

# Modelling and stability analysis of isolated fractional-order power system

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**Abstract**— This paper investigates the stability of an isolated fractional-order power system. A novel fractional-order mathematical model of the power system is built from an engineering application perspective. The model is described by state-space equations and is composed of the power system, governor, turbine equation. Based on stability theory for a fractional-order nonlinear system, the stable region of the power system is investigated by Matignon stability theory. The simulation results of fractional and integer order power system has been studied and compare.

**Keywords**—Fractional order, Power system, Modelling, Stability

## I. INTRODUCTION

In the last few decades, engineering scholars have paid a lot of attention to fractional-order systems (FOS) because of the growth of the theory of fractional calculus. These kinds of systems have been used in many real-world situations, such as power systems, brushless DC motor systems, and chaotic systems [1]. Everyone knows that the fractional-order dynamic model of the system can describe physical events more accurately than the integer-order model [1-3]. Based on what was found in the past [2], the power system is part of an FOS. In the past, however, most research was done on power systems with integer orders [4,5]. Fractional-order controllers (FOC) [4, 5] have recently been studied and used in power systems. In [4, 5], the FOC were looked at from both a theoretical and a practical implementation point of view. In [5], an adaptive FOC was made to solve the problem of controlling the grid frequency in a power system with a wind turbine. But in these works [4, 5], the only model of the system that is looked at is the order of integers. So, even though research is done on the stability and fractional-order control of integer-order power systems, the stability of fractional-order power systems is still not well understood.

Novel contribution of presented work described below:

- i. A novel fractional-order mathematical model of an isolated power system is built.
- ii. The stability of the fractional-order power system is analyzed based on stability theory for a fractional-order nonlinear system.
- iii. The stable region of the fractional-order power system is investigated in detail.
- iv. Fractional-order modeling of isolated power system based on fractional calculus (using 'FOTF' and 'FDE 12' MATLAB command).

The remainder of the paper is structured as follows. Section 2 presents the mathematical model of the fractional-order power system. Section 3 discusses the fractional-order system stability. Section 4 contains the simulation results and a comparative commentary. Section 5 provides conclusion of the presented work.

## II. ISOLATED FRACTIONAL-ORDER MODEL OF POWER SYSTEM

The linearized model can be developed to study the dynamic performance of isolated FoPS. The controller designed for frequency regulation has been derived from the linearized model. The conventional single area thermal power system model is shown in Fig. 1

$$D^\alpha (\Delta f_i(t)) = -\frac{1}{T_p} \Delta f(t) + \frac{K_p}{T_p} \Delta P_g(t) - \frac{K_p}{T_p} \Delta D(t) + \frac{K_p}{T_p} P_w(t) \quad (1)$$

$$D^\alpha (\Delta P_i(t)) = \frac{1}{T_i} \Delta P_i(t) + \frac{1}{T_i} \Delta G_g(t) \quad (2)$$

$$D^\alpha (\Delta G_g(t)) = \frac{1}{RT_g} \Delta f(t) - \frac{1}{T_g} \Delta G_g(t) - \frac{1}{T_g} \Delta I_e(t) + \frac{1}{T_g} u(t) \quad (3)$$

$$D^\alpha (\Delta I(t)) = K_e \Delta f(t) \quad (4)$$

Where , and are the deviation in turbine output, frequency, governor valve position, and integral control, respectively, is the deviation in load, and are the time constants of power system, governor, and turbine, respectively.

The state-space equation of FO system is given as

$$D^\alpha x(t) = Ax(t) + Bu(t) + F\Delta d(t) \quad (5)$$

$$A = \begin{bmatrix} -\frac{1}{T_p} & \frac{K_p}{T_p} & 0 & 0 \\ 0 & -\frac{1}{T_i} & \frac{1}{T_i} & 0 \\ -\frac{1}{RT_g} & 0 & -\frac{1}{T_g} & -\frac{1}{T_g} \\ K_i & 0 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & \frac{1}{T_g} & 0 \end{bmatrix}^T, \quad F = \begin{bmatrix} -\frac{K_p}{T_p} & 0 & 0 & 0 \end{bmatrix}^T$$

