Triaxial Star

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Abstract: The idea of a "Triaxial Star" first proposed by Chandrasekhar in 1969. More than 50 years passed, the detection of triaxial star remains unreachable. Detection of gravitational waves would be a probe to the astronomers to investigate the properties of the compact objects in a new direction in the light of gravitational waves. Recent discovery of cosmic baby , i.e., swift J1818.0-1607 with age ~ 300 years, offers the astronomers an opportunity , through continuous observations, to increase our knowledge about the physics of the evolution of a magnetar from its new born phase to end stage, generation of ultra-strong magnetic field in its interior, source of continuous gravitational waves, physics of coupling between magnetic field decay and cooling for keeping the internal temperature ~ 10^8 K for a period of ~ 10^4 years, , etc. Cosmic Baby most probably the first detected Triaxial star.

Key Words: Ellipsoid, Triaxial Star, Gravitational Waves, Neutron Star, Magnetar

Content

1. Introduction: Compact Star and Professor Chandrasekhar

A compact star is a system inside which a struggle is continuing between gravity and degenerate pressure created by its constituents. Gravity tries to crash the star's material by pulling towards the center of star, while degenerate pressure, created among the material particles due compactness, tries to counteract and overcome the gravity pull. In his book "An Introduction to the study of Stellar structure (Dover, 1967)" Professor Chandrasekhar first gave a detailed picture of "what is a star, what is going on inside it, etc" in the form of mathematical language.

Before the discovery of neutrons in 1932, electrons were the latest discovered particles on that time. Professor Chandrasekhar thus considered the latest star, i.e., white dwarf. He calculated the maximum mass of a white dwarf full of electrons = 1.4 $M_\odot$ ($M_\odot$ being the solar mass) which is known as Chandrasekhar limit. In 1932 Chadwick discovered neutrons. Then T. Gold proposed the possible existence of a new compact star beyond white dwarf, called Neutron Star which is made full of neutrons.

In the year 1969 Professor Chandrasekhar theoretically proposed another new compact star, called "Triaxial Star". Note that, in 1967 Jocelyn Bell, a Ph.D. scholar of Cambridge University, UK, detected a peculiar type signal but its rhythm was very accurate. This star later identified as neutron star. However, the triaxial star was confined in theoretical works only although neutron stars (isolated , rotating neutron stars or pulsars, and magnetars) became a gold mine to the astronomers and scientists. In searching the gravitational wave sources, isolated neutron stars, millisecond pulsars became the efficient gravitational wave sources. Many theoretical studies suggested that the triaxial neutron stars, i.e. triaxially deformed neutron stars, are the most significant gravitational wave sources . So, triaxially deformation in star thus gave an impetus to the astronomers, scientists to search for a triaxial star.

2. What is Triaxialityin a Star

More than 50 years passed after the proposed concept of triaxial star but till date it is remained undetectable. Several theoretical works suggested that star with fast rotation can be deformed and would become a triaxial star .Classically, the figures of equilibrium of uniformly, rotating homogeneous masses pertain various sequences of ellipsoidal figures. The Maclaurin sequence is a sequence of oblate *spheroid*s along which the eccentricity ("e") of the meridional sections *increases from zero to one* (Chandrasekhar and Lebovitz 1964). Initially it was thought that the sequence of the square of the angular velocity of rotation (Ω^2) is not a parameter of unrestricted range. This means that for each value of Ω^2, (except less than a certain determinate maximum, there are two permissible spheroidal figures of equilibrium. But Jacobi first showed that a sequence of genuine triaxial ellipsoids of equilibrium diverges from the Maclaurin sequence. This means that there is a bifurcation point which clearly distinguish the permissible sequences of figures of equilibrium from Maclaurin spheroidal sequence to Jacobi ellipsoidal sequence.

Let us consider a situation of general ellipsoid (also called a triaxial ellipsoid, see fig.1) whose quadratic surface can be expressed in Cartesian co-ordinates by

$$(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1 \qquad (1)$$

where the semi – axes are of lengths a, b, c.

