A Survey Study of Some Graph Labeling Techniques.

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ABSTRACT

Graph theory is one of the well-known prosperous branch of Mathematics (Arithmetic). Mathematics gives the same name to different things. But graph theory creates a graph labeling techniques in which different labeling are given to same graphical structures. Graph is a mathematical structure describing dots, curves, bars, or traces.Graph labeling is the venture of integers to the vertices or edges, or each, subject to certain conditions. The concept of graph theory may be used to shape various mathematical models for applications (packages) in Operation Research, control, Engineering; specially in studies areas of laptop technological know-how which include statistics mining, photograph segmentation, clustering, photograph capturing, networking that's used in structural fashions and so forth. Graph labeling is one of the crucial areas of graph concept which has many applications in social community, verbal exchange (communication) community, circuit design, Database management, coding principle, radar, astronomy, X-ray crystallography. Depending on trouble scenario a type of graph is used for representing the hassle (problem) and by way of applying appropriate graph labeling technique the hassle may be solved. Graph labeling is a flourshing as well as application oriented area of research in the field of graph theory.
In this paper we study the different graph labeling techniques for different graphs.

Keywords- Graph, Graph labelling, Cordial,Magic, Mean,Radio, Power mean, Permutation,combination.

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1 Introduction

The field graph theory has been invented in 1735 with the Koinsberg Seven Bridge problem. The phrase graph derived from the Greek word 'graphein'. A "graph" in this paper consists of "vertices" or"nodes" and lines called edges that connect with vertices. Graph labeling have been introduced within the mid 1960's by Alexander Rosa. Over the past six decades the area of graph labeling developed very fast. Graph labeling is a flourishing as well as application oriented area of research in the field of graph theory. Until date more than 200 types of graph labeling had been studied. In this survey paper we try to collect some important graph labeling techniques with suitable graphs. Here we consider only simple , finite, connected and undirected graph. Here we follow the terminology and notations of graph from Harary's 'Graph Theory'.While we follow the terminology and notations of Graph labeling techniques from Gallian's dynamic survey of graph labeling techniques.

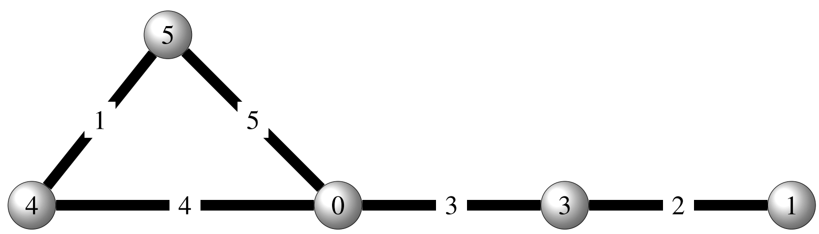
1.1 Graph :

Definition: 1. *A graph is an ordered triplet $G = (V(G), E(G), \phi_G)$,consisting of a non-empty set $V(G)$,whose elements are called vertices,a set $E(G)$ whose elements are called edges and an incidence function ϕ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.*
If e is an edge and u and v are vertices such that $\phi_G(e) = uv$ then e is said to join u and v ; the vertices u and v are called the end vertices of e.

Order of a graph: The cardinality of a vertex set $V(G)$ is called order of a graph.
Size of a graph : The cardinality of an edge set $E(G)$ is called size of a graph.
degree of a vertex: The degree of a vertex is the number of edges incident on a vertex.
degree of a graph: The degree of a graph G is the sum of degrees of all vertices in a graph G.