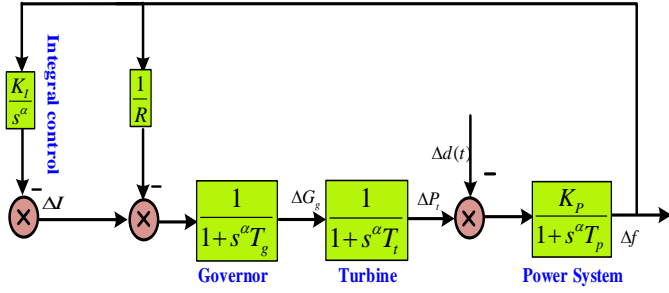


Fig. 1 Fractional-order power system model

### III. FRACTIONAL-ORDER SYSTEMR'S STABILITY

When discussing dynamical systems and their behaviours, one of the most frequently used terms in literature is "stability." In mathematics, stability theory is concerned with the convergence solutions of differential or difference equations. If the roots of the characteristics polynomial have a negative real part, the system (LTI) is said to be stable. The stability of a FO system (LTI) differs from that of an IO system. It is important to note that the roots of a FO system may lie on the right half of the complex plane (Fig. 2).

**Theorem [3]:** - As per the stability theory developed by Matignon, the FO transfer function(FOTF)

$$G(s) = \frac{n(s)}{d(s)} \text{ is stable if and only } \left| \arg(\sigma_j) \right| = \frac{q\pi}{2},$$

$n(s), d(s)$  are the numerator and denominator of FOTF

where  $\sigma = s^q, (0 < q < 2)$ , 's' is laplace oerator

with  $\forall \sigma_j \in \mathbb{C}, j^{\text{th}}$  root of  $d(\sigma) = 0$ .

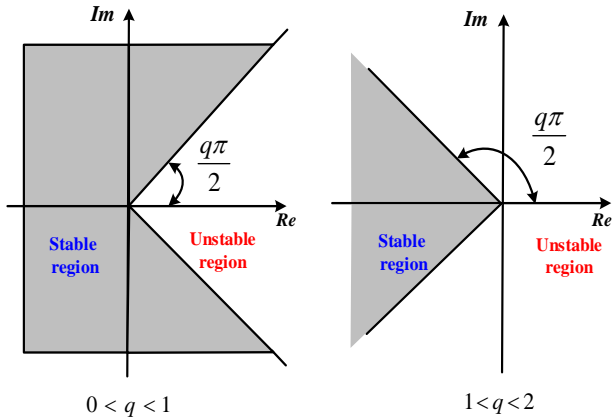


Fig. 2: Stable and unstable region of FO system [3]

### IV. SIMULATION RESULTS

The fractional-order transfer function model of isolated FOPS with fractional operator is given in Eq. 6 & 7

respectively. The pole locations of the system (illustrated in Figs. 2 & 3) reveal that no poles are situated in the complex plane's unstable zone, which is represented by the shaded region of the complex plane. As a result, it is certain that the system is stable

$$G(s)_{\alpha=0.95} = \frac{115s^{0.95}}{0.30375s^{3.8} + 5.1953s^{2.85} + 15.345s^{1.9} + 48.917s^{0.95} + 115} \quad (6)$$

$$G(s)_{\alpha=0.85} = \frac{115s^{0.85}}{0.30375s^{3.4} + 5.1953s^{2.55} + 15.345s^{1.7} + 48.917s^{0.85} + 115} \quad (7)$$

