“DISCUSSION FOR ECONOMIC AGRICULTURE THROUGH MATHEMATICAL PROGRAMMING”

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Abstract

The attempts to simulate the economy of agricultural production, including its dimensions, are where mathematical programming in agriculture first emerged. In light of economic research, we must talk about the design and resolution of single-period, deterministic linear programming models, which assist farmers in enhancing their economic growth and achieving happiness and success in life. The models have been improved in order to incorporate more economic theory and economic reality. Using a set of fixed farm restrictions, linear programming can be used to identify a combination of farm enterprises that will maximize profits.

Keywords: Linear Programming Model, Quantitative Methods, Agriculture Economics, Farm-level, Sector-level

Abbreviations: LPP-Linear Programming Problem, SQ- Statistical and Quantitative, LPM- Linear Programming Model, CFA- Common Feasible Area

Introduction

Since the beginning, statistical and quantitative approaches have been crucial in the study of agriculture, particularly in agricultural economics and other areas of applied economics. As a contribution, linear programming has emerged as one of the most useful methods for the study of resource allocation decisions at the business and sector level in the agriculture industry. For more than three decades, the field of agriculture economics has used this mathematical programming. It has been such a helpful analytical tool that its fundamental concepts are taught in all agricultural colleges and universities. In the past decade, there have also been several methodological advancements in the field of agriculture, and both developed and developing nations have adopted mathematical programming models for agriculture. The models have been improved in order to incorporate more economic theory and economic reality.

The models serve as a bridge between statistics and economic theory on the one hand and a realistic understanding of current issues and policy stances on the other. The attempts to model the economics of agricultural production, including its dimensions, are where mathematical programming in agriculture first emerged. Economic agriculture is a good fit for the mathematical programming structure also known as process analysis or activity analysis. In both rich and developing nations, agronomists and other agriculture specialists consider the agricultural inputs and outputs coefficient in terms of the annual crop cycle and per acre, hectare, or other units of land. In this scenario, the model turns into a tool for converting micro-level (farm-level) information into macro-level (sector-level) functions, which are more recognizable to many economists. At the sector level, parametric variations can be used to generate response functions that are implicit in the model's structure.

The equilibrium production and price levels that the sector would typically trend toward can be determined using a set of estimated supply and demand functions at the sector level. Data issues and changes in the underlying economic structure are the key issues with depend solely on econometrics. Cross-supply effects are crucial parts of the supply functions because, in many circumstances, numerous crops compete for the same fixed resources. As a result, the data problem develops. In a time series data collection, there are typically insufficient degrees of freedom to estimate both the own and cross-supply elastic tics. In addition to these factors, the supply functions of a programming model offer details on the response of inputs like labor, agrochemicals, and the like.

In its most basic form, linear programming is a technique for identifying a combination of farm companies that will maximize profits while taking into account a set of preset farm limitations. Early uses of linear programming in farm planning presupposed a single-period planning horizon (no expansion), profit-maximizing behavior, and a specific setting.

Structure of Linear Programming Model

Individual farmers frequently have to choose what products to produce, how to produce them, and during which seasonal times. These decisions are influenced by forecasted yields, costs, and prices for individual farm enterprises, as well as by the need for fixed resources within the enterprises and the total supply of fixed resources available. In its most basic form, linear programming is a technique for identifying a combination of farm companies that will maximize profits while taking into account a set of preset farm limitations.

For a given farm situation the linear programming model requires the specification of

- The alternative farm activities, their units of measurement, their resource requirements, and any specific constraints on their production.
- The fixed resource constraints of the farm
- The forecast activity returns net of variable costs, hereafter called gross margins.

To formulate the problem mathematically we introduce the following notation:

- X_j: The level of the j^{th} farm activity, such as the acreage of … grown

Let n denote the number of possible activities; then $j = 1\ to\ n$

- C_j: The forecasted gross margin of a unit of the j^{th} activity (*e.g.*, Rupee per ace)
- a_{ij}: The quantity of the i^{th} resource (*e.g.*, acres of land or days of labor required to produce one unit of the j^{th} activity)

Let m denote the number of resources; then $i = 1\ to\ m$

- b_i : The amount of the resource available (*e.g.*, acres of land or days of labor)

