RISHI TRANSFORM OF BESSEL FUNCTIONS OF FIRST KIND

[1*]Sudhanshu Aggarwal, [2]Rishi Kumar, [3]Jyotsna Chandel

[1*]Assistant Professor, Department of Mathematics, National Post Graduate College, Barhalganj, Gorakhpur-273402, Uttar Pradesh, India

Email: sudhanshu30187@gmail.com

[2]Research Scholar, Department of Mathematics, D.S. College, Aligarh, Uttar Pradesh, India

Email: rishi.saraswat1987@gmail.com

[3]Associate Professor, Department of Mathematics, D.S. College, Aligarh, Uttar Pradesh, India

Email: jyotsnaraghuvanshi5@gmail.com

*Corresponding Author

ABSTRACT: Various problems of Statistics, Mathematics, Radio Physics, Nuclear Physics, Atomic Physics, Fluid Mechanics, Engineering and Science can easily handle by applying integral transform techniques on their mathematical models. Problems of heat equation, wave equation, Laplace equation, Helmholtz equation and Schrodinger equation have solutions in terms of Bessel functions. To solve such equations by integral transform methods, we need to know the integral transform of Bessel functions. In this chapter, authors determine the Rishi transform of Bessel functions of first kind.

KEYWORDS: Rishi Transform; Bessel Function; Inverse Rishi Transform.

MATHEMATICS SUBJECT CLASSIFICATION: 35A22; 44A05; 44A35; 45A05; 45D05

1. INTRODUCTION: Nowadays integral transform methods have various applications to solve the problems of engineering and science [1-2]. Researchers used different integral transform methods and solved various ordinary differential equations [3-4]; partial differential equations [5]; Volterra integral equations [6-17, 40-42] and Volterra integro differential equations [18-33]. Aggarwal with different scholars [34-39] determined the Kamal; Mahgoub; Mohand; Aboodh; Elzaki and Sawi transforms of Bessel's functions. The motive of the present chapter is to determine the Rishi transform of Bessel function of first kind of orders zero, one and two.

2. NOMENCLATURE OF SYMBOLS:

Υ, Rishi transform operator;

Υ^{-1}, inverse Rishi transform operator;

N, the set of natural numbers;

$\in$, belongs to;

!, the usual factorial notation;

Γ, the classical Gamma function;

R, the set of real numbers;

$J_n(t)$, Bessel function of first kind of order n;

$J_0(t)$, Bessel function of first kind of order zero;

$J_1(t)$, Bessel function of first kind of order one;

$J_2(t)$, Bessel function of first kind of order two

3. DEFINITION OF RISHI TRANSFORM:

The Rishi transform of a piecewise continuous exponential order function $F(t), t \geq 0$ is given by [41]

$$\Upsilon\{F(t)\} = \left(\frac{\sigma}{\varepsilon}\right)\int_0^\infty F(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t}dt = T(\varepsilon,\sigma),\ \ \varepsilon > 0, \sigma > 0 \qquad (1)$$

4. INVERSE RISHI TRANSFORM [42]:

The inverse rishi transform of $T(\varepsilon,\sigma)$, designated by $\Upsilon^{-1}\{T(\varepsilon,\sigma)\}$, is another function $F(t)$ having the property that $\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$.

Some useful operational characteristics of Rishi transform, Rishi transforms of some fundamental functions and their inverse Rishi transforms are summarized in the Tables1-3 respectively.

