Intuitionistic Fuzzy Threshold Hypergraphs: Introduction and Review

Myithili. K. K (Associate Professor & Head),
Department of Mathematics(CA),
Vellalar College for Women, Erode,
TamilNadu, India.
mathsmyth@gmail.com

Nandhini. C (Research Scholar),
Department of Mathematics(CA),
Vellalar College for Women, Erode,
TamilNadu, India.
c.nandhini@vcw.ac.in

ABSTRACT

Intuitionistic fuzzy sets have gained significant attention in recent years due to their ability to handle uncertainty and imprecision in decision-making problems. Hypergraphs, on the other hand, provide a flexible framework for modeling complex relationships among multiple elements. In this review, we focus on the integration of intuitionistic fuzzy sets with hypergraphs through the concept of intuitionistic fuzzy threshold hypergraphs.

Keywords: intuitionistic fuzzy sets, threshold graph, intuitionistic fuzzy threshold hypergraph.

I. INTRODUCTION

Graph theory is a branch of mathematics that deals with the study of graphs, which are mathematical structures used to model pairwise relationships between objects. Graph theory has its roots in the 18th century and has since become a fundamental area of research with applications in various fields, including computer science, operations research, social networks and more.

The origins of graph theory can be traced back to the work of the Swiss Mathematician Leonhard Euler in the 18th century. In 1736, Euler solved the famous problem known as the Seven Bridges of Königsberg. The city of Königsberg (now Kaliningrad, Russia) was divided into four land masses connected by seven bridges. The problem was to determine if it was possible to take a walk through the city, crossing each bridge exactly once and returning to the starting point. Euler realized that he could represent the land masses and bridges as a graph, with land masses as vertices and bridges as edges. He then proved that it was impossible to find such a walk if there were more than two land masses with an odd number of bridges. This solution laid the foundations for graph theory.

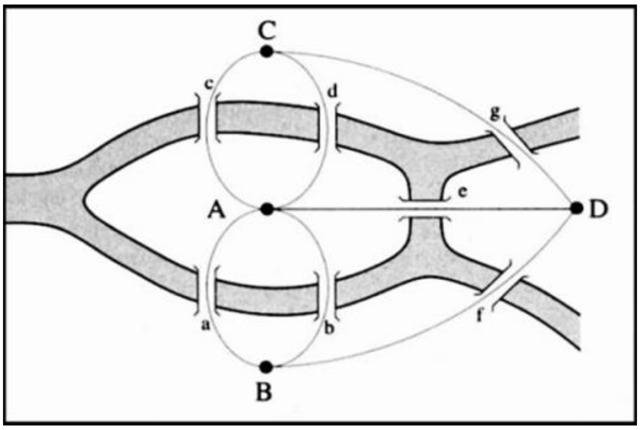

Figure 1: Seven Bridges of Königsberg

Following Euler's work, other mathematicians made significant contributions to the development of graph theory. In the 19th century, Augustin-Louis Cauchy and Gustav Kirchhoff developed the concept of a tree, which is a connected graph without any cycles. They also introduced the concept of a planar graph, which can be drawn on a plane without any edges crossing.

In the late 19th and early 20th centuries, important results in graph theory were obtained by Arthur Cayley, William Tutte and Percy Heawood. Cayley studied trees and introduced the concept of a Cayley graph, which represents the structure of a group. Tutte made significant contributions to the theory of planar graphs and graph coloring. Heawood made important advances in the study of map coloring and proved the Four Color Theorem, which states that any map on a plane can be colored using at most four colors in such a way that no two adjacent regions have the same color.

During the mid-20th century, graph theory experienced a rapid expansion, driven by the growing field of computer science. Many fundamental concepts and algorithms were developed during this time. Notable contributions include the development of algorithms for finding minimum spanning trees by Otakar Borůvka and Joseph Kruskal, the introduction of graph connectivity and algorithms for finding connected components by Edsger Dijkstra and Robert Tarjan and the development of the network flow algorithms by Ford and Fulkerson.

In the latter half of the 20th century, graph theory found numerous applications in computer science, optimization and network analysis. It became an essential tool for modeling and solving problems in various domains, including computer networks, social networks, logistics and operations research. Many advanced concepts and algorithms were developed, including spectral graph theory, random graph theory, graph clustering and algorithms for solving graph optimization problems.

Since then, graph theory has continued to evolve with the advent of new technologies and the increasing complexity of real-world networks. It remains an active area of research, with ongoing studies on topics such as network analysis, graph algorithms, social network analysis and machine learning on graphs. Graph theory has become an indispensable tool for understanding and analyzing the structure and dynamics of complex systems in a wide range of disciplines.

II. GRAPH & HYPERGRAPH

A. Graph

In graph theory, a graph, is a mathematical structure that consists of a set of objects called vertices (or nodes) and a set of connections between these vertices called edges (or arcs). This concept was first introduced by the Swiss Mathematician Leonhard Euler in the 18th century.

Formally, a *graph* is defined as an ordered pair $G = (V, E)$, where V represents the set of vertices and E represents the set of edges. The vertices can be any discrete objects or elements, such as cities, people or molecules. The edges represent the relationships or connections between these vertices.

