# Peristaltic flow of a Newtonian fluid through a porous medium in a two-dimensional channel with Hall effects

## 1.1 Introduction

Magnetohydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluid in the presence of amagnetic field. The motion of the conducting fluid across the magnetic field generateselectric currents that change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Ferraro, 1966). The magnetohydrodynamic (MHD) flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest inconnection with certain problems of the movement of conductive physiological fluids, e.g, the blood, blood pump machines and with the need for theoretical research on the operation of a peristaltic MHD compressor. Agrawal and Anwaruddin (1984) studied the effect of moving magnetic field on blood flow. They studied a simple mathematical model for blood through an equally branched channel with flexible outer walls executing peristaltic waves. The result revealed that the velocity of the fluid increases with anincrease in the magnetic field. Peristaltic transport of a Johnson-Segalman fluid under the effect of a magnetic field was developed by Elshahed and Haroun (2005). The peristaltic flow of a MHD fourth grade fluid in a planar channel has studied by Hayat et al. (2007). Ali et al. (2008) haveinvestigated theeffect of slip condition on the peristaltic flow of a Newtonian fluid with variable viscosity under the influence of magnetic field.Non-linear peristaltic motion of a Carreau fluid under the effect of a magnetic field in an inclined planar channel was studied by Subba Reddy and Gangadhar (2010). Subba Narasimhudu and Subba Reddy (2017) have studied the Hall effects on the peristaltic flow of a Newtonian fluid in the channel.

Moreover, flow through a porous medium has been studied by a number of researchers employing Darcy's law Scheidegger (1974). Several studies about this point have been given by Varshney (1979) and Raptis and Perdikis (1983).

The first study of peristaltic flow through a porous medium is presented by Elsehawey et al. (1999). Elsehawey et al. (2000) investigated the peristaltic motion of a generalized Newtonian fluid through a porous medium. Hayat et al. (2007) have first investigated the Hall effects on the peristaltic flow of a Maxwell fluid through the porous medium in channel. Peristaltic motion of the carreau fluid through a porous medium in a channel under the effect of a magnetic field was studied by Sudhakar Reddy et al. (2009). Subba Reddy and Prasnath Reddy (2010) has investigated the effect of variable viscosity on peristaltic flow of a Jeffrey fluid through a porous medium in the planar channel. Eldabe (2015) have studied the Hall Effect on peristaltic flow of third order fluid in the porous medium with heat and mass transfer.

In view of these, we studied the effect of hall on the peristaltic flow of a Newtonian fluid through a porous medium in a two dimensional channel under the assumption of long wavelength. A closed form solution is obtained for axial velocity, temperature field and pressure gradient. The effects of various emerging parameters on the pressure gradient, time-averaged volume flow level and temperature field are discussed with the help of graphs.

## 1.1 Mathematical Formulation

We consider the peristaltic pumping of a conducting Newtonian fluid flow through a porous medium in a channel of half-width $a$. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. For simplicity, we restrictour discussion to the half-width of the channel is shown in the Fig.1.1

