the weighted cosine similarity measure of two FNSs and also satisfies the following properties:

1. .
2. if .

Similar to the previous proof method, we can prove the above three properties.

In the following, we shall investigate the distance measure of the angle as

 It satisfies the following properties:

1. if ;
2. , if
3. if
4. if for any FNS .

Proof:

Obviously, satisfies the property (1) - (3). In the following, will be proved to satisfy the property (4).

 For any , , Since Equation (16) is the sum of terms, let us investigate the distance measures of the angle between the vectors:

 and

 where

 For three vectors ,

 in one plane, if Then, it is obvious that according to the triangle inequality. Combining the inequality with Equation (16), we can obtain Thus satisfies the property (4). So we finished the proof.

* 1. **Similarity measures of FNSs based on cosine function:**

Based on the cosine function, in this section, we shall propose two cosine similarity measures between FNSs and analyse their properties.

**Definition 3.2.1:**

Suppose that there are two FNSs and in the universe of discourse , we further propose the cosine similarity measures between FNSs as follows:

 --------------(19)

 --------------(20)

Where the symbol is the maximum operator.

**Proposition 3.2.2:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2. If is a FNS in and , then and .

Proof:

1. Since the value of the cosine function is within [0,1], the similarity measure based on the cosine function is also within [0,1]. Thus, there is .
2. For any two FNSs and in if , then and for Thus,

 So,

If this implies for Since cos(0) =1. Then, there are

 and for Hence

.

1. Proof is straightforward.
2. If then there are , and for Then, , and

Thus, we have

,

,

,

,

 and

.

Hence, and for as the cosine function is a decreasing function with the interval

Thus, the proofs of these properties are completed.

 In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cosine similarity measure between FNSs and is proposed as follows:

 --------------(21)

 --------------(22)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if , then the weighted cosine similarity measure reduces to cosine similarity measure. That is to say, if we take then there is .

Obviously, the weighted cosine similarity measures also satisfy the axiomatic requirements of similarity measures in Proposition 2.

**Proposition 3.2.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2.
3. If is a FNS in and , then W and .

By using similar proof in Proposition 1, we can give the proofs of these properties (1) – (4).

 **3.3 Similarity Measures of FNSs based on the Cotangent Function:**

In this section, we shall propose two cotangent similarity measures between FNSs.

**Definition 3.3.1**:

Suppose that there are two FNSs and in the universe of discourse , we further propose the cotangent similarity measures between FNSs as follows:

 --------------(23)

 --------------(24)

Where the symbol is the maximum operator.

**Proposition 3.3.2:**

For any two FNSs and in the cotangent similarity measures should satisfy the following properties (1) – (4):

1. If is a FNS in and , then and .

**Proof:**

1. Since,

 ,

It is obvious that the cotangent function are within 0 and 1.

1. It is obvious that the proposition is true.
2. When , then obviously . On the other hand if then,

 and for

This implies

1. If then we can write , and for Then, , and

The cotangent function is decreasing function within the interval .

Hence we can write and .

Thus, the proofs of these properties are completed.

 In many situations, the weight of the elements should be taken into account. For example, in Multiple Attribute Decision Making (MADM), the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, two weighted cotangent similarity measure between FNSs and is proposed as follows:

 --------------(25)

 ----------------(26)

Where is the weight vector of , with , and the symbol is the maximum operator. In particular, if

, then the weighted cotangent similarity measure reduces to cotangent similarity measure. That is to say, if we take then there is .

**Proposition 3.3.3:**

For any two FNSs and in the cosine similarity measures should satisfy the following properties (1) – (4):

1.
2.
3. If is a FNS in and , then W and .