Now,

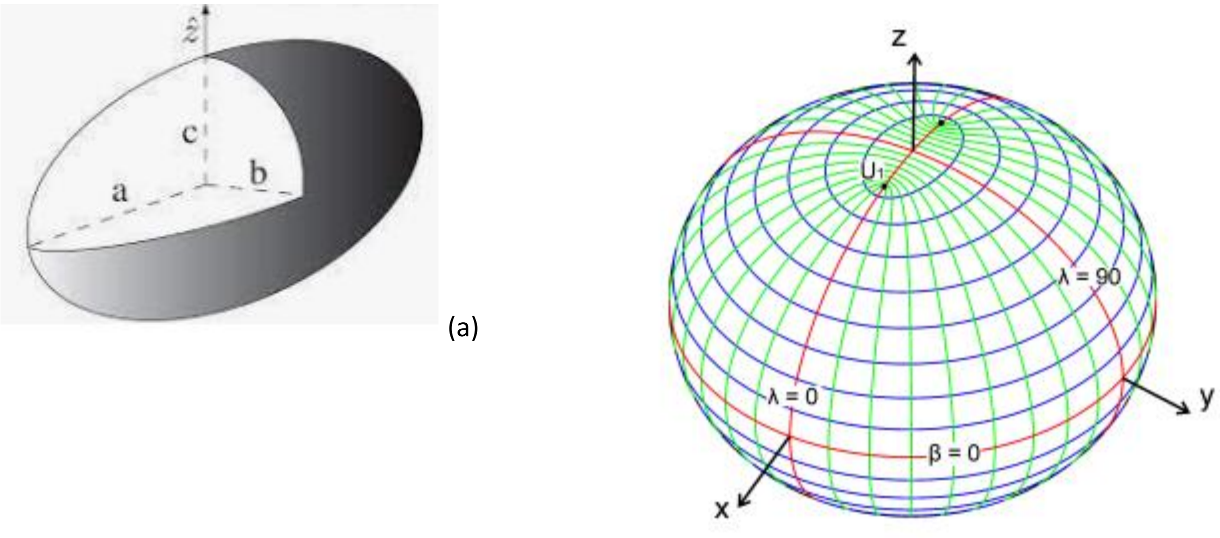
i) If all the three are same, i.e., $a = b = c$ then it is a sphere.

ii) If the length of two axes of an ellipsoid are the same, then the figure is called a spheroid , i.e., an oblate spheroid or a prolate spheroid depending on whether $c < a$ or $c > a$, respectively.

In spherical coordinate system (r, θ, ϕ) the eqn. (1) becomes

$$\frac{r^2\cos^2\theta\sin^2\phi}{a^2} + \frac{r^2\sin^2\theta\sin^2\phi}{b^2} + \frac{r^2\cos^2\phi}{c^2} = 1. \qquad (2)$$

<u>Geodesic form</u>



(a)

(b)

Fig. 1 : (a) Ellipsoid in Cartesian Coordinates (b) Ellipsoid in Geodesic Coordinates (Panou 2013)

In this case we consider a triaxial ellipsoid where Cartesian coordinates are (x, y, z) but three semi-axes are $a = a_x$, $b = a_y$, and $c = b$ then eqn.(1) takes the form

$$x^2 / a_x^2 + y^2 / a_y^2 + z^2 / b^2 = 1 , \quad 0 < b < a_y < a_x \qquad (3)$$

The linear eccentricities can be calculated as

$$h_x = \sqrt{a_x^2 - b^2}, \quad h_y = \sqrt{a_y^2 - b^2}, \quad h_e = \sqrt{a_x^2 - a_y^2}. \qquad (4)$$

and $h_e^2 = h_x^2 - h_y^2$.

This triaxial ellipsoid can be parameterized as

$$x = a_x \left(\cos^2 \beta + \frac{h_e^2}{h_x^2} \sin^2 \beta \right)^{1/2} \cos \lambda$$

$$y = a_y \cos \beta \sin \lambda$$

$$z = b \sin \beta \left(1 - \frac{h_e^2}{h_x^2} \cos^2 \lambda \right)^{1/2} \qquad (5)$$

where $-\pi/2 \leqslant \beta \leqslant +\pi/2$ and $-\pi < \lambda \leqslant +\pi.$ Ellipsoidal latitude and ellipsoidal longitude can therefore be interpreted with the help of this parameter. Consequently, the initial fundamental coefficients in this parameterization, E, F, and G, can be written as (Panou 2013)

$$E = \mathrm{B} \left(h_y^2 \cos^2 \beta + h_e^2 \sin^2 \lambda \right)$$

$$F = 0$$

$$G = \Lambda \left(h_y^2 \cos^2 \beta + h_e^2 \sin^2 \lambda \right) \qquad (6)$$

where

$$\mathrm{B} = \frac{a_y^2 \sin^2 \beta + b^2 \cos^2 \beta}{h_x^2 - h_y^2 \sin^2 \beta}$$

$$\Lambda = \frac{a_x^2 \sin^2 \lambda + a_y^2 \cos^2 \lambda}{h_x^2 - h_e^2 \cos^2 \lambda}. \qquad (7)$$

This implies that

i) $F = 0$ indicates that the β-curves and λ-curves are orthogonal.
ii) $\mathrm{B} \neq 0$ and $\Lambda \neq 0$ for all points and $E = G = 0$ means umbrella shape.
iii) For an orthogonal parameterization the line element “ds” on the triaxial ellipsoid is

$$ds^2 = E \, d\beta^2 + G \, d\lambda^2 \qquad (8)$$

3. General Properties of Equilibrium Figure

A relative equilibrium figures concern, in general, the liquid figures such that an equatorial symmetry plane exists. It has a similar situation in the case galaxies. This means that for liquid and stellar figures the internal flows are absent. Naturally, it is believed that for all figures of

equilibrium figures a much stranger inequality is present, namely, angular velocity $\Omega^2 / (\pi G \rho) \leq 0.45$ satisfying the validity of Maclaurin spheroid.