With the notation, the linear programming model can be written as follows:

$$\max Z = \sum_{j=1}^{n} C_j X_j \ldots \text{(eqn. 1)}$$

Such that,

$$\sum_{j=1}^{m} a_{ij} X_j \leq b_i \ \forall\ i = 1\ to\ m \ \ldots \text{(eqn. 2)}$$

And,

$$X_j \geq 0 \ \forall\ j = 1\ to\ n \ldots \text{(eqn. 3)}$$

The challenge is to identify the farm plan with the highest overall gross margin***Z***, that does not break any of the fixed resource limitations (eqn. 2) or entail any negative activity levels (eqn. 3). The farm plan is specified by a set of activity levels, X_j, $j = 1\ to\ n$). The primary linear programming problem is the name given to this issue.

Table 1: A Linear Programming Tableau

Structure of Linear Programming					
Objective function Resource constraints	X_1	X_2	…	X_n	**RHS**
1	c_1	c_2	…	c_n	**Maximize**
2	a_{11}	a_{11}	…	a_{11}	$\leq b_1$
3	a_{21}	a_{22}	…	a_{2n}	$\leq b_2$
. . .	. . .	. . .	. . .	. . .	. . .
m	a_{m1}	a_{m2}		a_{mn}	$\leq b_{mn}$

A matrix displaying each coefficient in the model's algebraic formulation. By custom, linear programming structures are presented in this manner. Several conventions have been introduced in Table 1.

First, the equation to be maximized is called the objective function. In the current problem, the objective is the total gross margin (eqn. 1) but other objective functions are also possible.

Second, the constraints are called rows and the activities are called columns.

Third, the Right Hand Side (or RHS) of the problem is defined as the fixed resource supply and the maximize coefficient. Though it is also possible to incorporate equality constraints (=) or greater than or equal (≥) constraints, they have all been specified as less than or equal (≤) constraints.

Assumptions

The linear programming model in equations (1) through equation (3) makes a variety of assumptions that change the nature of the production process, the resources, and the activities.

Optimization: Either maximization or minimization of a suitable objective function is used.

Fixedness: A non-zero RHS coefficient exists for at least one restriction.

Finiteness: If there are only a limited number of actions and limitations that need to be taken into account, the solution may be sought.

Determinism: In the model, all constants are a_{ij} , $b_{i,}$and c_j

Continuity: Activities and resource usage can both be done in fractional unit quantities.

Homogeneity: The same resource or activity has identical units throughout.

Additives: When two or more activities are used, their combined output equals the sum of their individual outputs.

Proportionality: Regardless of the degree of the activity used, it is assumed that the gross margin and resource requirements per unit of activity remain constant.

Let$Z = f(b)$, and constant returns to scale mean that if all the fixed resources are increased by a factor of proportionality k, then the value of the objective function Z also increases by k.

Specifically, $f(kb) = kf(b) = kZ$

Since,

$$Z = \sum_j c_j X_j$$

If the c_j coefficient is constant, it follows that

$$kZ = \sum_j c_j \, (kX_j)$$

Thus, the ideal activity levels will therefore increase by k, if the fixed factor supply is increased by a factor of proportionality$k$.

According to this, the total output is equal to the sum of the factors times their respective marginal products if each factor is valued at its marginal product. Linearity in the activities, namely linear programming and the assumptions underlying the linear programming model, are defined by the additive and proportionality hypotheses used together. These presumptions must be true for all rows and columns of a model, but not necessarily for the actual farm production processes. It is feasible to increase the model's flexibility in a variety of inventive ways without going against the assumptions.

The fixedness assumption can be relaxed through dynamic multi-period specifications which allow growth and changes in the resource constraints over time. Also, methods have been developed for modeling stochastic a_{ij}, $b_{i,}$ and c_j coefficients and for incorporating less than perfectly elastic input supplies.