The wall deformation is given by

$$H(X,t) = a + b\sin\left[\frac{2\pi}{\lambda}(X - ct)\right] \qquad (1.2.1)$$

Where$b$ is the amplitude, $\lambda$ the wavelength and $c$ is the wave speed

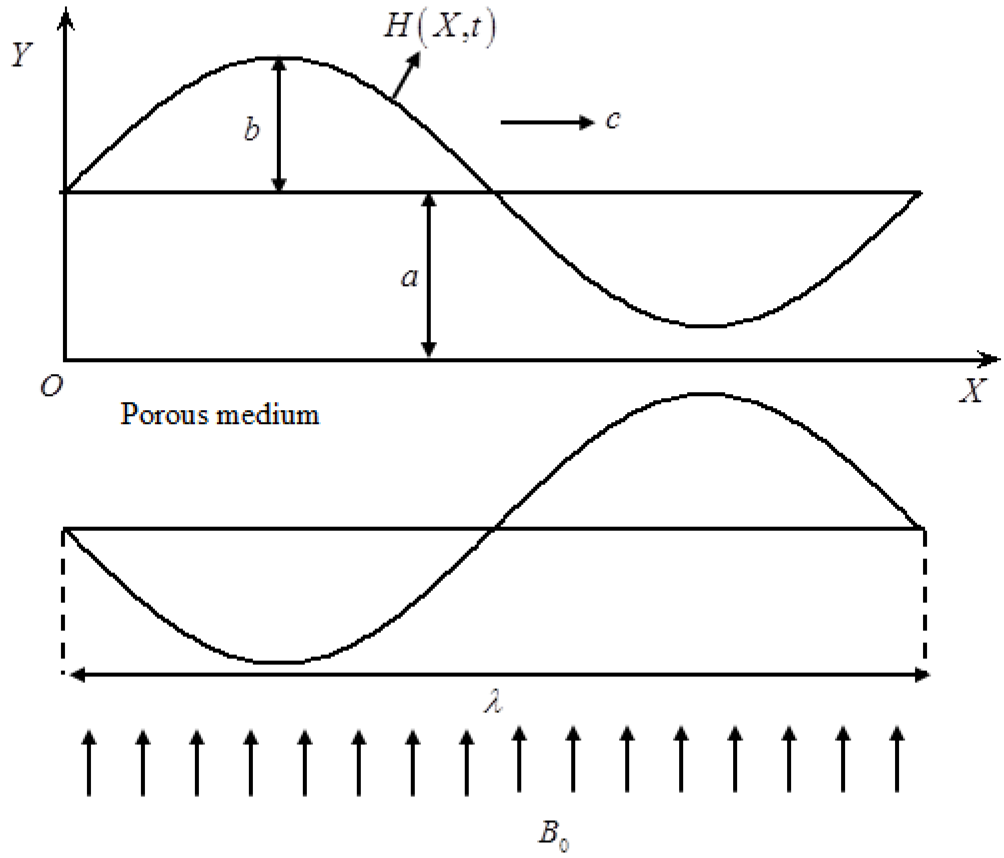


**Fig.1. 1** Physical Model

Under the assumptions that the channel length is an integral multiple of the wavelength $\lambda$ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame $(x, y)$ moving with velocity c away from the fixed (laboratory) frame $(X, Y)$. The transformation between these two frames is given by

$$x = X - c\,t,\ y = Y,\ \ u = U - c,\ \ v = V \text{ and } p(x) = P(X, t), \qquad (1.2.2)$$

Where $(u, v)$ and $(U, V)$ are the velocity factors, $p$ and $P$ were pressures in the wave and fixed frames of reference, respectively. The equations governing the flow in wave frame are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1.2.3)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{\sigma B_0^2}{1+m^2}\left(mv-(u+c)\right)-\frac{\mu}{k}(u+c) \quad (1.2.4)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\sigma B_0^2}{1+m^2}\left(m(u+c)+v\right)-\frac{\mu}{k}v \quad (1.2.5)$$

Where $\rho$ is the density $\sigma$ is the electrical conductivity, $B_0$ is the magnetic field strength, $m$ is the Hall parameter, $k$ is the permeability of the porous medium.

The dimensional boundary conditions are

$$u=-c \quad \text{at} \quad y=H \quad (1.2.6)$$

$$\frac{\partial u}{\partial y}=0 \quad \text{at} \quad y=0 \quad (1.2.7)$$

Introducing the non-dimensionalq quantities

$$\bar{x}=\frac{x}{\lambda},\bar{y}=\frac{y}{a},\bar{u}=\frac{u}{c},\bar{v}=\frac{v}{c\delta},\delta=\frac{a}{\lambda},\bar{p}=\frac{pa^2}{\mu c\lambda},\bar{t}=\frac{ct}{\lambda},h=\frac{H}{a},\phi=\frac{b}{a},$$

$$\bar{q}=\frac{q}{ac},M^2=\frac{\sigma a^2 B_0^2}{\mu},\ Da=\frac{k}{a^2}$$

Into equations (1.2.3) to (1.2.5), we get

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1.2.8)$$

$$\mathrm{Re}\,\delta\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\frac{M^2}{1+m^2}\left(m\delta v-(u+1)\right)-\frac{1}{Da}(u+1) \quad (1.2.9)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\delta M^2}{1+m^2}\left(m(u+1)+\delta v\right)-\frac{\delta^2}{Da}v \quad (1.2.10)$$

Here Re is the Reynolds number, $M$ is the Hartmann number and $Da$ is the Darcy number.

Using long wavelength ( i.e., $\delta \ll 1$) approximation, the equations ( 1.2.9) and (1.2.10) become

$$\frac{\partial^2 u}{\partial y^2} - \beta^2 u = \frac{\partial p}{\partial x} + \beta^2 \qquad (1.2.11)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (1.2.12)$$

Where

$$\beta = \sqrt{\frac{M^2}{1+m^2} + \frac{1}{Da}}.$$

From Eq. (1. 2.12), it is clear that $p$ is independenti of $y$. Therefore Eq. (1.2.11) can be rewritten as

$$\frac{\partial^2 u}{\partial y^2} - \beta^2 u = \frac{dp}{dx} + \beta^2 \qquad (1.2.13)$$