By using similar proof in Proposition 3, we can give the proofs of these properties (1) – (4)

4.**APPLICATIONS**

In this section, the cosine and cotangent similarity measures for FNSs are applied to pattern recognition and medical diagnosis to illustrate the feasibility of the proposed methods and deliver a comparative analysis

* 1. **Example 1: Pattern Recognition**

Let us consider, a three known patterns which are represented by the FNSs: in the feature space as

Consider an unknown pattern that will be recognized, where

 The purpose of this problem is classify the pattern in one classes and . For it, the proposed similarities degrees have been computed from to and are given in Table 1.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8704 | 0.8320 | **0.8992** |
|  | 0.8747 | 0.8360 | **0.9041** |
|  | 0.9104 | 0.8863 | **0.9328** |
|  | 0.5967 | 0.5533 | **0.654** |
|  | 0.7695 | 0.7327 | **0.7898** |

Table 1: The similarity measures between and

 From the numerical results presented in Table 1, we know that the degree of similarity between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

 If we consider the weight of are 0.5, 0.3 and 0.2 respectively. Then we use the proposed weighted similarities measures have been computed from to and are given in Table 2.

|  |  |  |  |
| --- | --- | --- | --- |
| Similarity Measures |  |  |  |
|  | 0.8244 | 0.8145 | **0.8692** |
|  | 0.8631 | 0.8622 | **0.8808** |
|  | 0.8938 | 0.8975 | **0.9221** |
|  | 0.5818 | 0.5877 | **0.6161** |
|  | 0.7503 | 0.7426 | **0.7753** |

Table 2: The weighted similarity measures between and

 From the numerical results presented in Table 2, we know that the weighted similarity measures between and is the largest one as derived by five similarity measures. That is, all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficient method [10], we can get the same result that all the 5 similarity measures assign the unknown class to the known class according to the principle of maximum degree of similarity between FNSs.

* 1. **Example 2: Medical Diagnosis**

Let us consider a set of diagnosis

 and a set of symptoms

 Suppose that a patient, with respect to all symptoms, can be depicted by the following FNS:

 ( (

And then each diagnoses can be viewed as FNSs with respect to all the symptoms as follows:

 (

The purpose of this problem is classify the pattern in one classes . For this, the proposed similarities measures have been computed from to and are given in Table 3.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5811 | 0.8863 | 0.9288 | 0.9420 | **0.9469** |
|  | 0.7440 | 0.8752 | 0.8911 | **0.9208** | 0.8685 |
|  | 0.7644 | 0.9236 | 0.9200 | **0.9355** | 0.9224 |
|  | 0.5083 | 0.6002 | 0.6328 | **0.7018** | 0.6005 |
|  | 0.6628 | 0.7741 | 0.7700 | **0.8155** | 0.7717 |

Table 3: The similarity measures between and

From the numerical results presented in Table 3, expect for the , we know that the similarity measures between and is the largest one as derived by five similarity measures. That is, the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs. Compared with Garg’s correlation coefficients method [10] we can get same result that the four similarity measures assign the unknown class to the known class according to the principle of the maximum degree of similarity between FNSs expect for the

If we consider the weight of is respectively. Then we apply the proposed weighted similarities measures, which have been computed from to and are given in Table 4.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Similarity Measures |  |  |  |  |  |
|  | 0.5608 | 0.7994 | 0.8251 | 0.8235 | **0.8517** |
|  | 0.6889 | 0.7881 | 0.8020 | **0.8204** | 0.7699 |
|  | 0.7083 | 0.8280 | 0.8201 | **0.8382** | 0.8206 |
|  | 0.4830 | 0.5465 | 0.5780 | **0.6245** | 0.5304 |
|  | 0.6149 | 0.6986 | 0.6892 | **0.7327** | 0.6865 |

Table 4: The weighted similarity measures between and

From the numerical results presented in table 4, we get the following results:

1. For similarity measures , the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs.
2. For similarity measures the degree of similarity between and P is the largest one, so the pattern P should belong to the class of known diagnoses according to the principle of the maximum degree of similarity between FNSs. At the same time, for this case compared with Garg’s correlation coeeficients method [10], we can get the same result that the pattern P should belong to the class of the known diagnoses according to the principle of the maximum degree of similarity between FNSs.
3. **CONCLUSION**

 In this paper, we presented another form of five similarity measures between FNSs based on the cosine function between FNSs by considering the degree of membership, degree of non-membership and the degree of hesitation in FNSs. Then, we applied these similarity measures and weighted similarity measures between FNSs to pattern recognition and medical diagnosis. Finally, two illustrative examples are given to demonstrate the efficiency of the similarity measures for pattern recognition and medical diagnosis.

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