Table-1: Some operational characteristics of Rishi transform [42]

S.N.	Name of Characteristic	Mathematical Form
1	Linearity	$\Upsilon\{\sum_{i=1}^{n} k_i F_i(t)\} = \sum_{i=1}^{n} k_i \Upsilon\{F_i(t)\}$, where k_i are arbitrary

		constants
2	Change of Scale	If $\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$ then $\Upsilon\{F(kt)\} = \frac{1}{k^2} T\left(\frac{\varepsilon}{k},\sigma\right)$
3	Translation	If $\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$ then $\left\{\Upsilon\{e^{kt}F(t)\} = \left(\frac{\varepsilon - k\sigma}{\varepsilon}\right) T(\varepsilon - k\sigma,\sigma)\right\}$
4	Convolution	If $\Upsilon\{F_1(t)\} = T_1(\varepsilon,\sigma)$ and $\Upsilon\{F_2(t)\} = T_2(\varepsilon,\sigma)$ then $\left\{\Upsilon\{F_1(t) * F_2(t)\} = \left[\left(\frac{\varepsilon}{\sigma}\right) T_1(\varepsilon,\sigma)\, T_2(\varepsilon,\sigma)\right]\right\}$

Table-2: Some fundamental functions and their Rishi transform [40-43]

S.N.	$F(t), t > 0$	$\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$
1	1	$\left(\frac{\sigma}{\varepsilon}\right)^2$
2	e^{lt}	$\frac{\sigma^2}{\varepsilon(\varepsilon - l\sigma)}$
3	$t^\rho, \rho \in N$	$\rho!\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}$
4	$t^\rho, \rho > -1, \rho \in R$	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2} \Gamma(\rho+1)$
5	$sin(lt)$	$\frac{l\sigma^3}{\varepsilon(\varepsilon^2 + \sigma^2 l^2)}$
6	$cos(lt)$	$\frac{\sigma^2}{(\varepsilon^2 + \sigma^2 l^2)}$
7	$sinh(lt)$	$\frac{l\sigma^3}{\varepsilon(\varepsilon^2 - \sigma^2 l^2)}$
8	$cosh(lt)$	$\frac{\sigma^2}{(\varepsilon^2 - \sigma^2 l^2)}$

Table-3: Inverse Rishi transformations of some fundamental functions [42]

S.N.	$T(\varepsilon,\sigma)$	$F(t) = \Upsilon^{-1}\{T(\varepsilon,\sigma)\}$
1	$\left(\frac{\sigma}{\varepsilon}\right)^2$	1
2	$\frac{\sigma^2}{\varepsilon(\varepsilon - l\sigma)}$	e^{lt}
3	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}, \rho \in N$	$\frac{t^\rho}{\rho!}$
4	$\left(\frac{\sigma}{\varepsilon}\right)^{\rho+2}, \rho > -1, \rho \in R$	$\frac{t^\rho}{\Gamma(\rho+1)}$
5	$\frac{\sigma^3}{\varepsilon(\varepsilon^2 + \sigma^2 l^2)}$	$\frac{sin(lt)}{l}$
6	$\frac{\sigma^2}{(\varepsilon^2 + \sigma^2 l^2)}$	$cos(lt)$
7	$\frac{\sigma^3}{\varepsilon(\varepsilon^2 - \sigma^2 l^2)}$	$\frac{sinh(lt)}{l}$
8	$\frac{\sigma^2}{(\varepsilon^2 - \sigma^2 l^2)}$	$cosh(lt)$

5. RISHI TRANSFORM OF DERIVATIVES OF A FUNCTION [43]:

If $\Upsilon\{F(t)\} = T(\varepsilon,\sigma)$ then

a) $\Upsilon\{F'(t)\} = \left(\frac{\varepsilon}{\sigma}\right) T(\varepsilon,\sigma) - \left(\frac{\varepsilon}{\sigma}\right)^{-1} F(0)$.

b) $\Upsilon\{F''(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^{2} T(\varepsilon,\sigma) - F(0) - \left(\frac{\varepsilon}{\sigma}\right)^{-1} F'(0)$.

c) $\Upsilon\{F'''(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^{3} T(\varepsilon,\sigma) - \left(\frac{\varepsilon}{\sigma}\right) F(0) - F'(0) - \left(\frac{\varepsilon}{\sigma}\right)^{-1} F''(0)$.