The corresponding non- dimensional boundary conditions are given as

$$u = -1 \quad \text{at} \quad y = h = 1 + \phi \sin 2\pi x \qquad (1.2.14)$$

$$\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \qquad (1.2.15)$$

Knowing the velocity, the volume flow rate $q$ in a wave frame of reference is given by

$$q = \int_0^h u dy. \qquad (1.2.16)$$

The instantaneous flow Q$(X,t)$ in the laboratory frame is

$$Q(X,t) = \int_0^h UdY = \int_0^h (u+1) dy = q + h \qquad (1.2.17)$$

The time averaged volume flow rate $\overline{Q}$ over one period $T\left(=\frac{\lambda}{.c}\right)$ of the peristaltic wave is given by

$$\overline{Q} = \frac{1}{T}\int_0^T Q dt = q + 1 \qquad (1.2.18)$$

## 1.3 Solution

Solving Eq. (1.2.13) together with the boundary conditions (1.2.14) and (1.2.15), we get

$$u = \frac{1}{\beta^2}\frac{dp}{dx}\left[\frac{\cosh \beta y}{\cosh \beta h} - 1\right] - 1 \tag{1.3.1}$$

The volume flow rate $q$ in a wave frame of reference is mentioned by

$$q = \frac{1}{\beta^3}\frac{dp}{dx}\left[\frac{\sinh \beta h - \beta h \cosh \beta h}{\cosh \beta h}\right] - h \tag{1.3.2}$$

From Eq. (1.3.2), we write

$$\frac{dp}{dx} = \frac{(q+h)\beta^3 \cosh \beta h}{\sinh \beta h - \beta h \cosh \beta h} \tag{1.3.3}$$

The dimensionless pressure rise per one wavelength in the wave frame is defined as

$$\Delta p = \int_0^1 \frac{dp}{dx} dx \tag{1.3.4}$$

As $Da \to \infty$, our results coincides with the results of Subbanarasimhudu and Subba Reddy (2017).

## 1.4 Results and Discussion

Fig.1.2 depicts the variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number $M$ for $Da = 0.1$, $\phi = 0.6$ and $m = 0.3$. It is found that, the axial pressure gradient $\frac{dp}{dx}$ increases with increasing $M$.

The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall parameter $m$ for $Da = 0.1$, $\phi = 0.6$ and $M = 1$ is depicted in Fig 1.3. It is observed that, the axial pressure gradient $\frac{dp}{dx}$ decreases with increasing $m$.

Fig 1.4 illustrates the variation of axial pressure gradient $\frac{dp}{dx}$ with Darcy number $Da$ for $\phi = 0.6$, $M = 1$ and $m = 0.3$. It is noted that, the axial pressure gradient $\frac{dp}{dx}$ decreases on increasing $Da$.

The variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio $\phi$ for $Da = 0.1$, $M = 1$ and $m = 0.3$ is shown in Fig. 1.5. It is noticed that, the axial pressure gradient $\frac{dp}{dx}$ increases on increasing $\phi$.

Fig.1.6 depicts the variation of pressure rise $\Delta p$ with time- averaged flow rate $\bar{Q}$ for different values of Hartmann number $M$ with $Da = 0.1$, $\phi = 0.6$ and $m = 0.3$. It is found that, the time-averaged flow rate $\bar{Q}$ increases in the pumping region $(\Delta p > 0)$ with increasing $M$, while it decreases in both the free-pumping $(\Delta p = 0)$ and co-pumping $(\Delta p < 0)$ regions with increasing $M$.

The variation of pressure rise $\Delta p$ with time- averaged flow rate $\bar{Q}$ for different values of Hall parameter $m$ with $Da = 0.1$, $\phi = 0.6$ and $M = 1$ is depicted in Fig.1.7. It is found that, the time- averaged flow rate $\bar{Q}$ decreases in the pumping region on increasing $m$, while it increases in both the free-pumping and co- pumping regions on increasing $m$.

Fig.1.8 illustrates the variation of pressure rise $\Delta p$ with time- averaged flow rate $\bar{Q}$ for different values of Darcy parameter $Da$ with $m = 0.3$, $\phi = 0.6$ and $M = 1$. It is found that, the time-averaged flow rate $\bar{Q}$ decreases in the pumping region with an increase in $Da$, while it increases in both the free-pumping and co-pumping regions with increasing $Da$.

The variation of pressure rise $\Delta p$ with time-averaged flow rate $\bar{Q}$ for different values of amplitude ratio $\phi$ with $Da = 0.1$, $M = 1$ and $m = 0.3$ is shown in

Fig.1.9 . It is found that the time-averaged flow rate $\overline{Q}$ increases with increasing amplitude ratio $\phi$ in both the pumping and free pumping regions, while it decreases with increasing amplitude ratio $\phi$ in the co-pumping region for chosen $\Delta p(<0)$.