Now, we can consider rotation in equilibrium figure. In that case, the liquid figures of relative equilibrium can rotate only around the least axis. In triaxial liquid figure, i.e., with internal flow, can rotate around the middle axis and possess an oblique rotation. In stellar system this equilibrium figures can rotate around any of the three axes turn into so called “interia ellipsoid” or “inclined rotation”. Although there is no direct relation between an angular velocity and flattening for equilibrium figures of relative equilibrium but direction of internal flows w.r.t. rotation of the figure plays an important role in astrophysics for modeling of stars and galaxies where consideration of complex interrelation (i.e., anisotropy) is essential.

As all the gravitating equilibrium figures are of negative heat capacity, an interpretation of bifurcation point, creates an interest for astrophysical degenerate systems or compact objects. The ratio of rotational kinetic energy (T) and gravitational binding / potential energy (W) , i.e., T/ |W| therefore indicates whether the system with the angular momentum distribution for axisymmetric (i.e., for Maclaurin spheroid) or non-axisymmetric (Jacobian ellipsoid). We know that a stable star can be deformed if it rotates faster. Rapid rotation in a star (i.e, for self-gravitating, incompressible fluids in equilibrium) differentiates its new figure from the earlier one i.e.,

(i) Figure of axisymmetric Macllaurin spheroid (earlier) ;
(ii) Figure of non-axisymmetric, i.e., Triaxial Jacobi ellipsoids (new figure)

As soon as the T/|W| ratio approaches approximately 0.14 (Bonazzola et al 1998), we can argue that this new configuration is a spinning compact star (or simply a "Triaxial Star") that is triaxially deformed, as opposed to an exact ellipsoid in relativistic gravity or for compressible fluids. I would like to know why triaxial figures are significant. The straight forward explanation is as follows: To model the relativistic neutron star as an axisymmetric uniformly rotating structure associated with the equation of state (EoS) of high density nuclear matter, relativistic astrophysics can allow for fluid compressibility through triaxiality.

3.1 Triaxiality and Gravitational Wave

Let us examine a reasonably stiff (piecewise) polytropic equation of state with a triaxial configuration that has tri-planar symmetry with respect to three orthogonal X, Y, and Z planes. The scenario that could arise in this instance is that the masses derived from the supra-massive triaxial solution would always be smaller than those of axisymmetric equilibrium, even if they would surpass the maximum mass of the spherical solution. In this context, two crucial facts are as follows:

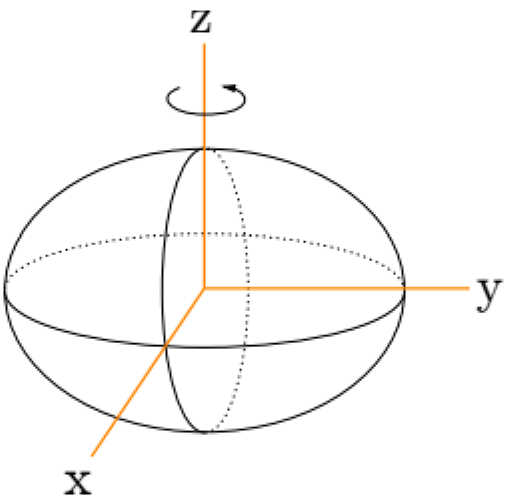


Fig. 3: : Diagram of a rotating triaxial ellipsoid, with its major axes aligned to a rectangular coordinate system, spinning about the z axis (Neunzert 2019)

(i) The equation of state (EoS) determines the variation in the maximum masses of the axisymmetric and triaxial star equilibrium solutions;

(ii) There will be strong evidence of significantly softer EoS of high density materials in the neutron star core if this difference turns out to be merely 10%.

Another important situation we have to consider for a newly born neutron star which is supernova fall back accretion and condition for gravitational collapse. We now consider a situation. We are aware that a neutron star is created when a supernova bursts with a magnetic field as strong as $B \leq 5 \times 10^{14}$ G. It rotates at a speed of $T/|W| \sim 0.14$ for 50–200 seconds before collapsing into a black hole. All that exists in this hypothetical scenario is a compact star that is triaxially distorted and may have generated triaxially via enormous stellar core collapse. If this occurs, that is, once a triaxial star of that kind forms, a huge number of gravitational waves would be released. This offers us the chance to extract attributes from high density nuclear materials located inside compact stars, such as neutron stars. For instance, the following relation can be used to estimate the typical values of the amplitude (h) of gravitational waves emitted from a triaxial star:

$$h \sim 9.1 \times 10^{-21} \, (30 \text{ Mpc} / D) \, (M / 1.4 M_o)^{3/4} \, (R / 10\text{Km})^{1/4} f^{-1/5} \quad (9)$$

where D, M, R, and f stand for the wave frequency in Hz, the mean radius, the source mass, and the distance to the source, respectively. The detectability of gravity waves emitted by triaxially compact stars was calculated by Piro and Thrane (2012) to be approximately 17 Mpc for the Advanced LIGO detector under the fallback accretion scenario (Harry 2010).

