

A STUDY OF NEUTROSOPHIC \mathbf{R}_g – CLOSED SETS

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Abstract - This paper introduces the idea of neutrosophic \mathbb{R}_g closed (Regular Generalised Closed) sets, which are new neutrosophic closed sets in topological spaces. Additionally, some of its connections to other neutrosophic closed sets that already exist have been analysed, and some of their characteristics have been examined.

Keywords: *Neutrosophic R-closed, Neutrosophic g-open, Neutrosophic \mathbb{R}_g closure*

I. INTRODUCTION

Zadeh [1] first proposed the fuzzy set theory who also researched truth (\mathfrak{T}), the degree of membership, and defined it. Atanassov [2,3,4] presented the falsity (\mathfrak{F}), often known as the degree of nonmembership, in an intuitionistic fuzzy set. The intuitionistic fuzzy topology was created by Coker [5]. Smarandache [6,7] first proposed the concept of neutrality (\mathfrak{S}), or the degree of uncertainty, in 1998. Additionally, he described the neutrosophic set as consisting of three elements: truth, falsehood, and indeterminacy. Salama et al.'s translation of the neutrosophic crisp set notion into neutrosophic topological spaces may be found in [8]. As a result, a wide range of research on neutrosophic topology and its application in decision-making algorithms became possible. In neutrosophic topological spaces, Arokiarani et al. [9] introduced and investigated α -open sets. Devi et al. [10,11,12] presented generally $\alpha\psi$ -closed sets. This study introduces the idea of Neutrosophic \mathbf{R}_g -closed sets and Neutrosophic \mathbf{R}_g -open sets in Neutrosophic topological space and studies some of their characteristics.

II. PRELIMINARIES

Throughout this paper, \mathfrak{X} denote the neutrosophic topological space $(\mathfrak{X}, \mathfrak{N}_\tau)$ and for a subset \mathfrak{N}_A of (\mathfrak{X}, τ) the closure of \mathfrak{N}_A , interior of \mathfrak{N}_A , regular closure of \mathfrak{N}_A denoted by $\text{cl}(\mathfrak{N}_A)$, $\text{int}(\mathfrak{N}_A)$, $\text{rcl}(\mathfrak{N}_A)$ respectively.

Explanation: 2.1 A Subset \mathfrak{N}_A of $(\mathfrak{X}, \mathfrak{N}_\tau)$ is called if

- (i) Regular neutrosophic Closed(r- closed) Set [9] if $\text{cl}(\text{int}(\mathfrak{N}_A)) = \mathfrak{N}_A$.

- (ii) Regular generalized neutrosophic closed (briefly neutrosophic rg – closed) set [6] if $cl(\mathfrak{N}\check{A}) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is regular neutrosophic open in \mathfrak{X} .
- (iii) Neutrosophic δ -closed set [10] if $\mathfrak{N}\check{A} = cl_{\delta}(\mathfrak{N}\check{A})$, where $cl_{\delta}(A) = \{u \in \mathfrak{X} : int(cl(Z)) \cap \mathfrak{N}\check{A} \neq \emptyset, Z \in \tau \text{ and } u \in Z\}$
- (iv) Weakly π -generalized neutrosophic closed (briefly $w\pi g$ – closed) [7] if $cl(int(\mathfrak{N}\check{A})) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is neutrosophic π -open in \mathfrak{X} .
- (v) Regular Feebly Generalized neutrosophic closed (briefly RFG – closed) set [11] if $fcl(\mathfrak{N}\check{A}) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is regular generalized neutrosophic open (rg – open) set in \mathfrak{X} .
- (vi) semi-closed [4] if $int(cl(\mathfrak{N}\check{A})) \subseteq \mathfrak{N}\check{A}$.

Explanation: 2.2 A Subset $\mathfrak{N}\check{A}$ of a neutrosophic topological space $(\mathfrak{X}, \mathfrak{N}_{\tau})$ is called

1. Generalized neutrosophic closed set (briefly g-closed) [3] if $cl(\mathfrak{N}\check{A}) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is open in $(\mathfrak{X}, \mathfrak{N}_{\tau})$.
2. Weakly generalized neutrosophic closed (briefly wg-closed) [5] if $cl(int(A)) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is open in \mathfrak{X} .
3. regular weakly generalized (briefly neutrosophic rwg -closed) [5] if $cl(int(\mathfrak{N}\check{A})) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is regular neutrosophic open in \mathfrak{X} .

Explanation: 2.3 Let \mathfrak{X} be a neutrosophic topological space. The finite union of regular neutrosophic open sets in \mathfrak{X} is said to be neutrosophic π -open set [2]. The complement of a neutrosophic π -open set is said to be neutrosophic π -closed set [2].