6. BESSEL FUNCTIONS OF FIRST KIND [35-39]:

Bessel's function of first kind of order n, where $n\epsilon N$ is given by

$$J_n(t) = \frac{t^n}{2^n\, n!}\left\{1 - \frac{t^2}{2.(2n+2)} + \frac{t^4}{2.4.(2n+2)(2n+4)} - \frac{t^6}{2.4.6.(2n+2)(2n+4)(2n+6)} + \cdots \ldots.\right\} \quad (2)$$

Bessel's function of first kind of zero order is given by

$$J_0(t) = \left\{1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots\right\} \quad (3)$$

Bessel's function of first kind of order one is given by

$$J_1(t) = \left\{\frac{t}{2} - \frac{t^3}{2^2.4} + \frac{t^5}{2^2.4^2.6} - \frac{t^7}{2^2.4^2.6^2.8} + \cdots..\right\} \quad (4)$$

Equation (4) can also be written as

$$J_1(t) = \left\{\frac{t}{2} - \frac{t^3}{2^3.2!} + \frac{t^5}{2^5.2!.3!} - \frac{t^7}{2^7.3!.4!} + \cdots\right\} \quad (5)$$

Bessel's function of first kind of order is given by

$$J_2(t) = \left\{\frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} - \frac{t^8}{2^2.4^2.6^2.8.10} + \cdots \ldots.\right\} \quad (6)$$

7. RELATION BETWEEN $J_0(t)$ AND $J_1(t)$ [35]:

$$\frac{d}{dt}[J_0(t)] = -J_1(t) \quad (7)$$

8. RELATION BETWEEN $J_0(t)$ AND $J_2(t)$ [38]:

$$J_2(t) = J_0(t) + 2J_0''(t) \quad (8)$$

9. RISHI TRANSFORM OF BESSEL FUNCTIONS OF FIRST KIND:

9.1 RISHI TRANSFORM OF BESSEL FUNCTION OF FIRST KIND OF ORDER ZERO $J_0(t)$:

Operating Rishi transform on both sides of (3), we have

$$\Upsilon\{J_0(t)\} = \Upsilon\left\{1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots\right\} \quad (9)$$

Use of linearity property of Rishi transform on (9) gives

$$\Upsilon\{J_0(t)\} = \Upsilon\{1\} - \Upsilon\left\{\frac{t^2}{2^2}\right\} + \Upsilon\left\{\frac{t^4}{2^2.4^2}\right\} - \Upsilon\left\{\frac{t^6}{2^2.4^2.6^2}\right\} + \cdots.$$

$$\Rightarrow \Upsilon\{J_0(t)\} = \left(\frac{\sigma}{\varepsilon}\right)^2 - \frac{1}{2^2}2!\left(\frac{\sigma}{\varepsilon}\right)^4 + \frac{1}{2^2.4^2}4!\left(\frac{\sigma}{\varepsilon}\right)^6 - \frac{1}{2^2.4^2.6^2}6!\left(\frac{\sigma}{\varepsilon}\right)^8 + \cdots.$$

$$\Rightarrow \Upsilon\{J_0(t)\} = \left(\frac{\sigma}{\varepsilon}\right)^2\left[1 - \frac{1}{2}\left(\frac{\sigma}{\varepsilon}\right)^2 + \frac{1.3}{2.4}\left(\frac{\sigma}{\varepsilon}\right)^4 - \frac{1.3.5}{2.4.6}\left(\frac{\sigma}{\varepsilon}\right)^6 + \cdots.\right]$$

$$\Rightarrow \Upsilon\{J_0(t)\} = \left(\frac{\sigma}{\varepsilon}\right)^2\left[\frac{1}{\sqrt{1+\left(\frac{\sigma}{\varepsilon}\right)^2}}\right] = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] \quad (10)$$