Deformation of a star due strong magnetic fields

Theoretical models of neutron stars, in general assumed that they are made entirely of neutrons. In general relativity, the equilibrium configuration of neutron star (i.e. by solving Einstein-Maxwell equations) is coupled with both rotation and magnetic field i.e., consideration of magnetic field is essential. This means in a spherical symmetric back ground perturbation due to internal magnetic field in the shape of neutron star surface structure would appear. Therefore, a neutron star or magnetar (the special type of neutron star with internal strong magnetic field) can significantly be deformed by both rotation and strong magnetic field. Theoretical calculation (Bonanno et al 2003) hints that at the end of the collapse of the core of a massive star, differential rotation could generate a strong toroidal magnetic fields $\sim 10^{16} - 10^{17}$ G inside the proto-neutron star. Of course, the relativistic models of magnetized stars (i.e., neutron stars, magnetars) require the simultaneous of both poloidal and toroidal field components. The combined effects of magnetic field and rotation could make:

(a) An apparent morphological alteration, or surface deformation, that is pertinent to the electromagnetic emission

(b) Regarding the quadruple distortion, or internal matter distribution, which is pertinent to the gravitational wave emission.

4. The Ellipticity of the Star

The stellar deformation can be expressed in terms of stellar ellipticity or simply “ellipticity”(ϵ). There are two definitions relevant for two conceptually different quantities — i) the surface ellipticity($\epsilon_{surface}$)and ii) the quadruple elliptcity (ϵ_{qud}).

(A) The Surface Ellipticity

The surface ellipticity can be defined as (Chandrasekhar and Miller 1974)

$$\epsilon_{surface} = \underline{(\text{ Equatorial radius } R_e \text{ - (Polar radius } R_p \text{)}} \text{ / (Polar radius } R_p \text{)} \qquad (10)$$

In the case of neutron star, to know the degree of deformation we can write

$$\epsilon = \sqrt{\{ 1 - (R_p / R_e)^2 \}} \qquad (11)$$

The equ.(1) describes the geometrical shape of the star. This means the surface ellipticity describes the external appearance of the star.

In case of neutron star, the above equ. (2) gives us a relationship between the mass and radius of the neutron star. The known facts are :

(a) If the polar radius R_p of the neutron star increases, then it's mass decreases.
(b) If the equatorial radius R_e of the neutron star increases, the mass increases.
(c) The maximum deformation appears at the ellipticity $\epsilon \sim 0.257$. In this case $R_e > R_p$ such that the neutron stars are maximally in oblate form.

(B) The Quadrupole Ellipticity

It is a measure of the mass quadruple of the star (Bonazzola and Gourgouljan 1996, Haskell et al 2007) which can be expressed as

$$\epsilon_{Quad} = - Q / I \qquad (12)$$

where I = the mean value of the moment of inertia of the star, and Q = its mass – energy quadruple moment.

This quadruple ellipticity can also be written in terms of inertia components:

$$\epsilon_{Quad} = (I_{zz} - I_{yy}) / I_{zz} \qquad (13)$$

In general, the quadruple ellipticity is a measure of the entire stellar bulk deformation. It is used to evaluate the gravitational wave emission of a rotating neutron star.

5. Magnetar and Triaxiality

A magnetar is a type of neutron star characterized by an exceptionally powerful magnetic field. These magnetic fields reach strengths of approximately 10^{15} Gauss, which is a magnitude of a thousand trillion times greater than Earth's magnetic field and between 100 to 1,000 times stronger than the magnetic field of a typical radio pulsar. This extreme magnetic intensity renders magnetars the most magnetically potent objects currently identified.

Magnetars share similarities with other neutron stars, including a diameter of approximately 20 kilometers and a mass of around 1.4 times that of the Sun (1.4 $M_\odot$). They originate from the gravitational collapse of a star with a mass within the range of 10 to 25 times that of the Sun (10–25 $M_\odot$). What sets magnetars apart from other neutron stars is their significantly more potent magnetic fields and relatively slower rotation. While most observed magnetars have rotation periods ranging from 2 to 12 seconds (Kaspi 2010), typical neutron stars, like those observed as radio pulsars, typically rotate at rates of one to ten times per second. (Condon and Ransom 2021). Extremely powerful and distinctive bursts of gamma and X-ray radiation are produced by the significant characteristics of a magnetar's magnetic field. In contrast to other celestial bodies, a magnetar's active life is brief. After roughly 10,000 years, their powerful magnetic fields begin to weaken, which is when the activity and intense X-ray emission stop.

The formation of magnetars is thought to follow the same process as that of all neutron stars, originating from the core-collapse of a massive star during a supernova explosion. However, the precise conditions leading to the creation of a magnetar rather than an ordinary neutron star or pulsar remain somewhat unclear. Some theories propose that, to attain such intense magnetic fields, the neutron star must initially rotate at rates ranging from 100 to 1,000 times per second.

The prevailing explanation for the powerful magnetic fields of magnetars is that they arise from a magneto-hydrodynamic dynamo process occurring within the turbulent and incredibly dense conducting fluid present before the neutron star stabilizes into its equilibrium configuration. (Thompson and Duncan 1993). These magnetic fields persist due to ongoing currents in a proton-superconductor phase of matter at a mid-depth in the neutron star, where mass is primarily composed of neutrons. When two neutron stars combine, a similar magnetohydrodynamic dynamo mechanism creates even stronger transient fields. (Price and Rosswog 2006). However, an alternate theory posits that they originate from the collapse of stars with unusually robust magnetic fields (Zhou et al 2019).