Graphs can be classified into two main types based on their characteristics:

In a *directed graph*, each edge has a specific direction associated with it. This means that the edges have an origin vertex and a destination vertex and the connection is one-way. For example, if there is an edge from vertex 1 to vertex 2, it does not necessarily imply the existence of an edge from 2 to 1.

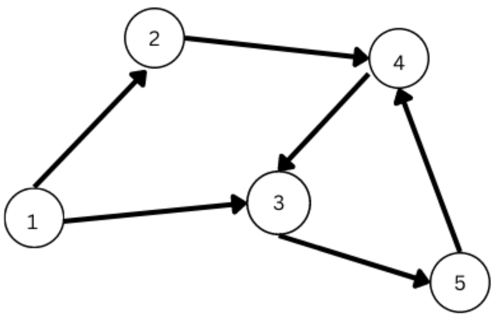

Figure 2: Digraph

In an *undirected graph*, the edges do not have any specific direction. The connections between the vertices are bidirectional, meaning that if there is an edge between vertex 1 and vertex 2, there is also an edge between 2 and 1.

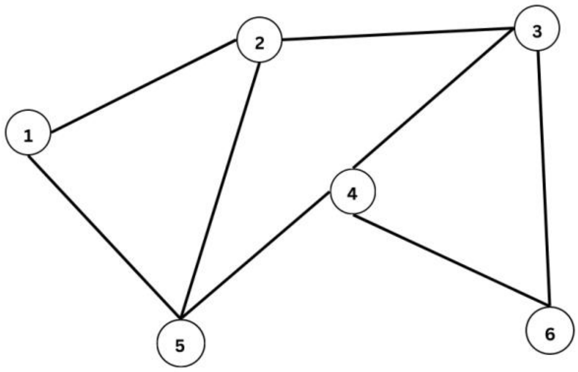

Figure 3: Undirected graph

The transition from mathematics to graph theory can be attributed to several reasons:

- Graph theory provides a visual representation of mathematical concepts and relationships. By using graphs and networks, it becomes easier to analyze and understand complex structures and patterns. This visual aspect makes graph theory particularly useful in various fields.
- Graph theory simplifies the representation of complex systems and problems. It allows us to abstract real-world situations into a set of nodes (vertices) and connections (edges) between them. This abstraction enables us to focus on the underlying relationships and properties of the system, making

problem-solving more manageable.

- Graph theory emphasizes the study of connectivity and relationships between entities. By representing entities as nodes and relationships as edges, graph theory provides a powerful framework for analyzing interconnectedness, communication networks, dependencies and flows in various domains. This approach helps in understanding the behavior of systems and optimizing their performance.
- Graph theory has found applications in numerous disciplines, including computer science, physics, biology, sociology, transportation planning and many others. Its versatility and ability to model diverse systems have made it a fundamental tool for researchers and practitioners across multiple fields.
- Graph theory is deeply rooted in combinatorial mathematics, which deals with discrete structures and counting techniques. By focusing on combinatorial aspects, graph theory provides a foundation for solving problems related to permutations, combinations, optimization and enumeration, making it a valuable branch of mathematics in its own right.

Graph theory provides a mathematical framework to study the properties and relationships within graphs, allowing for the analysis of various real-world problems. It is widely used in computer science, operations research, social network analysis and many other fields.

Leonhard Euler's[11] seminal paper on graph theory is titled as Solution of a Problem in the Geometry of Position was published in 1736 in the Journal Commentaries of the St. Petersburg Academy of Sciences. The Königsberg problem was to determine whether it was possible to take a walk through the city in such a way that one would cross each bridge exactly once and return to the starting point. Graph theory with applications by J. A. Bondy and U. S. R. Murty[2] covers fundamental concepts such as graph representation, connectivity, cycles, trees, planarity and graph coloring. The book also explores more advanced topics like graph algorithms, network flows and matchings. A first course in graph theory by S. A. Choudum[3] covers various topics in graph theory such as Eulerian and Hamiltonian graphs, planarity, colouring and digraph. A complete vector spaces associated with graphs, rarely found in textbooks is an important feature of this book.

B. Threshold Graph

A threshold graph is defined based on a threshold function. Threshold graphs were first introduced by Chvatal and Hammer in 1977. A threshold graph can be defined as follows: Given a set of vertices V and a set of threshold values s(v) associated with each vertex v, an edge exists between two vertices v and u if and only if the sum of their threshold values exceeds the threshold sum of their common neighbors.

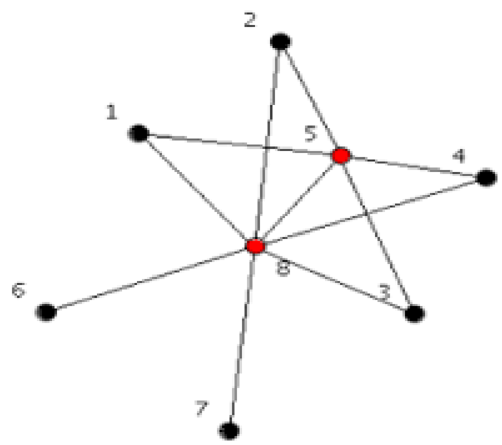

Figure 6: Threshold graph

Threshold function: The threshold function assigns a real value to each vertex, indicating its threshold or willingness to form an edge. The values are typically in the range [0, 1]. Vertices with higher threshold values are more likely to be connected to other vertices.