9.2 RISHI TRANSFORM OF BESSEL FUNCTION OF FIRST KIND OF ORDER ONE $J_1(t)$:

Operating Rishi transform on both sides of (7), we have

$$\Upsilon\left\{\frac{d}{dt}[J_0(t)]\right\} = -\Upsilon\{J_1(t)\} \quad (11)$$

Use of Rishi transform of derivatives of a function property in (11) gives

$$\left(\frac{\varepsilon}{\sigma}\right)\Upsilon\{J_0(t)\} - \left(\frac{\varepsilon}{\sigma}\right)^{-1} J_0(0) = -\Upsilon\{J_1(t)\} \quad (12)$$

Use of (3) and (10) in (12) provides

$$\left(\frac{\varepsilon}{\sigma}\right)\left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] - \left(\frac{\varepsilon}{\sigma}\right)^{-1} = -\Upsilon\{J_1(t)\}$$

$$\Rightarrow \Upsilon\{J_1(t)\} = \left(\frac{\varepsilon}{\sigma}\right)^{-1} - \left(\frac{\varepsilon}{\sigma}\right)\left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{J_1(t)\} = \left(\frac{\sigma}{\varepsilon}\right) - \left[\frac{\sigma}{\sqrt{\varepsilon^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{J_1(t)\} = \left[\frac{\sigma\left(\sqrt{\varepsilon^2+\sigma^2}-\varepsilon\right)}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] \tag{13}$$

9.3 RISHI TRANSFORM OF BESSEL FUNCTION OF FIRST KIND OF ORDER TWO $J_2(t)$:

Operating Rishi transform on both sides of (8), we have

$$\Upsilon\{J_2(t)\} = \Upsilon\{J_0(t) + 2J_0''(t)\} \tag{14}$$

Use of linearity property of Rishi transform on (14) gives

$$\Upsilon\{J_2(t)\} = \Upsilon\{J_0(t)\} + 2\Upsilon\{J_0''(t)\} \tag{15}$$

Use of Rishi transform of derivatives of a function property in (15) gives

$$\Upsilon\{J_2(t)\} = \Upsilon\{J_0(t)\} + 2\left[\left(\frac{\varepsilon}{\sigma}\right)^2 \Upsilon\{J_0(t)\} - J_0(0) - \left(\frac{\varepsilon}{\sigma}\right)^{-1} J_0'(0)\right] \tag{16}$$

Use of (3), (7) and (10) in (16) provides

$$\Upsilon\{J_2(t)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] + 2\left[\left(\frac{\varepsilon}{\sigma}\right)^2\left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] - 1 + \left(\frac{\varepsilon}{\sigma}\right)^{-1} J_1(0)\right] \tag{17}$$

Using (4) in (17), we have

$$\Upsilon\{J_2(t)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] + 2\left[\left[\frac{\varepsilon}{\sqrt{\varepsilon^2+\sigma^2}}\right] - 1\right]$$

$$\Rightarrow \Upsilon\{J_2(t)\} = \left[\frac{\sigma^2+2\varepsilon^2-2\varepsilon\sqrt{\varepsilon^2+\sigma^2}}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right] \tag{18}$$

9.4 RISHI TRANSFORM OF $e^{kt}J_0(t)$:

From (10), we have

$$\Upsilon\{J_0(t)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using translation property of Rishi transform on above equation, we have

$$\Upsilon\{e^{kt}J_0(t)\} = \left(\frac{\varepsilon-k\sigma}{\varepsilon}\right)\left[\frac{\sigma^2}{(\varepsilon-k\sigma)\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{e^{kt}J_0(t)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right] \tag{19}$$

9.5 RISHI TRANSFORM OF $e^{kt}J_1(t)$:

From (13), we have

$$\Upsilon\{J_1(t)\} = \left[\frac{\sigma\left(\sqrt{\varepsilon^2+\sigma^2}-\varepsilon\right)}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using translation property of Rishi transform on above equation, we have

$$\Upsilon\{e^{kt}J_1(t)\} = \left(\frac{\varepsilon-k\sigma}{\varepsilon}\right)\left[\frac{\sigma\left(\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}-(\varepsilon-k\sigma)\right)}{(\varepsilon-k\sigma)\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{e^{kt}J_1(t)\} = \left[\frac{\sigma\left(\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}-(\varepsilon-k\sigma)\right)}{\varepsilon\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right] \tag{20}$$

