Lower Bounds on the Multiplicative S-index of Various Operations on Graph

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ABSTRACT

The mathematical modeling of chemical phenomena is done using graph theory by the field of mathematical chemistry known as the theory of chemical graphs. In chemical graph theory, a molecular graph is a simple graph where the vertices and edges stand in for individual atoms and the chemical bonds that connect them. Based on a certain topological feature of the relevant molecular graph, it is discovered that there is a strong association between the qualities of chemical compounds and their molecular structure. Topological index is a numerical description of a molecule. We introduce the Multiplicative S-index of a graph, a brand-new graph invariant. We define the lower bounds for the multiplicative S-index of the graph operations Join, Cartesian product, Composition, Strong product, Corona product, and Corona in this work.

Keywords— Zagreb, S-index, Graph operations

# INTRODUCTION

Graph theory has provided chemists with a wealth of useful tools, such as topological indices. Molecular graphs are commonly used to represent molecules and molecular compounds. A topological index is analogous to converting a chemical structure into a real number. Topological and graph invariants based on graph vertex distances are widely used for characterizing molecular graphs, establishing structural and property relationships, predicting biological activities of chemical compounds, and developing chemical applications. Topological indices are significant because they can be utilized directly as simple numerical descriptors in Quantitative Structure Property Relationships (QSPR) and Quantitative Structure Activity Relationships (QSAR) in comparison with physical, chemical, or biological characteristics of molecules. Topological indices come in a variety of forms, such as degree-based, distance-based, counting-related polynomials 2, and graph indices, among others. The practice of numerically coding chemical structures using topological indices or topological coindices has recently gained popularity in medicinal chemistry and bioinformatics [6,12,19].

Let's take a look at two simple connected graphs with disjoint vertex and edge sets, and . and stand for the number of vertices and edges for . For each vertex , the degree is given as , where is the number of edges incident on the vertex .

I. Gutman and N. Trinajstic [2] defined the first and second Zagreb indexes of a graph in 1972 as follows:

The F-index was [5] according to B. Furtula and I. Gutman in 2015:

The Y-index was first introduced in 2020 by Abdu Alameri and Noman AI-Naggar [10]. It is defined as:

The S-index was established as [20] in 2021 by S. Nagarajan, G. Kayalvizhi, and G. Priyadharsini:

R. Todeschini and D. Ballabio [8] established the first and second Multiplicative Zagreb indices of a graph in 2010.

The Multiplicative Forgotten Topological Index was introduced in 2019 by Asghar Yousefi and Ali Iranmanesh [1]. It is defined as:

In (2013) C.D. Kinkar and Y. Aysum [9] derived graph operations in Multiplicative Zagreb indices . The Multiplicative Zagreb coindices were calculated by K. Xu and K.C. Das [3] in (2013). In [1] Y. Asghar and Ali Iranmanesh (2019) derived the Multiplicative F-index of graph operations. M. Radhakrishnan and M. Suresh [15] derived the F-sum operations on graphs in multiplicative zagreb indices in [2020]. Liu J and Q. Zhang [16] defined upper bounds on multiplicative zagreb of connected graphs in [2012]. In [2015], M. Azari and A. Iranmanesh [17] presented lower bounds for the multiplicative sum zagreb index of graphs. M. Eliasi and D. Vukicevic [11] compared multiplicative indices in [2013]. In 2014, M. Azari [14] published the lower bounds on the narumi-katayama index of graphs. We investigated the lower bounds for the Multiplicative S-index of several graph operations in this article. Researchers who want to study more about graph operations might refer to [4,,7,10,13].

**Definition 1.1:** The Multiplicative S-index of a graph G is defined as the product of graph’s four degree vertices and is denoted by:

# MAIN RESULTS

We developed various graph operations in the lower bound on the multiplicative S-index in this section.

**Lemma 2.1:** [18] (AM-GM inequality)

Let be a nonnegative numbers. Then

holds with equality if and only if .

**Join:**

“The *join* of graphs and with vertex sets and and edge sets and is the graph union together with all the edges between and ”. Obviously, and.

**Theorem 2.2:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Cartesian Product:**

“The *Cartesian product* of graphs and has the vertex set and is an edge of if and , or and ”. Obviously, and .

**Theorem 2.3:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Composition:**

“The *Composition* of graphs and with disjoint vertex sets and edge sets is the graph with vertex set and is adjacent to whenever is adjacent to or and is adjacent to ”..

**Theorem 2.4:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Strong product:**

“The *Strong product* of a graphs and is a graph with vertex set and any two vertices and are adjacent if and only if or or ”. .

**Theorem 2.5:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Corona product:**

“The *Corona product*  of graphs and with disjoint vertex sets and and edge sets and is the graph derived by one copy of and copies of and joining the vertex of to each vertex in copy of ”. Obviously,.

**Theorem 2.6:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

**Corona join product**:

“Let and be simple connected graphs, and the Corona join graph of and is obtained by taking one copy of , copies of , and joining each vertex of the copy of with all vertices of ”. The *Corona join product* of and is denoted by

**Theorem 2.7:** The Multiplicative S-index of satisfies the inequality,

**Proof:** Utilizing the definition 1, we’ve

We now have, according to lemma 2.1,

We receive the complete result.

# Conclusion

In numerous disciplines, topological indices are defined and employed to study the characteristics of diverse things, such as atoms and molecules. Numerous topological indices have been defined and researched by mathematicians and chemists. In this paper, we looked at the lower bound of the Multiplicative S-index for a number of graph operations, including join, cartesian product, composition, strong product, corona product, and corona join product.

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