Note that since then, the magnetar model—which is predicated on the degradation of the magnetic field driving the emission of X-rays and gamma rays—has been able to satisfactorily explain both SGRs and anomalous X-ray pulsars. The spin-down rate of an SGR was measured from observed pulsations, and the results indicated that the object was a neutron star with a magnetic field strength of 8 × 10^{14}G. However, magnetars appear to be X-ray intense for a limited duration, as their pulse periods are grouped between 6 and 12 seconds. In addition, if magnetars remained active for a long time, we should also notice them with pulse periods of tens of seconds or longer.

6. Recently detected Magnetar, the Cosmic Baby

The Swift Burst Alert Telescope (BAT) on board the Neil Gehrels Swift Observatory (Ghurels et al., 2004) discovered a magnetar brief burst with typical characteristics on March 12, 2020, at 21:16:49 UT (Evans et al., 2020). Following observation for 64 seconds, the Swift X-ray Telescope (XRT) ultimately discovered a new, unidentified x-ray source that is currently known as Cosmic Baby. This source is named Swift 1818.0 – 1607. Table I displays the key parameters of this cosmic infant, which were derived from the timing analysis of preliminary observations at the moment of detection.

Table I : Various early observed / measured parameters of Swift J1818.0 – 1607 (Parui 2023,2023a)

Typical properties	Value	Reference
Characteristic age (shortest known)	~ 240 years	Esposito et al (2020)
Surface magnetic field	~ 2.7 x 10^{14} G	

Dipolar magnetic field at poles	$\approx 7 \times 10^{14}$ G	
Spin down Luminosity ($\dot{E}_{rot}$)	~ 1.4 x 10^{36} erg.s^{-1}	
Luminosity	~ 8 x 10^{34} erg.s^{-1}	
Coherent periodicity of x-ray signal	1.36 s	Enoto et al (2020)
Spin period derivative	~ 8.2 x 10^{-11} s.s^{-1}	Champion et al (2020)
Period derivative	~ 9 x 10^{-11} s.s^{-1}	
Spin Period	0.7333920 s	

Upon doing several observations at multiple wavelengths using different telescopes (such TMRT and NICER) till July 27, 2020, certain verified parameters of the cosmic infant magnetars become accessible. A few examples of these include the spin period derivative (~3.74 x 10^{-11} s^{-2}), spin down luminosity (~1.1 x 10^{36} erg. s^{-1}), and surface dipole magnetic field (~ 3 x 10^{14} G). Astronomers were able to obtain the first high resolution x-ray picture of the cosmic baby because to Chandra Observatory's (Blumer and Safi-Harb 2020) studies of J1818.0-1607, which started less than a month after the cosmic baby was discovered. Even still, there is a great deal of diversity in practically every cosmic infant property metric.

7. Ellipticity of Cosmic Baby Magnetar Swift J1818.0-1607

The idea of a magnetar emerged to explain the characteristics of transitory gamma-ray sources, or Soft Gamma Repeaters (SGRs). The aforementioned fact cannot be explained by rotational power, or energy. Therefore, the ultra-strong magnetic field could serve as a backup supply of energy for these very high energy sources. The basic idea regarding these peculiar high energy sources could be the ultra-strong magnetized neutron stars, magnetars with surface (dipole) fields in the range $10^{14} – 10^{16}$ G and internal magnetic fields >~10^{16} G (at least one order of magnitude stronger) (Paczynski 1992, Thompson and Duncan 1995). Massive flares and rapid radio bursts are just two examples of the high energy emissions produced by magnetars, which are isolated young neutron stars with strong magnetic fields. The compact object becomes anisotropic when the density of the neutron star core surpasses approximately 10^{15} g.cm^{-3} (Ruderman, 1972). This means that the internal pressure can be divided into two components: the transverse pressure (p_t), which is the orthogonal to the radial pressure (p_r), and the internal pressure (p_r). This pressure anisotropy affects the structure, stability, and physical properties of the star matter (Dev and Gleiser 2002). The extremely strong internal magnetic field of the stellar mass can also cause anisotropic pressure, which is the deformation of the spheroidal shape of a revolving neutron star and the emission resultant from such distortion.

The intrinsic field structure of a neutron star can function as a powerful and efficient source of gravitational wave emission, as initially noted by Cutler (2002). He claimed that a neutron star with a strong internal toroidal field—a neutron star that is magnetically distorted—becomes prolate and has the highest potential to emit strong gravitational waves. Therefore, a key component of the powerful gravitational wave source is the ellipticity of the magnetic deformation (σ) in the form of a star object. Specifically, the stellar body's triaxiality—its reliance on the strength of the internal strong magnetic field, the field's decay, and the dynamic shape shift to a triaxial ellipsoid form.