## 1.5 Conclusions

In this chapter, the effect of hall on the peristaltic flow of a conductingNewtonian fluid through a porous medium in a two- dimensional channel under the assumption of long wavelength approximation is investigated. The expressions for the velocity field and temperature field and pressure gradient are obtained analytically . It is observed that, the pressure gradient and the time- averaged flow rate in the pumping region are increases with increasing Hartmann number $M$ and amplitude ratio $\phi$ , while they decreases with increasing hall parameter $m$ and Darcy number $Da$.

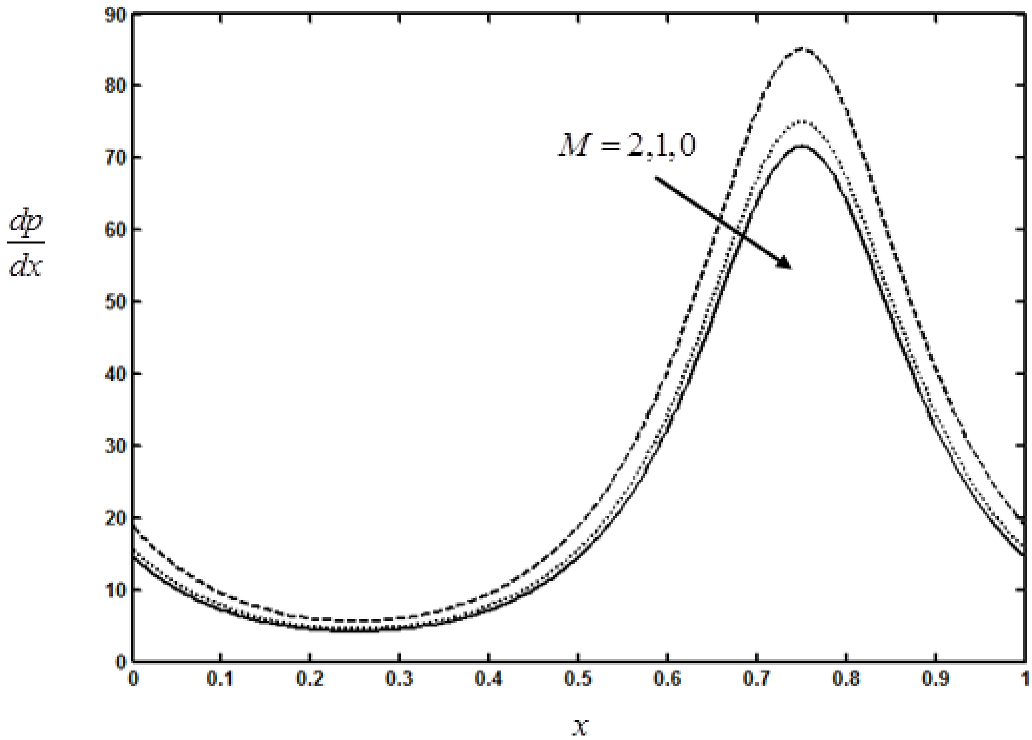


**Fig. 1.2** The variation of axial pressure gradient $\frac{dp}{dx}$ with Hartmann number $M$ for $\phi = 0.5$, $Da = 0.1$ and $m = 0.2$ .