Some important properties of threshold graphs include:

- Threshold graphs are closed under complementation and induced subgraphs
- Threshold graphs can be recognized in polynomial time
- Threshold graphs have a number of characterizations, such as intersection graphs of intervals, consecutive ones property matrices and comparability graphs of posets

Threshold graphs exhibit several structural properties that make them interesting and useful. These properties include:

- Threshold graphs are chordal, i.e., every induced cycle of length greater than three has a chord (an edge connecting two non-adjacent vertices)
- Threshold graphs have a perfect elimination ordering, which is an ordering of the vertices such that, at each step, the vertex being eliminated is simplicial (i.e., its neighbors form a clique)
- Threshold graphs have a modular decomposition, which allows them to be decomposed into smaller components with specific properties

Connectivity: Threshold graphs are connected graphs then there exists a path between any two vertices in the graph.

Transitivity: Threshold graphs are not transitive, which means that if vertex 1 is connected to vertex 2 and vertex 2 is connected to vertex 3, it does not guarantee that vertex 1 is directly connected to vertex 3. The lack of transitivity can lead to interesting and complex connectivity patterns in the graph.

Recognition: There are efficient algorithms for recognizing and constructing threshold graphs. The Chvatal and Hammer algorithm provides a polynomial-time algorithm for constructing a threshold graph from a threshold function.

Threshold graphs have applications in various domains, including social networks, scheduling problems, genetics and resource allocation. They provide a useful framework for modeling relationships and connectivity based on thresholds or preferences. Some well-known graph classes, such as interval graphs, bipartite graphs and comparability graphs are special cases of threshold graphs also highlighting the broad applicability of this graph class.

The paper "Aggregation of Inequalities in Integer Programming," was authored by Vaclav Chvatal and Peter L. Hammer[4] and published in 1977. Although this paper primarily focuses on the application of threshold graphs to integer programming, it provides important insights into the theory and properties of threshold graphs. In their paper, Chvatal and Hammer investigate the connection between threshold graphs and integer programming formulations. They introduce the concept of threshold graphs as a tool for modeling combinatorial problems and explore their structural properties. The authors demonstrate that certain classes of valid inequalities in integer programming can be systematically generated by exploiting the structure of threshold graphs.

Graduate students and researchers interested in graph theory may find the book titled as Threshold Graphs and Related Topics by N.V.R. Mahadev, U.N. Peled[13] has numerous open questions and research proposals appealing. But most importantly, threshold graphs and related topics is a great resource for anyone working in this field for information.

C. Hypergraph

A hypergraph is a mathematical structure that generalizes the concept of a graph. While a traditional graph consists of a set of vertices connected by edges, a hypergraph allows for hyperedges, which can connect multiple vertices simultaneously. In other words, hypergraphs can capture relationships among more than two elements.

Formally, a hypergraph H is defined as a pair (V, E), where V is a set of vertices or nodes and E is a set of hyperedges. Each hyperedge $e_i \in E,$ where i=1,2,3,4 is a subset of V, meaning it can include any number of vertices.

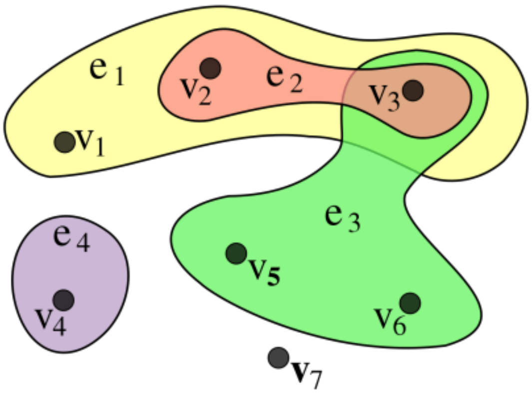

Figure 4: Hypergraph

Hypergraphs provide a flexible framework for modeling complex relationships and dependencies. They find applications in various fields, including computer science, mathematics, data mining, social network analysis and biology. For instance, in computer science, hypergraphs are useful for modeling data with higher-order dependencies, such as in knowledge representation, database design and constraint satisfaction problems.

Hypergraphs can be represented visually using diagrams, similar to graphs, where vertices are represented as dots and hyperedges are depicted as curves or lines connecting the relevant vertices. The study of hypergraphs involves exploring properties, algorithms and structures specific to this mathematical object, enabling researchers to understand and solve problems that go beyond the limitations of traditional graphs.