According to Lai and Shapiro (1991), a magnetar is an isolated neutron star that rotates slowly and has an extremely high internal magnetic field that can reach 10^{16}–10^{18} G and even up to 10^{20} G. A magnetar is a triaxial stellar entity in general, especially in its newborn period (Melatos 1999). One can estimate the internal ultra-strong magnetic field of a neutron star (or magnetar) based on the geometric distortion caused by a strong toroidal magnetic field. This is defined as the toroidal magnetic field being greater than or equal to at least one order of the surface dipolar magnetic field. Therefore, constraints on the ellipticities of magnetars can be used to constrain the toroidal magnetic field as a period or duration of GW emission. Therefore, if the field is dipolar, hydro-magnetic stresses arising from non-radial gradients of the super-strong internal magnetic field deform the magnetar, which is the area between the magnetic poles and the equator. The following formula can be used to express the fractional difference (ε) between the major moments of intertia: (Melatos 1999; Goldreich 1970; deCampli 1980)

$$\varepsilon \sim \delta p\, R^5 / I_1 \approx 2 \times 10^{-9} (B_{int} / 10^{10}\ \mathrm{T})^2 \qquad (14)$$

where δp = induced matter-density perturbation

$\sim B_{int}^2 / \mu_o C_s^2$,

R = the stellar radius,

C_s = the isothermal sound speed (= $3^{-1/2}$ c, “ c “ being the velocity of light),

B_{int} = the characteristic magnitude of the internal magnetic field such that

i) $\approx B_o$, if the internal magnetic field is confined to the stellar crust,

ii) $>\sim B_o$, if it is generated deep inside the star (i.e. convective dynamo model (Thopson and Duncan 1993),

iii) $B_{int} <\sim 10^9$ T in the case of rotation powered pulsars

The significant characteristics of the triaxial star are (Melatos 1999) :

a) Compared to the elastic deformation resulting from shear forces in the crystalline stellar crust of a revolving neutron star, the hydrodynamic deformation in a magnetar is substantially greater.;

b) In a revolving neutron star, the principal axes of inertia are aligned arbitrary with respect to the magnetic axis of the external magnetic dipole field.

Notably, the magnetic axis in the magnetar situation is approximately parallel to one of the primary axes of inertia (let's say e3). Put another way, the magnetic axis alignment is not exactly aligned with respect to the axis of inertia due to the complex internal field structure around the magnetar's generation site.

8. Origin and decay of core magnetic field of Swift J1818.0-1607

Imagining the magnetar as a neutron star with a small ellipticity and a uniform, slightly distorted ellipsoid shape

$\epsilon = (I_1 - I_2) / I_3$ (15)

The following constraint on the magnetarellipticity is obtained using equation (10) where I1, I2, and I3 are the principal moments of inertia of the neutron star, with the assumption that I3 is aligned with the spin axis (Moriya and Tauris 2016).

$| \epsilon | < (5 / 3G)^{1/2} \{ C R^3 P_o B_{dipole} / 2^4 \pi I \}$ (16)

Equation (16) becomes, using the conventional neutron star parameters specified by Cutler and Jones (2001), I = 1045 g.cm2, R = 10 km, Po = initial spin period, and the angle between the spin axis and the primary axis of the neutron star distortion = $\pi/2$.

$| \epsilon | \cong 3 \times 10^{-4} (B_{dipole} / 10^{14} G) (P_o / 1ms)$ (17)

We may now constrain the average value of this component by using the relation, assuming that the primary source of neutron star deformation is the magnetar's internal toroidal magnetic field component (Btoroidal). Carpenter (2002)

$| \epsilon | \sim 1.6 \times 10^{-4} (B_{toroidal} / 10^{16} G)^2$(18),

and $B_{toroidal}$ as (Moriya 2016)

$B_{toroidal} < \sim 1.4 \times 10^{16} G (B_{dipole} / 10^{14} G)^{1/2} (P_o / 1 ms)^{1/2}$ (19)

Using the available parameters, such as observed dipolar magnetic field strength $B_{dipole} = 7 \times 10^{14}$ G , spin period P_o = 1.36s of the Swift J1818.0-1607 we can estimate the ultra-strong internal toroidal field strength $B_{toroidal} <\sim 10^{18}$ G. This value is consistent with the value $10^{17} - 10^{18}$ G in the case of newly born proto-neutron stars (Del Zanna et al 2018; Ciolfi et al 2019; Franceschetti and Del Zanna2020) and also supports the model proposed by Dall"Osso et al (2012) that the internal magnetic field must be a very large initial value ($>\sim 10^{16}$ G) for the internal magnetic field decay.

The decay of Core Magnetic field

Theoretical research on the decay of magnetic fields in neutron star cores (Dall'Osso et al., 2012; Thompson and Duncan, 1996) suggests that the evolution and dissipation of magnetic fields in magnetar interiors are influenced by three distinct processes that may be involved: ohmic dissipation, ambipolar diffusion, and Hall drift. Specifically, ambipolar diffusion and ohmic dissipation contribute directly to dissipation, whereas Hall drift contributes indirectly. Additional research (Goldreich and Reisenegger 1992; Pons and Geppert 2007) also shows that following the Hall drift, the conservation of total energy essentially stays the same; that is, Hall diffusion will cause a new equilibrium configuration with a lower total energy to appear in the magnetar interior. Their experimental results, however, indicate that the initial stable magneto-hydrodynamic configurations remain quite close to the new equilibrium configuration. In terms of the early phase evolution of magnetars, which are those with ages significantly less than $\sim 10^4$ years, the primary mode of internal field degradation is predicted to be even the ambipolar diffusion in the neutron star core.