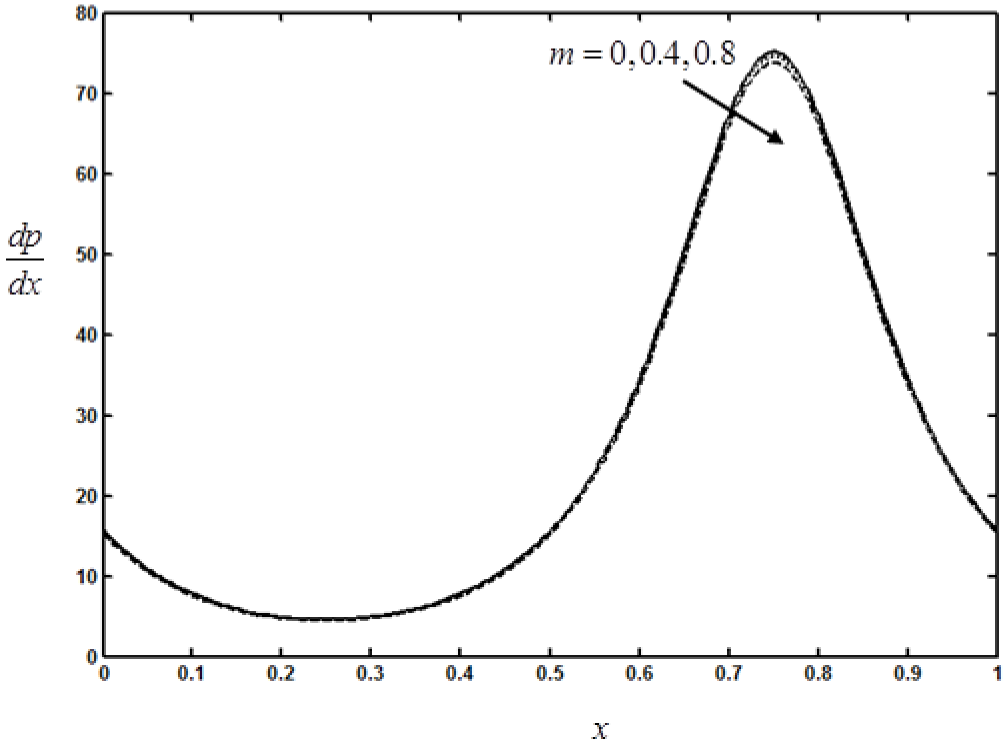


**Fig. 1.3.** The variation of axial pressure gradient $\frac{dp}{dx}$ with Hall Parameter $m$ for $\phi = 0.5.$, $Da = 0.1$ and $M = 1$ .

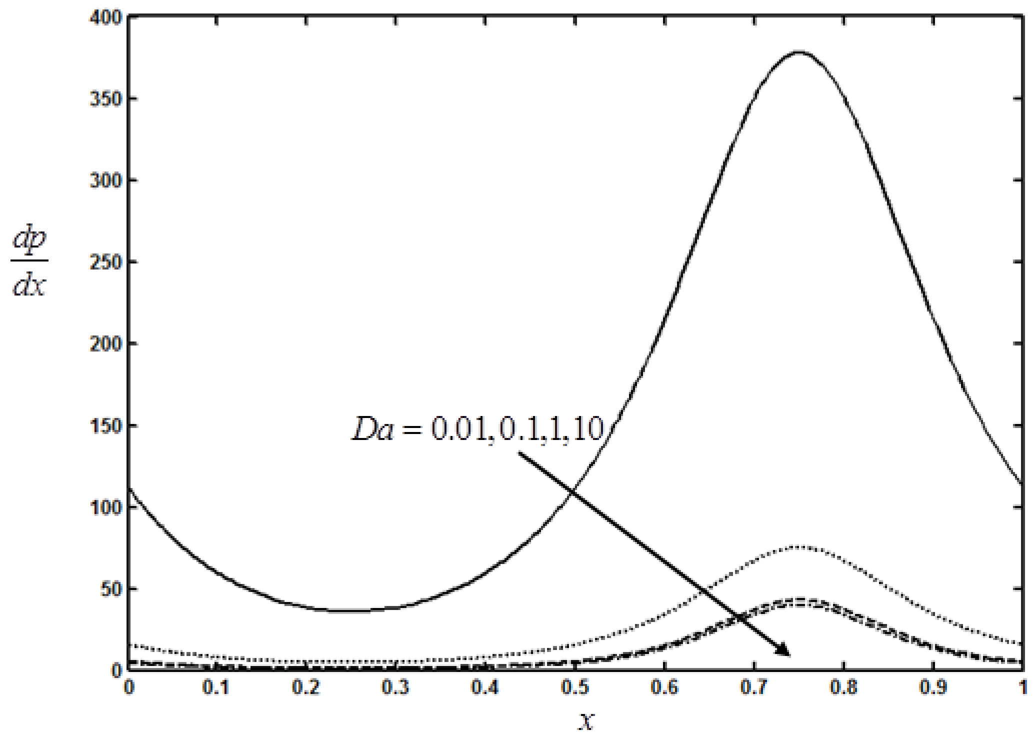


**Fig.1. 4.**The variation of axial pressure gradient $\frac{dp}{dx}$ with Darcy number $Da$ for $\phi = 0.5$, $m = 0.2$ and $M = 1$