Hypergraphs by Claude Berge[6] is a seminal book in the field of combinatorial mathematics and graph theory. First published in 1973, it remains a significant reference for researchers and students interested in hypergraphs and their applications. Berge's book provides a comprehensive and rigorous introduction to the theory of hypergraphs. The author begins by defining hypergraphs and exploring their basic properties, such as edges, vertices and its incidences. He then delves into various types of hypergraphs, including uniform hypergraphs, regular hypergraphs and bipartite hypergraphs among others. The exposition is clear and concise, making it accessible to readers with a solid mathematical background. Graphs and Hypergraphs by Claude Berge[5] is a classic textbook that provides a comprehensive introduction to the theory and applications of graphs and hypergraphs. The book starts by presenting the basic concepts and terminology of graph theory, such as graphs, vertices, edges, connectivity, planarity and graph coloring. It then delves into more advanced topics, including graph algorithms, graph embeddings and graph minors. Hypergraph Theory: An Introduction by Alain Bretto[1] book provides an introduction to hypergraphs and aims to overcome the lack of recent manuscripts on this theory. Hypergraphs come by a variety of other names in the literature, including set systems and families of sets. This work introduces and evaluates the most recent theories on hypergraphs as well as the theory of hypergraphs in its most original parts. Connection and separation in hypergraphs by Mohammad A. Bahmanian & Mateja Sajna[14] investigates different fundamental connectivity features of hypergraphs from a graph-theoretical perspective, with a focus on cut edges, cut vertices and blocks. They demonstrate a variety of new results involving these ideas. They specifically outline the exact connection between a hypergraph's incidence graph and its block decomposition. Hypergraphs: An introduction and review by Xavier Ouvrard[19] aims to provide some clues on the key findings that may find in the literature, both from a mathematical and practical usage perspective.

III. FUZZY SETS, FUZZY RELATION & FUZZY GRAPH

A *fuzzy set* is an extension of a classical set, where each element has a degree of membership between 0 and 1, representing the degree of belongingness to the set. Unlike crisp sets, which have binary membership (either 0 or 1), fuzzy sets allow for partial membership. This enables the representation of vague or uncertain information in a more flexible manner.

A *fuzzy relation* extends the concept of a crisp relation between two fuzzy sets . It describes the relationship between elements of two fuzzy sets by assigning a degree of membership to each pair of elements. Fuzzy relations are commonly represented as matrices, where each entry represents the degree of membership of the corresponding pair of elements.

A *fuzzy graph* combines the notions of a graph and fuzzy sets. It consists of a set of vertices and a set of edges, where each vertex and edge can have a degree of membership associated with it. The degree of membership represents the strength or intensity of the presence of a vertex or edge in the graph. Fuzzy graphs provide a powerful tool for modeling and analyzing complex systems with uncertain or imprecise relationships.

A *Fuzzy logic* employs linguistic variables, which are terms that represent imprecise or subjective concepts. These variables are defined by membership functions that assign a degree of membership to each possible value within the variable's range. For example, a linguistic variable like "temperature" could have values such as "hot," "warm," or "cold," each with a corresponding membership function indicating the degree to which a given temperature belongs to that category.

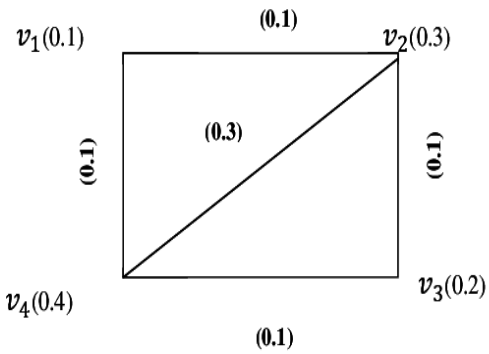

Figure 4: Fuzzy Graph

Here are a few of its applications.:

- Fuzzy logic provides a framework for handling imprecise information and is widely used in decision-making systems, particularly in areas where human expertise and subjective judgment are involved.
- Fuzzy logic controllers are used to control complex systems where the rules and relationships are not well-defined. They are employed in areas such as industrial automation, robotics and intelligent transportation systems.
- Fuzzy sets and fuzzy relations are used in pattern recognition algorithms to deal with uncertainties in feature measurements and classification tasks. They could help to handle ambiguous data.
- Fuzzy image processing techniques are applied in tasks like image enhancement, segmentation and object recognition. Fuzzy sets and fuzzy relations enable the modeling of fuzzy boundaries and regions in images.
- Fuzzy logic is used in data mining and machine learning to handle uncertain data. Fuzzy clustering algorithms and fuzzy association rule mining techniques can extract valuable patterns from imprecise datasets.

These are just a few examples of the broad range of applications of fuzzy sets, fuzzy relations and fuzzy logic.

They provide a powerful mathematical framework to tackle uncertainty and imprecision, making them invaluable tools in various fields.