Yet, it is proposed that ambipolar diffusion at high temperature in the core of a neutron star, or magnetar, plays a major impact (Goldreich and Reisenegger 1992; Pons and Geppert 2007). The temperature of the magnetar core material will be greater than 10^9 K since the core magnetic field of a neutron star, or magnetar, is less than 10^{18} G, or more than 10^{16} G. The field degradation is not frozen in this instance. This suggests that in the high-temperature region, an equilibrium state between heating and cooling may emerge.Therefore, magnetar core fields larger than that would have the capability to

i) release sufficient energy in addition to
ii) in order to counteract neutrino cooling during the early phase, when the effective solenoidal and irrotational modes are still degenerate..

As long as the temperature is high enough, it can be said that the field decay is negligible when the time scale for the decay occurs on the same time scale in both modes. (e.g., $> T_9$). The reason ambipolar diffusion matters is that

(a) It soon becomes active following the magnetar's creation, and
(b) It can stop the magnetar core from dropping below a certain temperature. $\sim 10^9$ K for a period of thousands yrs (at least 10^3yrs) (Zhou et al 2018).

(c) Coupling of heating and cooling — the decay of an internal magnetic field $>\sim 10^{16}$ G couples with the magnetar cooling at the early stage.

9. Ellipticity and Triaxiality of Swift J1818.0-1607

Rotational vs Magnetic Energies

It is believed that a newly born rotating compact star can also achieve higher value of T / |W| when it is born from core collapse supernova. In that case, for a triaxial neutron star, the ratio T / |W| is essentially constant along with the triaxial sequence for higher compactness (Zhou et al 2018).

Thirty magnetars have been found to yet, not including extremely fast J1818.0-1607. The 30 magnetars have spin and rotational periods ranging from 2 to 10 s. Based on the period derivatives, the surface dipolar fields are determined to be between 10^{13} and 10^{15} G (Kouveliotou et al 1999). Yet research (Jawor and Tauris, 2022) demonstrated that the magnetar's initial period had to be smaller than two seconds. Magnetars are young, with the majority of them having distinctive spin down ages of less than 104 years, according to analysis of observed data (White et al., 2022). They cannot be powered by spin-down energy losses since they are sluggish rotators. Here**, a significant finding is that the rotational effect is negligible for the magnetar** (Kiuchi et al 2011). As a result, magnetic induced deformation is an alternate for obtaining the triaxial sequence.

Their magnetic energy dissipating and rearranging is thought to be an alternate source. The deformation in shape and triaxial value of the magnetar are determined by the magnetar's internal structure, specifically, the cooling process and the equation of state (EoS) in the presence of strong magnetic field, high density, and strong gravity all at the same time. (Yakovlev et al 2005; Chamel and Haensel 2005).

Numerical simulations(Lindblom et al 1998; Doneva et al 2015) demonstrated how a young magnetar will spin quickly and have a large magnetic field, both of which cause star deformation. A newborn magnetar will spin down as a result of both gravitational wave quadrupole radiation and a magnetic dipole torque, which is why a magnetar can release detectable gravitational waves. Electro-magnetic radiation is also produced by the magnetar's spin-down evolution. Stated otherwise, the relationship between the magnetar spin down dynamic evolution and the theoretical breaking index (n) is such that

a) $n = 3$ when magnetic dipole radiation (i.e. electromagnetic phenomena) dominates the spin down of the magnetar ;

b) $n = 5$ when the GW radiation dominates the magnetar spin down.

We are able to constrain the initial spin period (Po), dipole magnetic fields (B_{dipole}), and ellipticity (ϵ) of the neutron star (also known as a magnetar) by comparing the observed spin down light curves with their corresponding models. For instance, the ellipticityϵ $\sim 10^{-3}$ is often found in magnetars with an initial period Po ~ 1 ms and a surface dipole magnetic field Bdipole $\sim 10^{14} – 10^{15}$ G. However, according to Koranda et al. (1997), the magnetar's minimum rotation period has a theoretical value of between 0.3 and 0.5 ms. The relationships that fit the best (Xie et al., 2022) are

$\log\epsilon = 3.79^{+0.52}_{-0.43} + (2.19^{+0.17}_{-0.15}) \log P_o$ (20)

and $\log\epsilon = -22.50^{+2.15}_{-2.22} + (1.29^{+0.15}_{-0.14}) \log B_{dipole}$ (21)

suggest that

a) A magnetar with a longer spin period and/or a greater magnetic field is associated with a longer ellipticity.

b) A stronger magnetic field is associated with a longer rotation period;the neutron star deformation is related to its surface dipole magnetic field to some extent.

However, it is suggested (Majid et al., 2022; Rizaldy et al., 2018) that a strong internal magnetic field (Bint) in the stellar core may generate neutron star deformation instead of a dipole magnetic field. This is done by the relation

$\epsilon \approx 10^{-8} (B_{int} / 10^{12}\ G)$ (22)

This relationship suggests that in order to obtain the ellipticity (ϵ) ~ $10^{-3} - 10^{-4}$, one must possess a very strong internal magnetic field (Bint ~ $10^{16} - 10^{17}$ G). Additionally, the strength of the internal core magnetic field must be at least one or two orders of magnitude greater than the surface (i.e., external) magnetic field (B_{dipole} ~ 10^{15} G).