Principles of Solving Mathematical Programming Problems

In terms of the ideal activity, the answer to a linear programming problem typically results in a distinct farm design. Consider the following linear programming problem:

$$\max Z = ax + by$$

Such that,

$$cx + dy \leq e, \text{ resource } b_1$$

$$fx + gy \leq h, \text{ resource } b_2$$

$$x, y \geq 0$$

If we plot the activity levels on the axes of a graph, the constraints of this problem can be portrayed as below figure:

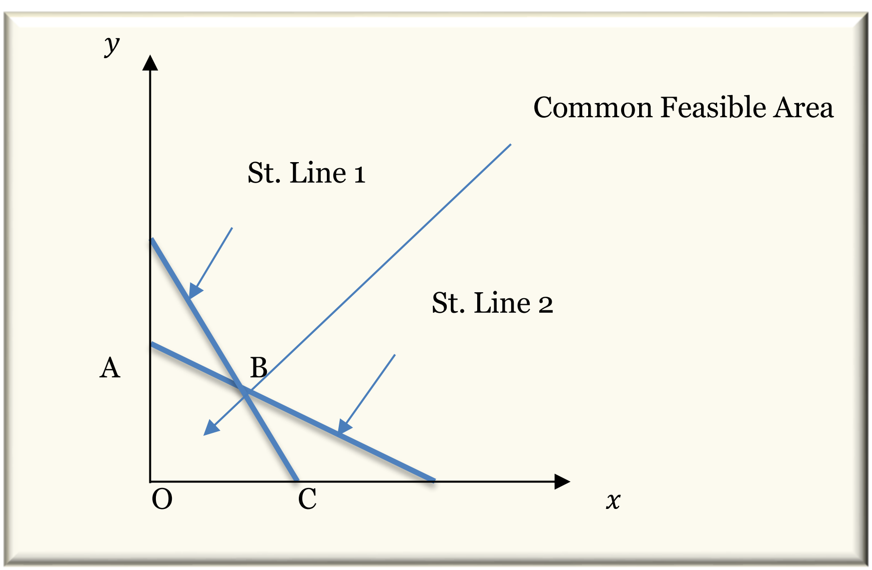

Figure 1: The Feasible Set

Each constraint is represented by a straight line that intersects the axes at the maximum level of each activity that can be produced with the assumed supply of the associated resource. For the jth activity and the ith resource, the maximum activity level is b_i / a_{ij}

Intermediate points along a constraint depict linear combinations of the activity that also exactly exhaust the resource. For a farm plan to be eligible for consideration as the optimal solution to the linear programming problem, it must be feasible for all the resource constraints. In this limits consideration to those combinations of x and y contained in the area OABC. ABC is known as the production possibility frontier, it defines the maximum amount of x and y that can be produced for all possible ratios of the levels of these activities. To identify the optimal farm plan, we need to introduce the objective$Z$. This is done by drawing revenue lines that define the combinations of x and y that can be used to attain some fixed amount of total gross margin Z. These revenue lines are always parallel, and the ones corresponding to larger values of Z always lie above and to the right of those corresponding to smaller values of Z.

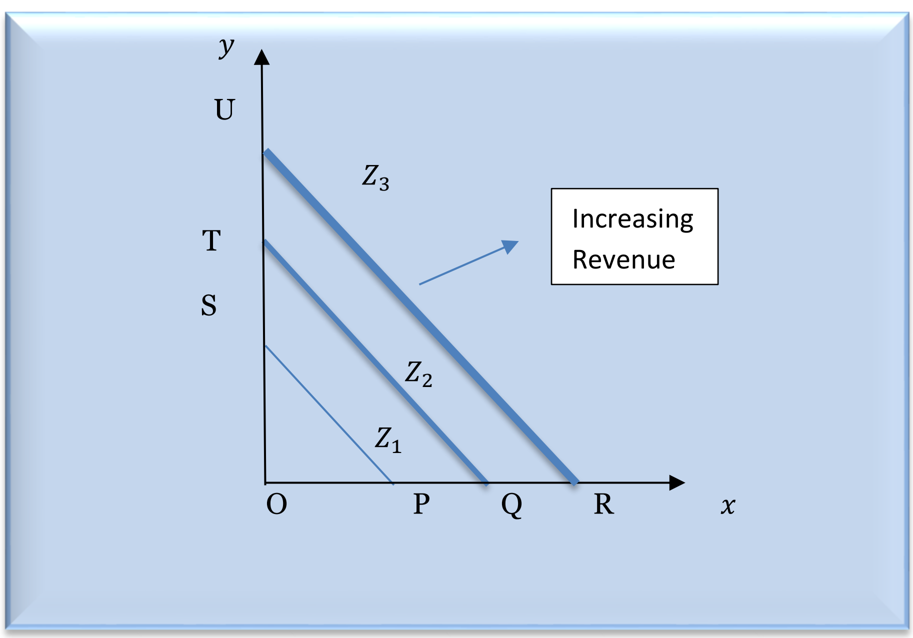

Figure 2: The Revenue Lines

At we want to maximize Z, the total optimal farm plan is clearly the feasible plan that lies on the highest attainable revenue line.