Fuzzy sets by L. A. Zadeh[20] discussed the concepts of inclusion, union, intersection, complement, relation, convexity, etc. and various features of these concepts are established in the context of fuzzy sets in 1965. Fuzzy Graph Theory by Sunil Mathew, John N. Mordeson, Davender S. Malik[18], is a book provides a thorough exploration of fuzzy graph theory, offering a balanced mix of theoretical foundations, methodologies and practical applications. It is suitable for both researchers and practitioners interested in understanding and applying fuzzy graph theory to real-world problems. Fuzzy Graphs and Fuzzy Hypergraphs by John N. Mordeson and Premchand S. Nair[7] book provides a comprehensive introduction to fuzzy graphs, covering basic concepts, properties and algorithms. It also discusses applications in areas such as computer science, social sciences and engineering. Modern Trends in Fuzzy Graph Theory by Madhumangal Pal, Sovan Samanta, Ganesh Ghorai[12], book offers a comprehensive set of methods for applying graph theory and fuzzy mathematics to practical issues. In order to focus on more advanced ideas like planarity in fuzzy graphs, fuzzy competition graphs, fuzzy threshold graphs, fuzzy tolerance graphs, fuzzy trees, colouring in fuzzy graphs, bipolar fuzzy graphs, intuitionistic fuzzy graphs and m-polar fuzzy graph. This book begins with existing fundamental theories such as connectivity, isomorphism, products of fuzzy graphs and different types of paths and arcs in fuzzy graphs. A number of important representative applications of the idea being explored are included in each chapter.

IV. INTUITIONISTIC FUZZY SET, RELATION & GRAPH

Intuitionistic fuzzy sets, relations and graphs were introduced by Krassimir T. Atanassov as extensions of classical fuzzy set theory. These concepts aim to capture uncertainty and ambiguity in a more nuanced and flexible manner. Here's a brief explanation of each:

An *Intuitionistic Fuzzy Set*(IFS) is a generalization of classical fuzzy sets. It allows for the representation of not only the degree of membership but also the degree of non-membership and hesitancy associated with each element. In an intuitionistic fuzzy set, each element is assigned a membership degree, a non-membership degree and a hesitation degree. This provides a more comprehensive description of the uncertainty and partial knowledge associated with the set of elements.

An *intuitionistic fuzzy relation* extends the notion of a fuzzy relation to capture the uncertainty and hesitancy in the relationships between elements of different sets. It assigns a membership degree, a non-membership degree and a hesitation degree to each pair of elements, indicating the degree of support, non-support and hesitancy associated with the relation between them.

An *intuitionistic fuzzy graph* is a graph structure where each edge is associated with membership, non-membership and hesitation degrees, reflecting the uncertainty and hesitancy in the relationship between connected nodes. Intuitionistic fuzzy graphs are useful in modeling complex systems where the relationships between entities are not precisely defined and involved uncertainty. They have applications in areas such as social networks, transportation networks and knowledge representation.

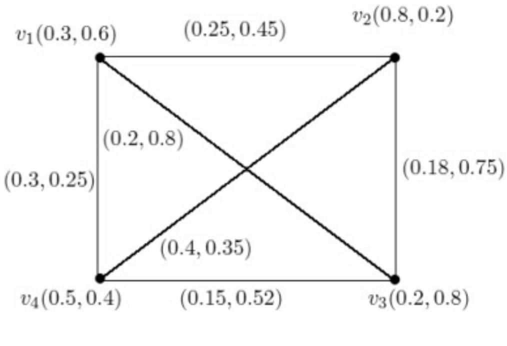

Figure 5: Intuitionistic Fuzzy Graph

Fuzzy graphs are useful for representing imprecise or uncertain information in a graph structure. They allow for degrees of membership to be assigned to vertices and edges, which enables modeling of gradual transitions and fuzzy relationships. However, fuzzy graphs primarily focus on capturing the membership values and do not explicitly consider uncertainty or hesitation in the assigned memberships.

Intuitionistic fuzzy graphs, on the other hand, extend the concept of fuzzy graphs by incorporating the notion of non-membership and hesitancy. Non-membership values provide a measure of the degree to which a vertex or an edge does not belong to a specific category, complementing the membership values. Hesitancy values quantify the degree of uncertainty in assigning a particular label or membership value. This additional information allows for a more nuanced representation of ambiguous situations.

By moving from a fuzzy graph to an intuitionistic fuzzy graph, to gain the ability to explicitly represent uncertainty, ambiguity and hesitation in the graph structure. This enhanced representation can be valuable in various domains and applications where the level of uncertainty needs to be considered explicitly. It provides a more comprehensive model for reasoning, decision-making and handling incomplete information in graph-based systems.

One of the key strengths of the book named as On intuitionistic fuzzy sets: Theory and Application by Krassimir T. Atanassov[9] lies in its clear and rigorous mathematical formalism. Atanassov presents the mathematical foundations of intuitionistic fuzzy sets with precision, providing definitions, axioms and theorems. The book also includes numerous examples and illustrations to aid in understanding the concepts and their applications. The later chapters of the book delve into the various applications of intuitionistic fuzzy sets. Atanassov explores their use in decision-making, pattern recognition, image processing, clustering and expert systems, and other multiple domains. He demonstrates how intuitionistic fuzzy sets can handle uncertainty, vagueness and imprecision more effectively than classical fuzzy sets, making them a valuable tool in real-world applications. The book's organization and structure are well-designed, allowing readers to follow the logical progression of concepts and build their knowledge systematically. Atanassov's writing style is clear and concise, making complex ideas accessible to a wide range of readers, from researchers and practitioners to students interested in the field.

The book on intuitionistic fuzzy set theory by Krassimir T. Atanassov[8], consists of the concept of IFS, operations and relation over IFS and geometrical interpretations of IFS. The book begins by introducing the concept of IFS as an extension of classical fuzzy sets. Atanassov presents a thorough discussion of the basic operations and properties of IFS, including the membership function, non-membership function and the hesitation function. He provides detailed explanations of these functions and their interpretations in the context of intuitionistic fuzzy sets.