Explanation: 2.4 A subset $\mathfrak{N}\check{A}$ of a neutrosophic topological space $(\mathfrak{X}, \mathfrak{N}_{\tau})$ is called

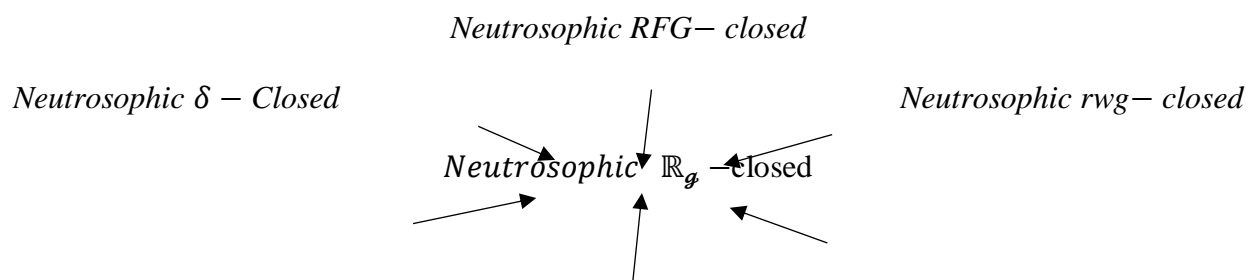
1. Neutrosophic Pre-closed set [8] if $cl(int(\mathfrak{N}\check{A})) \subseteq \mathfrak{N}\check{A}$.
2. Neutrosophic β – closed set [1] if $int(cl(int(\mathfrak{N}\check{A}))) \subseteq \mathfrak{N}\check{A}$.

The complements of the above mentioned neutrosophic closed sets are their respective neutrosophic open sets.

III. NEUTROSOPHIC \mathbb{R}_g – CLOSED SETS

Explanation: 3.1 A subset $\mathfrak{N}\check{A}$ of a neutrosophic topological space $(\mathfrak{X}, \mathfrak{N}_{\tau})$ is called a regular generalized neutrosophic closed set (briefly \mathbb{R}_g – closed) if $rcl(\mathfrak{N}\check{A}) \subseteq Z$ whenever $\mathfrak{N}\check{A} \subseteq Z$ and Z is neutrosophic g open in $(\mathfrak{X}, \mathfrak{N}_{\tau})$. The complement of a neutrosophic \mathbb{R}_g – closed set is neutrosophic \mathbb{R}_g – open set.

Remark: 3.1



Neutrosophic π –closed Neutrosophic $w\pi g$ – closed

Neutrosophic r – closed

Principium :3.1 Every neutrosophic closed sets are neutrosophic \mathbb{R}_g -closed sets.

Testament: Let $\mathfrak{N}\check{A}$ be any neutrosophic closed set in \mathfrak{X} . Suppose Z is neutrosophic τ -open. Since every neutrosophic τ -open set is neutrosophic g -open and $\mathfrak{N}\check{A}$ is neutrosophic closed,

we have $cl(\mathfrak{N}\check{A}) \subseteq rcl(\mathfrak{N}\check{A}) \subseteq Z$ implies $cl(\mathfrak{N}\check{A}) \subseteq Z$, Z is neutrosophic g -open. Hence $\mathfrak{N}\check{A}$ is neutrosophic R_g -closed.

Principium :3.2 Every neutrosophic RFG-closed sets are neutrosophic \mathbb{R}_g -closed sets.

Testament: Let $\mathfrak{N}\check{A}$ be any neutrosophic RFG-closed set in \mathfrak{X} . Suppose Z is $\mathfrak{N}\check{A}$ rg -open in \mathfrak{X} such that $\mathfrak{N}\check{A} \subseteq Z$. Since every neutrosophic g -open set is neutrosophic rg -open and $\mathfrak{N}\check{A}$ is neutrosophic RFG-closed, we have $rcl(\mathfrak{N}\check{A}) \subseteq fcl(\mathfrak{N}\check{A}) \subseteq Z$ implies $rcl \subseteq Z$, Z is g -open. Hence $\mathfrak{N}\check{A}$ is neutrosophic \mathbb{R}_g -closed.

The following Illustration 3.3 clears that the converse of the Principium 3.1 need not be true.

Illustration:3.3 Let $\mathfrak{X} = \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph)\}$

$\tau = \{\phi, \mathfrak{X}, \{a\}, \{b,c\} \{a,b,c\}\}$. Neutrosophic R_g -closed sets are

$\{\mathfrak{X}, \phi, \{(\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{(\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\},$

$\{(\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{ \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\} \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\},$

$\{(\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{ \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\} \},$

neutrosophic RFG-closed sets are $\{\mathfrak{X}, \phi, \{(\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{ \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\} \}, \{(\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}, \{ \{(\mathfrak{I}_\lambda, \mathfrak{S}_\lambda, \mathfrak{F}_\lambda), (\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\} \}$.

Here $\{(\mathfrak{I}_\aleph, \mathfrak{S}_\aleph, \mathfrak{F}_\aleph), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}$ and $\{(\mathfrak{I}_2, \mathfrak{S}_2, \mathfrak{F}_2), (\mathfrak{I}_\bullet, \mathfrak{S}_\bullet, \mathfrak{F}_\bullet)\}$ are neutrosophic \mathbb{R}_g -closed but not neutrosophic RFG-closed.

Principium :3.4 Every neutrosophic \mathbb{R}_g -closed sets are neutrosophic rwg -closed sets.

Testament: Let $\mathfrak{N}\check{A}$ be any neutrosophic \mathbb{R}_g -closed set in \mathfrak{X} . Suppose Z is neutrosophic r -open in \mathfrak{X} . Since every neutrosophic r -open set is neutrosophic g -open in X and $\mathfrak{N}\check{A}$ is neutrosophic \mathbb{R}_g -

ACKNOWLEDGMENT

I would like to express my special thanks of gratitude to those who helped me in completing the work. I have come to know about so many new things. I am really thankful to them.

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