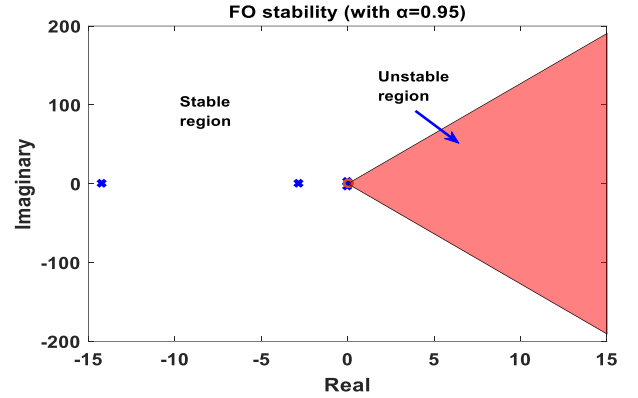


Fig. 3 FO system's stability with  $\alpha = 0.95$

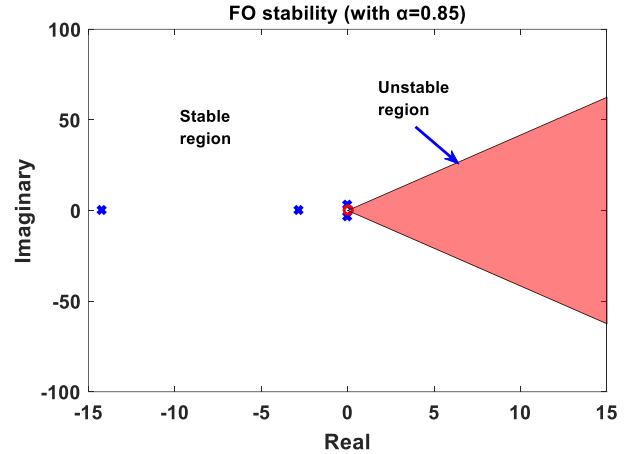


Fig. 4 FO system's stability with  $\alpha = 0.85$

For the verification of stability and accuracy of FO calculus, we have plotted the output of isolated FOPS (given in Fig. 5 and Fig. 6). The frequency deviation of IOPS with IO calculus ('ODE 23' MATLAB command) and FO calculus ('FDE 12' MATLAB command) is same, which confirms the FO calculus accuracy. Furthermore, a plot of frequency deviation FOPS with different values of fractional parameters such as is given in Fig. 6.

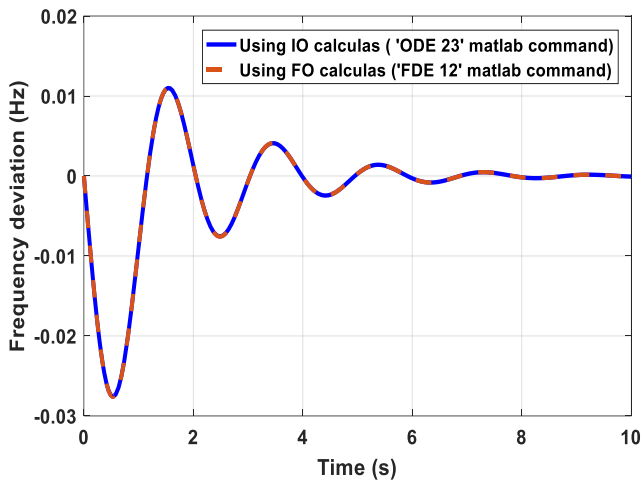


Fig. 5 Frequency deviation of FoPS with  $\alpha = 1$

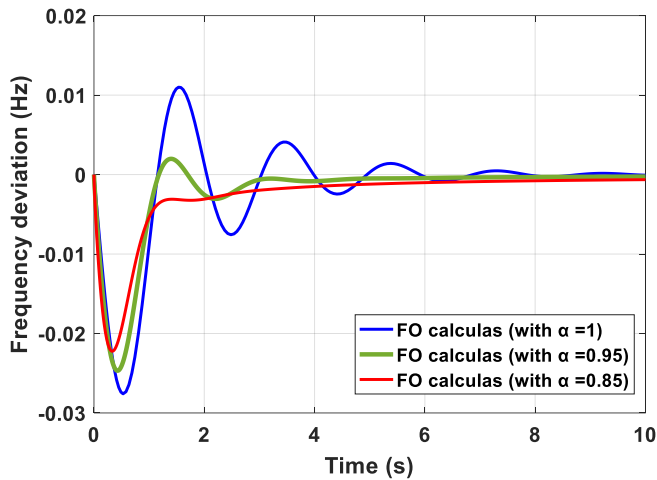


Fig. 6 Frequency deviation of FoPS with  $\alpha = 1, 0.95, 0.85$

## V. CONCLUSION

In this brief, modelling and stability of isolated fractional-order power system. The relevant stability results were obtained by using fractional-order stability theory [3]. The pole locations of the system reveal that no poles are situated in the complex plane's unstable zone, which is represented by the shaded region of the complex plane. As a result, it is certain that the system is stable. The frequency deviation of IOPS with IO calculus and FO calculus is same, which confirms the accuracy of FO calculus. Furthermore, plot of frequency deviation FOPS with different value of fractional parameters has plotted

## Appendix

### System parameters

$$T_g=0.075s, T_i=0.27s, K_p=115, R=2.4, T_p=15s, K_e=0.6$$

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