V. INTUITIONISTIC FUZZY THRESHOLD GRAPHS

Intuitionistic fuzzy threshold graphs is an extension of threshold graphs, where the assigned values to the vertices and the threshold condition are represented using intuitionistic fuzzy numbers. This allows for a more nuanced representation of uncertainty and vagueness in the graph structure. It seems possible to consider intuitionistic fuzzy threshold graph to be a special IFG. It is therefore worthwhile in order to develop IFG theories, intuitionistic fuzzy threshold graphs is used subsequently. According to these, studying intuitionistic fuzzy threshold graphs (IFTGs) as compared to threshold graphs is essential because they use intuitionistic fuzzy sets to explain uncertainty and ambiguity rather than sets.

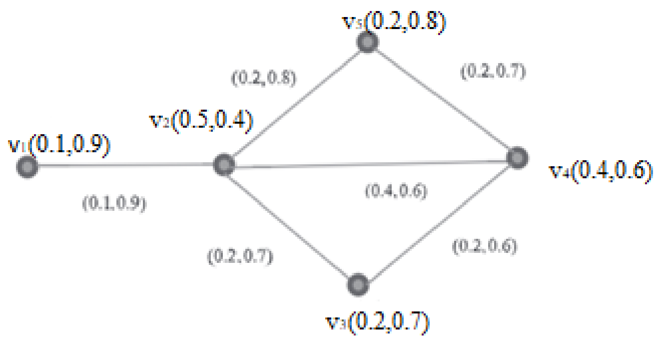

Figure 7: Intuitionistic Fuzzy Threshold Graph

The paper Intuitionistic fuzzy threshold graphs by Lanzhen Yang and Hua Mao[10], they provide three concepts of intuitionistic fuzzy threshold graphs, intuitionistic fuzzy alternating 4-cycle and threshold dimension of intuitionistic fuzzy graphs and provide an extension of threshold graphs. Then they discuss various characteristics of intuitionistic fuzzy graphs and come to two primary conclusions. The first is that an intuitionistic fuzzy threshold graph is equivalent to an intuitionistic fuzzy graph in some circumstances.

VI. INTUITIONISTIC FUZZY HYPERGRAPH

Intuitionistic fuzzy hypergraphs are an extension of classical hypergraphs that incorporate the concept of intuitionistic fuzzy sets. Intuitionistic fuzzy sets, on the other hand, are an extension of fuzzy sets that allow for the modeling of uncertainty and hesitation in the membership and non-membership degrees of elements. In intuitionistic fuzzy hypergraphs, the vertices represent elements and the edges represent relationships or connections between these elements. However, in intuitionistic fuzzy hypergraphs, an edge is connected to an intuitionistic fuzzy set that describes the membership and non-membership degrees of the vertices, in contrast to classical hypergraphs where a hyperedge can connect any number of vertices.

The concept of intuitionistic fuzzy hypergraphs provides a powerful tool for modeling complex and uncertain relationships. By incorporating the notion of hesitation and uncertainty in the membership and non-membership degrees, it allows for a more realistic representation of real-world situations where the exact membership status of elements may not be known. Intuitionistic fuzzy hypergraphs have found applications in various fields such as clustering, data mining, expert systems and pattern recognition. They offer a flexible framework for handling uncertain and imprecise information, providing a more comprehensive understanding of complex systems.

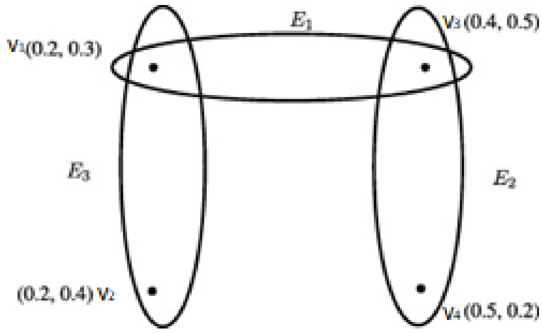

Figure 8: Intuitionistic Fuzzy Hypergraph

R. Parvathi, S. Thilagavathi & M.G.Karunambigai[17] was the first to introduce the intuitionistic fuzzy hypergraph. The terms dual intuitionistic fuzzy hypergraph (DIFHG) and intuitionistic fuzzy hypergraph (IFHG) are explained in intuitionistic fuzzy hypergraph. The ideas of (α, β)-cut hypergraphs and edge strength have also been introduced in IFHG. Then Muhammad Akram and Wieslaw A. Dudek[15] explained about Intuitionistic fuzzy hypergraphs with applications. Some types of intuitionistic fuzzy directed hypergraphs are discussed in [16].

VII. INTUITIONISTIC FUZZY THRESHOLD HYPERGRAPH

Intuitionistic fuzzy threshold hypergraphs (IFTHs) are an extension of fuzzy hypergraphs that incorporate the concept of intuitionistic fuzzy sets. These hypergraphs are used to model complex systems, decision-making processes and information representation. The importance of intuitionistic fuzzy threshold hypergraphs lies in their ability to handle imprecision in a more expressive and flexible manner.