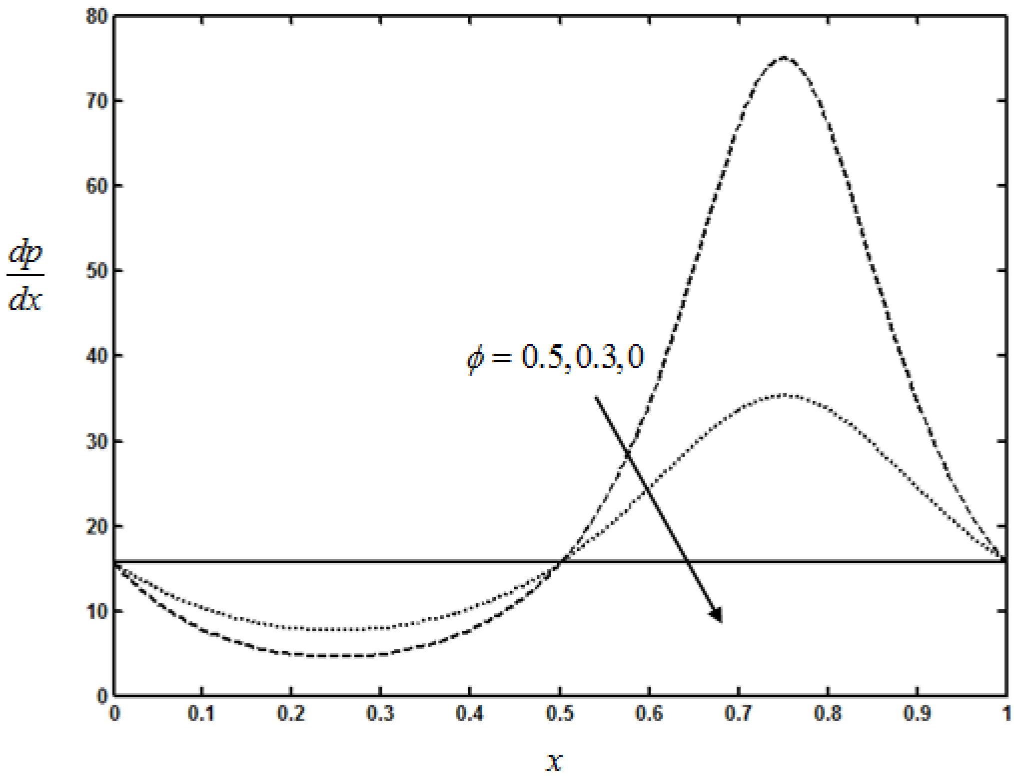


**Fig.1. 5.** The variation of axial pressure gradient $\frac{dp}{dx}$ with amplitude ratio $\phi$ for $M = 1$, $Da = 0.1$ and $m = 0.2$.

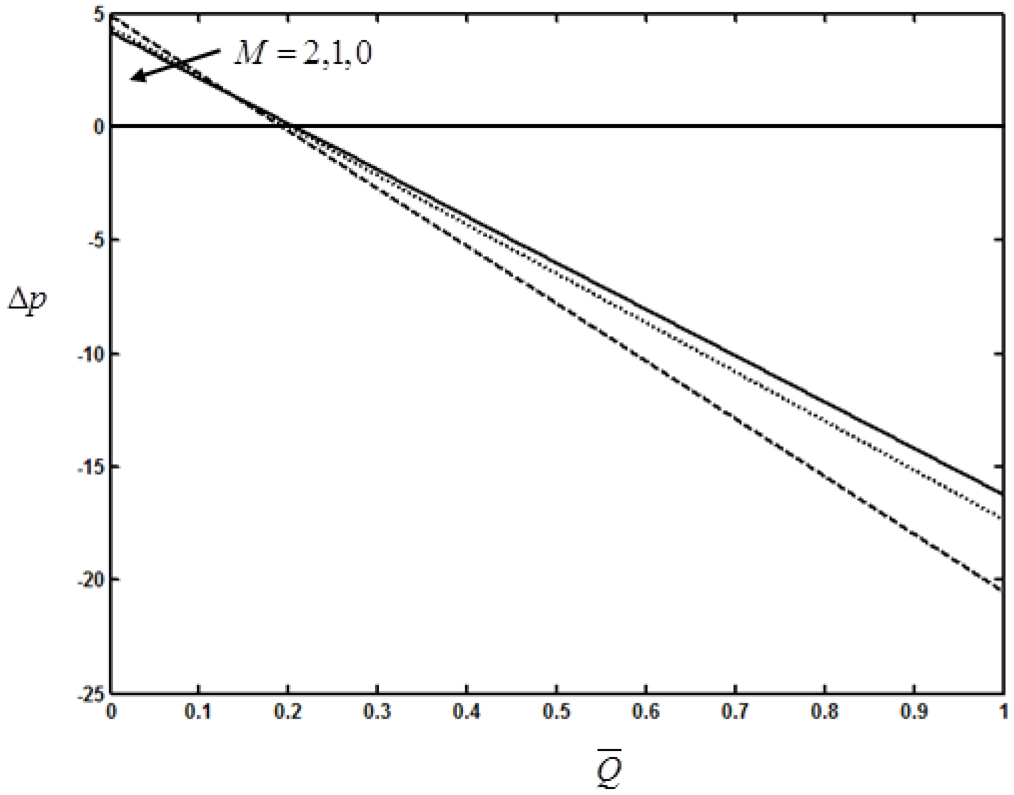


**Fig. 1.6.** The variation of pressure rise $\Delta p$ with time-averaged flow rate $\overline{Q}$ for different values of Hartmann number $M$ with $Da = 0.1$, $\phi = 0.5$ and $m = 0.2$.