9.6 RISHI TRANSFORM OF $e^{kt}J_2(t)$:

From (18), we have

$$\Upsilon\{J_2(t)\} = \left[\frac{\sigma^2+2\varepsilon^2-2\varepsilon\sqrt{\varepsilon^2+\sigma^2}}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using translation property of Rishi transform on above equation, we have

$$\Upsilon\{e^{kt}J_2(t)\} = \left(\frac{\varepsilon-k\sigma}{\varepsilon}\right)\left[\frac{\sigma^2+2(\varepsilon-k\sigma)^2-2(\varepsilon-k\sigma)\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}{(\varepsilon-k\sigma)\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{e^{kt}J_2(t)\} = \left[\frac{\sigma^2+2(\varepsilon-k\sigma)^2-2(\varepsilon-k\sigma)\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}{\varepsilon\sqrt{(\varepsilon-k\sigma)^2+\sigma^2}}\right] \qquad (21)$$

9.7 RISHI TRANSFORM OF $J_0(kt)$:

From (10), we have

$$\Upsilon\{J_0(t)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using change of scale property of Rishi transform on above equation, we have

$$\Upsilon\{J_0(kt)\} = \frac{1}{k^2}\left[\frac{\sigma^2}{\left(\frac{\varepsilon}{k}\right)\sqrt{\left(\frac{\varepsilon}{k}\right)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{J_0(kt)\} = \left[\frac{\sigma^2}{\varepsilon\sqrt{\varepsilon^2+k^2\sigma^2}}\right] \qquad (22)$$

9.8 RISHI TRANSFORM OF $J_1(kt)$:

From (13), we have

$$\Upsilon\{J_1(t)\} = \left[\frac{\sigma\left(\sqrt{\varepsilon^2+\sigma^2}-\varepsilon\right)}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using change of scale property of Rishi transform on above equation, we have

$$\Upsilon\{J_1(kt)\} = \frac{1}{k^2}\left[\frac{\sigma\left(\sqrt{\left(\frac{\varepsilon}{k}\right)^2+\sigma^2}-\left(\frac{\varepsilon}{k}\right)\right)}{\left(\frac{\varepsilon}{k}\right)\sqrt{\left(\frac{\varepsilon}{k}\right)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{J_1(kt)\} = \frac{1}{k}\left[\frac{\sigma\left(\sqrt{\varepsilon^2+k^2\sigma^2}-\varepsilon\right)}{\varepsilon\sqrt{\varepsilon^2+k^2\sigma^2}}\right] \qquad (23)$$

9.9 RISHI TRANSFORM OF $J_2(kt)$:

From (18), we have

$$\Upsilon\{J_2(t)\} = \left[\frac{\sigma^2+2\varepsilon^2-2\varepsilon\sqrt{\varepsilon^2+\sigma^2}}{\varepsilon\sqrt{\varepsilon^2+\sigma^2}}\right]$$

Using change of scale property of Rishi transform on above equation, we have

$$\Upsilon\{J_2(kt)\} = \frac{1}{k^2}\left[\frac{\sigma^2+2\left(\frac{\varepsilon}{k}\right)^2-2\left(\frac{\varepsilon}{k}\right)\sqrt{\left(\frac{\varepsilon}{k}\right)^2+\sigma^2}}{\left(\frac{\varepsilon}{k}\right)\sqrt{\left(\frac{\varepsilon}{k}\right)^2+\sigma^2}}\right]$$

$$\Rightarrow \Upsilon\{J_2(kt)\} = \frac{1}{k^2}\left[\frac{k^2\sigma^2+2\varepsilon^2-2\varepsilon\sqrt{\varepsilon^2+k^2\sigma^2}}{\varepsilon\sqrt{\varepsilon^2+k^2\sigma^2}}\right] \qquad (24)$$