Intuitionistic fuzzy threshold hypergraphs (IFTHGs) are a mathematical tool that extends the concept of fuzzy hypergraphs by allowing the representation of degrees of membership and non-membership to a hyperedge. This makes IFTHGs a suitable framework for modeling complex systems where uncertainty and ambiguity are present.

The *intuitionistic fuzzy threshold hypergraph*(IFTHG) is defined as $H_G = (V, E, s_1, s_2)$ where,
(i) $V = \{v_1, v_2, \dots, v_n\}$ is a finite set of intuitionistic fuzzy vertices,
(ii) $E = \{E_1, E_2, \dots, E_m\}$ is a family of crisp subsets of V,
(iii)$E_j = \{u_i, \mu_j(v_i), \nu_j(v_i) \big| 0 \le \mu_j(v_i) + \nu_j(v_i) \le 1\}, j = 1,2,\dots,m,$
(iv) $E_j \neq \emptyset, j = 1,2,\dots,m,$
(v) $\bigcup_j supp(E_j) = V, j = 1,2,\dots,m,$
(vi) An independent set $U \subseteq V$ has a set of all distinct combinations of a non-adjacent vertices in H_G iff there exists a threshold values $s_1, s_2 > 0$ such that $\sum_{v_i \in U} \mu_j(v_i) \le s_1, \sum_{v_i \in U} 1 - \nu_j(v_i) \le s_2$.

Example:

Consider an IFTHG ($\boldsymbol{H_G}$) such that $\boldsymbol{V = \{v_1, v_2, v_3, v_4, v_5\}}$, $\boldsymbol{E = \{E_1, E_2, E_3, E_4, E_5\}}$, where $\boldsymbol{E_1 = \{v_1, v_2\}, E_2 = \{v_4, v_5\}, E_3 = \{v_3, v_4\}, E_4 = \{v_3, v_5\}, E_5 = \{v_2, v_3\}, E_6 = \{v_2, v_5\}}$ with threshold value <0.4,0.5>.

The IFTHG $\boldsymbol{H_G = (V, E, 0.4, 0.5)}$ has been given below.

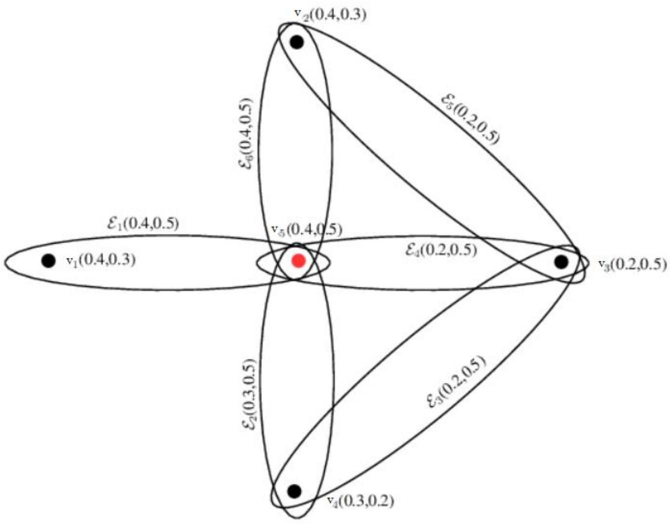

Figure 9: Intuitionistic Fuzzy Threshold Graph

Incidence matrix of IFTHG is given below

$$\begin{bmatrix} <\mathbf{0.4,0.3}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> \\ <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0.4,0.3}> & <\mathbf{0.4,0.3}> \\ <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0.2,0.5}> & <\mathbf{0.2,0.5}> & <\mathbf{0.2,0.5}> & <\mathbf{0,1}> \\ <\mathbf{0,1}> & <\mathbf{0.3,0.2}> & <\mathbf{0.3,0.2}> & <\mathbf{0,1}> & <\mathbf{0,1}> & <\mathbf{0,1}> \\ <\mathbf{0.4,0.5}> & <\mathbf{0.4,0.5}> & <\mathbf{0.4,0.5}> & <\mathbf{0.4,0.5}> & <\mathbf{0.4,0.5}> & <\mathbf{0.4,0.5}> \end{bmatrix}$$

Here are some key points highlighting their significance:

Intuitionistic fuzzy sets, used in IFTHGs, allow for a more complicated representation of uncertainty. Unlike traditional fuzzy sets that only consider the degree of membership, intuitionistic fuzzy sets consider both the degree of membership and the degree of non-membership. This additional degree of freedom enables a more accurate modeling of uncertainty, which is crucial in decision-making processes.

- IFTHGs provide a flexible framework for representing and analyzing complex systems. The use of hypergraphs allows for the representation of multiple relationships among elements, which may not be adequately captured by other graph-based models. The threshold concept in IFTHGs enables the modeling of different levels of significance or relevance for hyperedges, allowing for a more fine-grained representation of relationships.