<u>The Estimation of theTriaxiality of Swift J1818.0-1607</u>

We can estimate the triaxiality of cosmic baby by using the relation in equ. (22) with the observed parameter at the time of its discovery on 12th March 2020:

i.e. rotational period = 1.36 s, surface dipolar magnetic field = 3 x 10^{14} G, surface magnetic field at poles = 7 x 10^{14} G, characteristic age ~ 300 years

Using equations (17), (18), (19) and (22) and with the above parameters as input we calculate **<u>the ellipticity, internal core magnetic field of Swift J1818.0-1607 and found ~ 9 x 10^{-3} and 8.9424 x 10^{17} G</u>, respectively**.

Interestingly, Rizaldy and Sulaksono's (2018) Numerical Simulation study of magnetized deformation of neutron stars indicates an interesting finding for low mass neutron stars: the influence of magnetic field is larger for internal magnetic fields > 4 × 10^{18} G for B_{int}. Rizaldy and Sulakseno (2018) assert that the balance between gravity and magnetic field is notably different for different orientations in the case of a modest mass rather than a huge neutron star. Even the magnetic field's gravitational pressure on the z-axis is noticeably more than it is on the other axes, giving low mass neutron stars their oblate shape. When compared to less massive neutron stars, the oblate shape of a big neutron star is much smaller. Stated otherwise, the internal toroidal magnetic field exhibits more effectiveness compared to the poloidal field. ***<u>The</u>***

deformation associated to the poloidal field ($B_p \approx 10^{14}$ and 10^{15} G) and the corresponding correction in ellipticity (i.e. ~ $10^{-4} – 10^{-2}$, respectively) is negligible (Morasi et al 2011).

Though the recent observation of magnetized deformation of neutron stars, or magnetars, is due to the interaction of both toroidal and poloidal magnetic fields, or a combined magnetic field. Since the poloidal field component's impact to deformation is minimal, we will only be examining the toroidal magnetic field's effect. Since our main objective is to evaluate the ellipticity and stability of the deformed neutron star, magnetar Swift 1818.0-1607. Heras (2012) conducted a comparative investigation between pulsars and magnetars and discovered that, in a realistic scenario, the initial magnetic fields inside newborn neutron stars fall between 10^{14} and 10^{16} G. Since ambipolar radiation is active, it inhibits both the cooling of the neutron star, or magnetar (since the effect is the same and applies to both types of stars), allowing the core temperature of the magnetar to remain higher than several times 108 K for a few thousand years (at least 10^3 years). It is possible that the ellipticity of the newly formed magnetar will not vary significantly during the next million years. Given that the Swift J1818.0 – 1607 is only approximately 300 years old, or in its infant phase compared to thousands of years old, it will undoubtedly display triaxility, or triaxial activity, at least until it reaches the age of 1000 years.

This magnetar's estimated ellipticity falls within the range of triaxiality, and it will stay there for thousands of years before demonstrating triaxiality. Therefore, the Swift J1818.0-1607 can be regarded as a triaxial star, or alternatively as a triaxial magnetar.

10. Conclusion : What do we want from Triaxial Cosmic Baby

When the ratio T / |W| surpasses a certain threshold, newly formed rotating neutron stars have the ability to spontaneously violate their axial symmetry. Magnetars are a special class of somewhat slow-rotating neutron stars with very powerful magnetic fields. Incorporating a magnetic field component parallel to the rotation axis results in a non-dissipative phenomenon known as spontaneous symmetry breaking, which violates circular conservation. Swift J1818.0-1607 is a juvenile magnetar with an approximate age of 300 years and a strong internal core magnetic field of 8.9424 x 1017G. With a spin or rotational period of around 1.36 seconds, it is the fastest of the 31 magnetars that have been found. The rotating axes are not aligned with its magnetic fields. The fast J1818.0-1607's aforementioned characteristics suggest that it is a perfect triaxial magnetar, or compact object, for testing and studying the discovery of a triaxial star and its peculiar characteristics. Despite being excessively powerful, its internal core magnetic field has a slow decline mode through ambipolar diffusion that activates shortly after formation (birth). We will thus have an opportunity to understand our understanding of the evolution of magnetar magnetic fields through continuous observation of Swift J1818.0 – 1607, as this process can prevent the magnetar core from cooling below a few times of 10^8 K (i.e. < 10^9 K) for thousands of years.

Finally, one can conclude that Cosmic Baby is most probably the detected “**First Triaxial Star”**

The frequency of the continuous gravitational waves emitted by this triaxial baby magnetar (i.e., the swift J1818.0-1607) would be very low due to its 1.36-second rotational period, which falls within the range of 1 to 10 seconds (Sieniawska and Bejger 2019; Ibrahim et al 2023). The author thus invites the Gravitational Wave Community to keep a close eye on this magnetar while they monitor other compact objects through their electromagnetic counterparts. It is believed that further observations would help us better understand the physics driving the evolution of the magnetars.

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