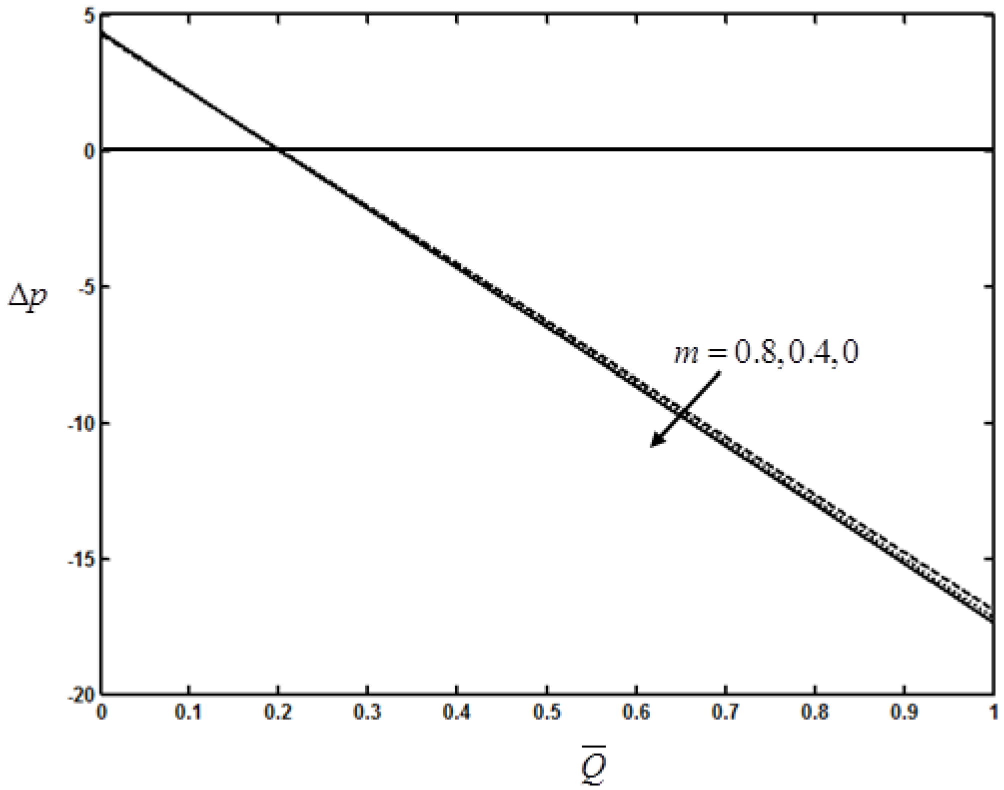


**Fig. 1.7.** The variation of pressure rise $\Delta p$ with time-averaged flow rate $\overline{Q}$ for different values of Hall parameter $m$ with $Da = 0.1$, $\phi = 0.5$ and $M = 1$ .