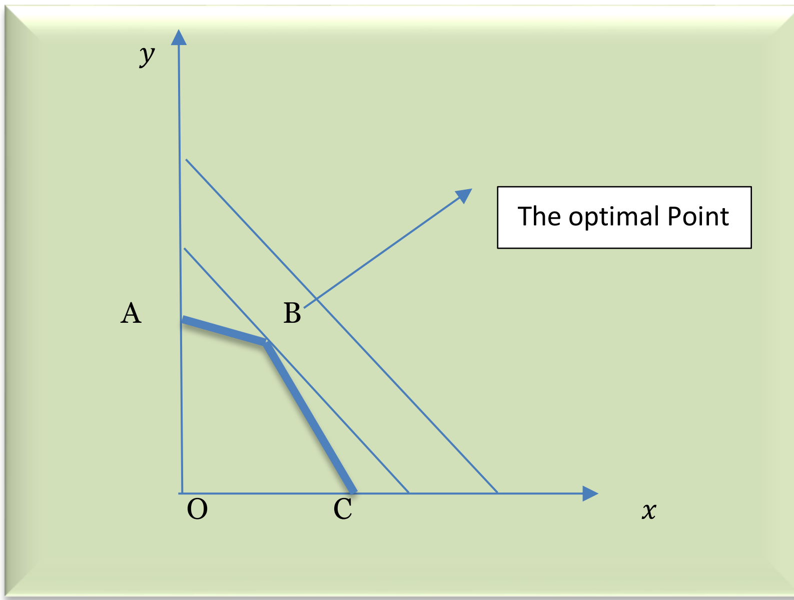

Figure 3: The Optimal Solution

The set of realistic farm plans from Figure 1 has been placed in Figure 3 with revenue lines. The best answer to the linear programming issue is consequently the farm plan with the highest revenue from the set of workable farm plans from the frontier at B. The simplex method's key is to narrow down the pool of potential farm plans to a manageable quantity. In Figure 3, we were to rotate the revenue line to reflect alternative ratios of the activity gross margin. It should be clear that the optimal solution for each rotational shift must not only lie on the production possibility frontier ABC, but it will be either points A, B, or C. Some ambiguity arises if the revenue lines are exactly parallel to either segment yielding the same value of Z, so one can choose a plan lying at the end of the segment as the optimal solution The end-point solutions are again A, B, and C.

Problems with Computation in Mathematical Programming

Problems in mathematical programming may not always have an optimal solution, or they may have an optimal solution but the simplex process may converge slowly or not at all due to degeneracy issues. We have to discuss each problem as below:

Infeasibility: If there isn't a single solution to a linear programming problem that meets every requirement, it is said to be infeasible. In reality, infeasibilities typically result from errors made when preparing the data for a linear programming task. They may also occur in huge and complex issues when the analyst does not adequately account for all of the logical relationships present in the model.

Unboundedness: If there is a workable solution with an infinite value for the objective function, the problem is said to be unbounded. The most frequent cause of unboundedness is incorrect data preparation for a linear programming task.

Degeneracy: If the value of the objective function remains constant from one iteration to the next, the problem is said to be degenerate. When the best incoming activity can only enter the basis at level zero, this occurs. Ties are yet another issue related to degeneracy. When two or more potential incoming activities at a given iteration are equally beneficial in terms of the growth in the objective functions that result, a tie is created.

Post-optimality Analysis

In solving a linear programming problem, all the a_{ij} , $b_{i,}$and c_j coefficients are assumed to be known constant. However, the user may not always be sure of his data, particularly his forecasts of activity gross margins. Some coefficients, such as price and yields, may also vary from year to year because of weather or economic changes beyond the farmer's control. One way of dealing with these uncertainties in the data is to solve the model for different but realistic sets of assumptions about the data to determine the stability or robustness of the optimal farm plan. Such post-optimality analysis is also useful for evaluating longer-term farm decisions, or changes in the economic and technological environment, which affect the fixed constraints of the farm.

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