10. CONCLUSION: In this chapter, authors successfully determined the Rishi transform of Bessel's functions of first kind of order zero, one and two i.e. $J_0(t), J_1(t)$ and $J_3(t)$. Authors also obtained the Rishi transform of $e^{kt}J_0(t), e^{kt}J_1(t), e^{kt}J_2(t), J_0(kt), J_1(kt)$ and $J_2(kt)$ using translation and change of scale properties of Rishi transform. These results are important for determining the solutions of Bessel's equations and evaluating the improper integrals which contain Bessel's functions.

REFERENCES:

1. Wazwaz, A.M., Linear and nonlinear integral equations: Methods and applications, Higher Education Press, Beijing, 2011.
2. Jerri, A., Introduction to integral equations with applications, Wiley, New York, 1999.

3. Aggarwal, S., Sharma, N., Chauhan, R., Gupta, A.R. and Khandelwal, A., A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients, Journal of Computer and Mathematical Sciences, 9(6), 520-525, 2018.
4. S. Maitama and W. Zhao, New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations, International Journal of Analysis and Applications, 17(2), 167-190, 2019. https://doi.org/10.1186/s13662-019-2068-6
5. Ahmed, S.A., Elzaki, T.M., Elbadri, M. and Mohamed, M.Z., Solution of partial differential equations by new double integral transform (Laplace-Sumudu transform), Ain Shams Engineering Journal, 12(2021), 4045-4049. https://doi.org/10.1016/j.asej.2021.02.032
6. Chauhan, R. and Aggarwal, S., Laplace transform for convolution type linear Volterra integral equation of second kind, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 1-7, 2019.
7. Aggarwal, S. and Sharma, N., Laplace transform for the solution of first kind linear Volterra integral equation, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 16-23, 2019.
8. Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Kamal transform for solving linear Volterra integral equations, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(4), 138-140, 2018.
9. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Kamal transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(8), 2081-2088, 2018.
10. Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Mahgoub transform for solving linear Volterra integral equations, Asian Resonance, 7(2), 46-48, 2018.
11. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Mahgoub transform for solving linear Volterra integral equations of first kind, Global Journal of Engineering Science and Researches, 5(9), 154-161, 2018.
12. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integral equations of second kind using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3098-3102, 2018.
13. Aggarwal, S., Sharma, N. and Chauhan, R., A new application of Aboodh transform for solving linear Volterra integral equations, Asian Resonance, 7(3), 156-158, 2018.
14. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3745-3753, 2018.
15. Aggarwal, S., Chauhan, R. and Sharma, N., Application of Elzaki transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3687-3692, 2018.
16. Aggarwal, S., Gupta, A.R. and Sharma, S.D., A new application of Shehu transform for handling Volterra integral equations of first kind, International Journal of Research in Advent Technology, 7(4), 439-445, 2019.
17. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of second kind linear Volterra integral equation using Shehu transform, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(8), 26-32, 2020.
18. Aggarwal, S. and Gupta, A.R., Solution of linear Volterra integro-differential equations of second kind using Kamal transform, Journal of Emerging Technologies and Innovative Research, 6(1), 741-747, 2019.
19. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(5), 173-176, 2018.
20. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integro-differential equations of second kind, International Journal of Research in Advent Technology, 6(6), 1186-1190, 2018.