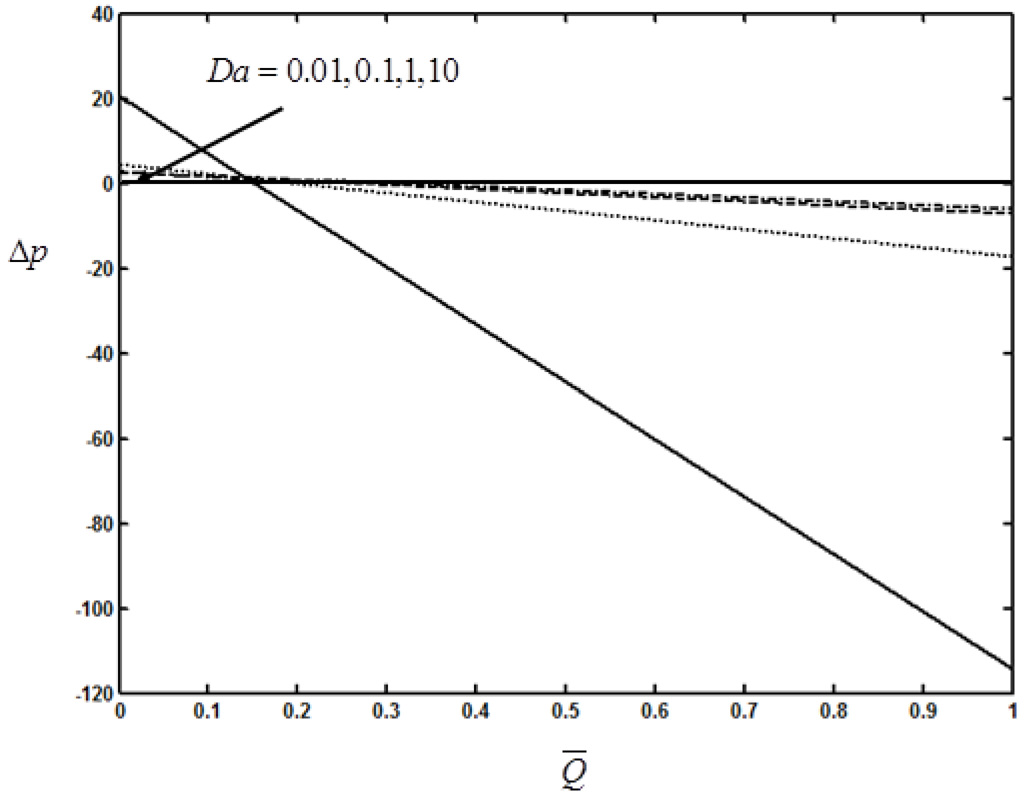


**Fig.1. 8.** The variation of pressure rise $\Delta p$ by time-averaged flowrate $\overline{Q}$ for different values of Darcy number $Da$ with $m = 0.2$, $\phi = 0.5$ and $M = 1$.

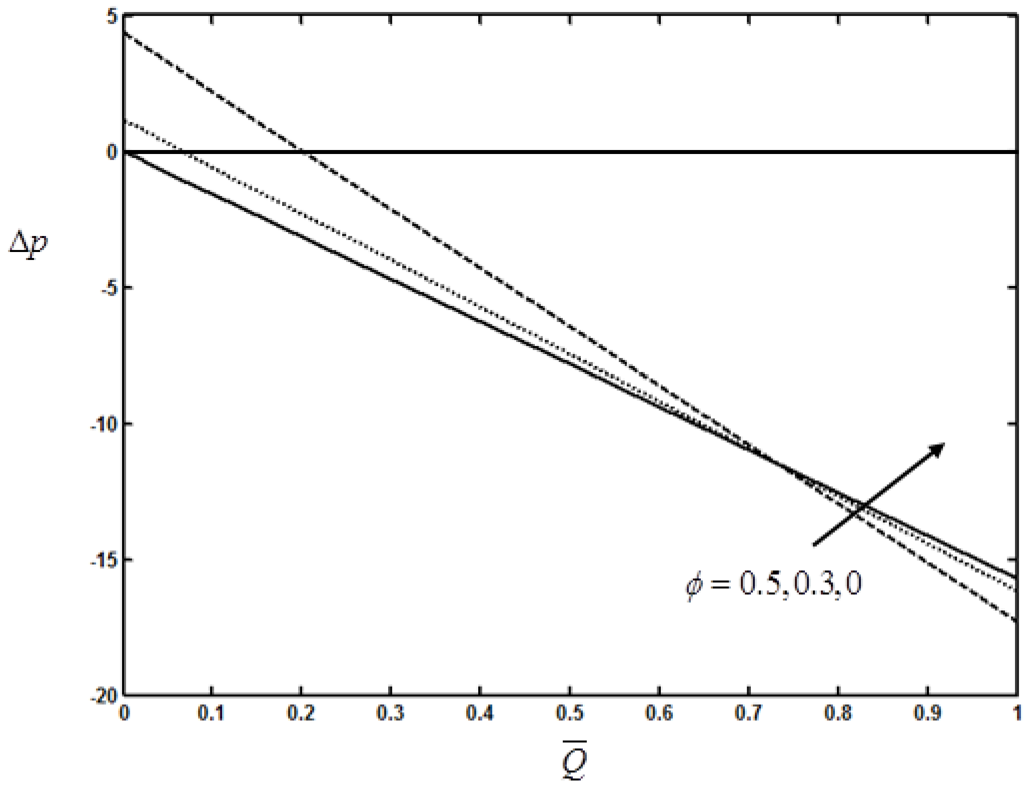


**Fig.1. 9.** The variation of pressure rise $\Delta p$ with time-averaged flow rate $\overline{Q}$ for different values of amplitude ratio $\phi$ with $M = 1$, and $m = 0.2$.