- Intuitionistic fuzzy threshold hypergraphs have been widely used in decision-making processes, especially when dealing with uncertain and imprecise information. By incorporating intuitionistic fuzzy sets, IFTHGs can handle incomplete or uncertain data, making them suitable for decision-making problems with vague or imprecise criteria. The threshold mechanism provides a way to control the influence of hyperedges based on their significance, allowing decision-makers to prioritize and weigh different factors accordingly.

- IFTHGs offer a powerful framework for fusing and aggregating information from multiple sources. The use of intuitionistic fuzzy sets allows for the combination of different sources of information with varying degrees of uncertainty, thus providing a more comprehensive and reliable result. The threshold mechanism can be used to determine the level of significance for each information source, enabling a more effective fusion and aggregation process.

- Intuitionistic fuzzy threshold hypergraphs have found applications in various domains, including engineering, medicine, finance and decision analysis. They have been applied to problems such as pattern recognition, expert systems, risk assessment, multi-criteria decision-making and optimization. The ability of IFTHGs to handle uncertainty and ambiguity makes them particularly useful in real-world situations where precise and deterministic models may not be appropriate.

In summary, the importance of intuitionistic fuzzy threshold hypergraphs lies in their ability to handle uncertainty, provide flexible representation, support decision-making processes, facilitate information fusion and find applications in various domains. These characteristics make IFTHGs a valuable tool for modeling and analyzing complex systems and dealing with uncertain and imprecise information.

CONCLUSION

The paper emphasizes the importance of IFTHGs in dealing with uncertainty in a variety of fields. Because IFTHGs can integrate intuitionistic fuzzy sets, they can model uncertainty more accurately, which makes them beneficial to information representation and making choices. A flexible foundation for expressing complicated systems is provided by IFTHGs. The threshold concept allows for the modeling of different levels of significance or relevance for hyperedges, enabling a more fine-grained representation of relationships. This flexibility makes IFTHGs suitable for applications that require a complicated understanding of interconnections and dependencies.

REFERENCES

[1] Alain Bretto, "Hypergraph Theory: An Introduction", Springer International Publishing, Switzerland, 2013.
[2] J. A. Bondy, U. S. R. Murty , "Graph theory with applications", Elsevier Science Publishing Co., Newyork, 1976.
[3] S. A. Choudum, "A first course in graph theory", Laxmi Publications, Macmillan India, 2000.
[4] Chvatal Vaclav, Hammer and L Peter, "Aggregation of inequalities in integer programming", Annals of Discrete Mathematics, vol. 1, Amsterdam: North-Holland, pp. 145–162, 1977.
[5] Claude Berge, "Graphs and Hypergraphs", North-Holland Publishing Company, North-Holland, 1973.
[6] Claude Berge, "Hypergraphs: Combinatorics of finite sets", Elsevier Science Publishing Company, New York, 1989.
[7] John N. Mordeson, Premchand S. Nair, "Fuzzy Graphs and Fuzzy Hypergraphs", Physica-Verlag, 2012.
[8] Krassimir T. Atanassov, "On intuitionistic fuzzy set theory", Springer-Verlag Berlin Heidelberg, 2012.
[9] Krassimir T. Atanassov, "On intuitionistic fuzzy sets: Theory and Applications", Springer-Verlag Berlin Heidelberg, 1999.
[10] Lanzhen Yang, Hua Mao, "Intuitionistic fuzzy threshold graphs, Journal of Intelligent & Fuzzy Systems", vol. 36, pp.6641–6651, 2019.
[11] Leonhard Euler, "Solution of a Problem in the Geometry of Position", The Journal Commentaries of the St. Petersburg Academy of Sciences, 1763.
[12] Madhumangal Pal, Sovan Samanta and Ganesh Ghorai, "Modern Trends in Fuzzy Graph Theory", Springer Nature Singapore Pvt. Ltd., 2020.
[13] N. V. R. Mahadev, U. N. Peled, "Threshold Graphs and Related Topics", Elsevier Science, Amsterdam, The Netherlands, 1995.
[14] Mohammad A. Bahmanian and Mateja Sajna, "Connection and separation in hypergraphs", Theory and Applications of Graphs, vol. 2(2), 2015, pp.1-25.
[15] Muhammad Akram, Wieslaw A. Dudek, "Intuitionistic fuzzy hypergraphs with applications", Information Sciences, vol 218, 2013, pp. 182-193.
[16] K. K. Myithili, R. Parvathi ,M. Akram, "Certain Types of Intuitionistic Fuzzy Directed Hypergraphs", International Journal of Machine Learning and Cybernetics, Springer-Verlag, Berlin Heidelberg, vol. 7(2), 2016, pp.287-295.
[17] R. Parvathi, S. Thilagavathi and M. G. Karunambigai, "Intuitionistic Fuzzy Hypergraphs", Bulgarian Academy of Sciences, Cybernetics and Information Technologies, vol. 9(2), 2009, pp. 6641–6651.
[18] Sunil Mathew, John N. Mordeson and Davender S. Malik, "Fuzzy Graph Theory", Springer International Publishing, Switzerland 2018.
[19] Xavier Ouvrard, "Hypergraphs: An introduction and review", unpublished, 2020.
[20] L.A.Zadeh, "Fuzzy sets", Information and Control, vol. 8, 1965, pp. 338-353.