21. Aggarwal, S., Gupta, A.R. and Sharma, S.D., Application of Sadik transform for handling linear Volterra integro-differential equations of second kind, Universal Review, 10(7), 177-187, July 2019.
22. Aggarwal, S., Sharma, S.D. and Vyas, A., Aboodh transform for solving first kind faltung type Volterra integro-differential equation, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 71-77, 2020.
23. Aggarwal, S., Vyas, A. and Sharma, S.D., Mohand transform for handling convolution type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 78-84, 2020.
24. Aggarwal, S., Sharma, S.D. and Vyas, A., Solution of convolution type Volterra integro-differential equation of first kind using Kamal transformation, International Journal of Research and Innovation in Applied Science, 5 (7), 137-143, 2020.
25. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of faltung type Volterra integro-differential equation of first kind using Elzaki transform, International Journal of Research and Innovation in Applied Science, 5(7), 144-150, 2020.
26. Aggarwal, S., Sharma, S.D. and Vyas, A., Laplace-Carson transform for the primitive of convolution type Volterra integro-differential equation of first kind, International Journal of Research and Innovation in Applied Science, 5(7), 30-36, 2020.
27. Aggarwal, S., Sharma, S.D. and Vyas, A., Laplace transformation for the solution of faltung type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 5-11, 2020.
28. Aggarwal, S., Vyas, A. and Sharma, S.D., Analytical solution of first kind Volterra integro-differential equation using Sadik transform, International Journal of Research and Innovation in Applied Science, 5(8), 73-80, 2020.
29. Aggarwal, S., Sharma, S.D. and Vyas, A., Application of Sawi transform for solving convolution type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(8), 13-19, 2020.
30. Aggarwal, S., Sharma, S.D. and Vyas, A., Application of Sumudu transform for handling Volterra integro-differential equation of first kind with convolution kernel, International Journal of Research and Innovation in Applied Science, 5(8), 24-30, 2020.
31. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of first kind Volterra integro-differential equation using Shehu transform, International Journal of Research and Innovation in Applied Science, 5(8), 31-38, 2020.
32. Aggarwal, S., Sharma, S.D. and Vyas, A., Application of Sumudu transform for handling Volterra integro-differential equation of first kind with convolution kernel, International Journal of Research and Innovation in Applied Science, 5(8), 24-30, 2020.
33. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of first kind Volterra integro-differential equation using Shehu transform, International Journal of Research and Innovation in Applied Science, 5(8), 31-38, 2020.
34. Aggarwal, S., Kamal transform of Bessel's functions, International Journal of Research and Innovation in Applied Science, 3(7), 1-4, 2018.
35. Aggarwal, S., Sharma, N. and Chauhan, R., Mahgoub transform of Bessel's functions, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(8), 32-36, 2018.
36. Aggarwal, S., Chauhan, R. and Sharma, N., Mohand transform of Bessel's functions, International Journal of Research in Advent Technology, 6(11), 3034-3038, 2018.
37. Aggarwal, S., Gupta, A.R. and Agrawal, D., Aboodh transform of Bessel's functions, Journal of Advanced Research in Applied Mathematics and Statistics, 3(3), 1-5, 2018.
38. Aggarwal, S., Elzaki transform of Bessel's functions, Global Journal of Engineering Science and Researches, 5(8), 45-51, 2018.

39. Aggarwal, S., Sharma, S.D. and Vyas, A., Sawi transform of Bessel's functions with application for evaluating definite integrals, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 12-18, 2020.
40. Aggarwal, S., Kumar, R. and Chandel, J., Solution of linear Volterra integral equation of second kind via Rishi transform, Journal of Scientific Research, 15(1), 111-119, 2023. http://dx.doi.org/10.3329/jsr.v15i1.60337
41. Kumar, R., Chandel, J. and Aggarwal, S., A new integral transform "Rishi Transform" with application, Journal of Scientific Research, 14(2), 521-532, 2022. http://dx.doi.org/10.3329/jsr.v14i2.56545
42. Aggarwal, S., Kumar, R. and Chandel, J., Exact solution of non-linear Volterra integral equation of first kind using Rishi transform, Bulletin of Pure and Applied Sciences, Section-E-Mathematics & Statistics, 41E(2), 159-166, 2022.
43. Priyanka and Aggarwal, S., Solution of the model of the bacteria growth via Rishi transform, Journal of Advanced Research in Applied Mathematics and Statistics, 7(1&2), 5-11, 2022. https://doi.org/10.24321/2455.7021.202202