1.2 Graph Labeling :

Definition: 2. *A Graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions.*

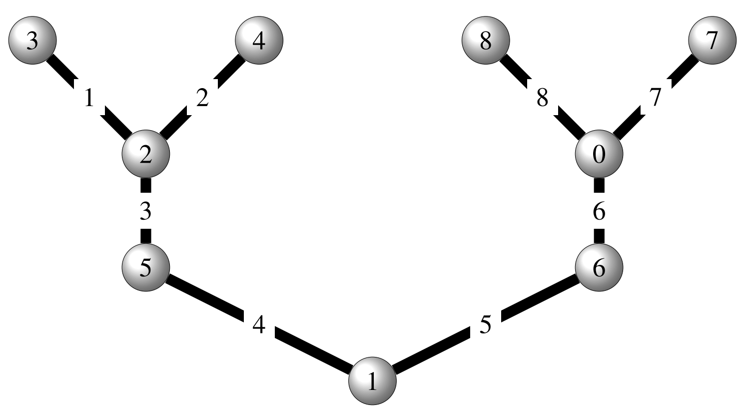


1.3 Different Graph Labeling Techniques :

1.3.1 Graceful Labeling:

Definition: 3. *A simple and finite graph $G = (V, E)$ is called graceful if there exists an injection $F : V(G) \to \{0, 1, 2, ..., |E(G|\}$ such that the induced function $F^*(uv) = |F(u) - F(v)|, uv \in E(G)$ is a bijection.The injection F is called a graceful labeling of G. The values of $F(u)$ and $F^*(uv)$ are called graceful labels of vertex u and the edge uv resp.*

Exa. Banana tree is graceful. ,trees with vertices ≤ 35,Caterpillars,bananas etc. are always graceful.

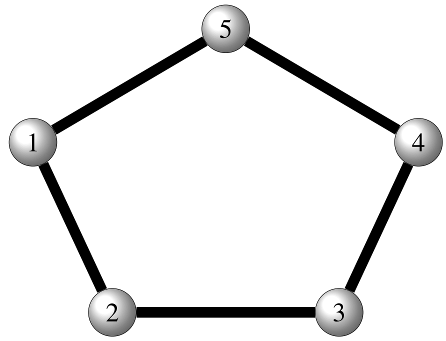


$B_{2,4}$

Examples of graph having Graceful Labeling.

- All symmetrical trees
- Trees with vertices ≤ 35.
- Caterpillars, banana trees etc.

Non-graceful graphs: A simple graph G is said to be non-graceful if there does not exists any graceful labeling.



$5 - cycle$

Result

In graceful graph with q (q is positive integer) number of edges and $q+1$ number of vertices the following holds:

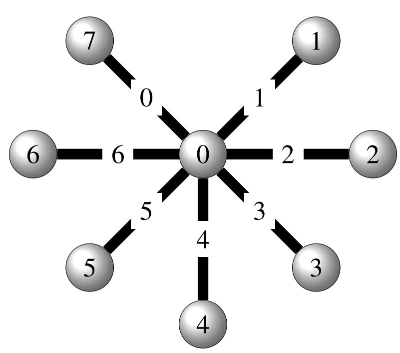
- The graceful labeling of a graph is not unique.
- In any graceful graph the vertices with labels 0 and q are always adjacent.
- If the graph has q edges then each graceful labeling must contains vertex label as q.
- The vertices having labels 0 and q are always adjacent.
- Graceful labeling graph may contains a triangle graph.
- The complementarity property for graceful labeling is satisfied. That is for a given graph with graceful labeling if we swap every vertex label q with $q-k$, the resulting labeling is also graceful since the edge labels will not have changed the end vertices of an edge with labels a and b become $q-a$ and $q-b$.

1.3.2 Harmonious labeling:

Graham and Sloane [33]

Definition: 4. *Let G be a graph with q edges. A function f is called as harmonious labeling of graph G if* $f: V(G) \to \{0,1,2,...,q-1\}$, *is injective and the induced function* $f^*: V(G) \to \{0,1,2,...,q\}$, *defined by* $f^*(e=uv) = (f(u)+f(v))(mod q)$ *is bijective.*
A graph which admits harmonious labeling is called a harmonious graphs.

Exa. $K_{1,7}$



$K_{1,7}$

Examples of graph having Harmonious Labeling.

- The Cycle $C_n (n \geq 3)$ is harmonious if and only if n is odd.
- All ladders except L_2 are harmonious.
- Friendship graph F_n is harmonious except $n \equiv 2\ mod 4$

- The fan graph f_n is harmonious.
- The graph g_n is harmonious $(n \geq 2)$

Chang, Hsu, and Rogers [12] and Grace [31], [32] have investigated subclasses of harmonious graphs. Chang et al. define an injective labeling f of a graph G with q vertices to be strongly c-harmonious if the vertex labels are from $\{0, 1, ..., q-1\}$ and the edge labels induced by $f(u) + f(v)$ for each edge (u, v) are $c, ..., c+q-1$.
Strongly 1-harmoinious labeling are more simply called strongly harmonious. Grace called such a labeling sequential.
Grace proved that caterpillars with a pedent edge, odd cycle with zero or more pendent edges, trees with α-labelings,wheels W_{2n+1} and P_n^2 are sequential.
Result

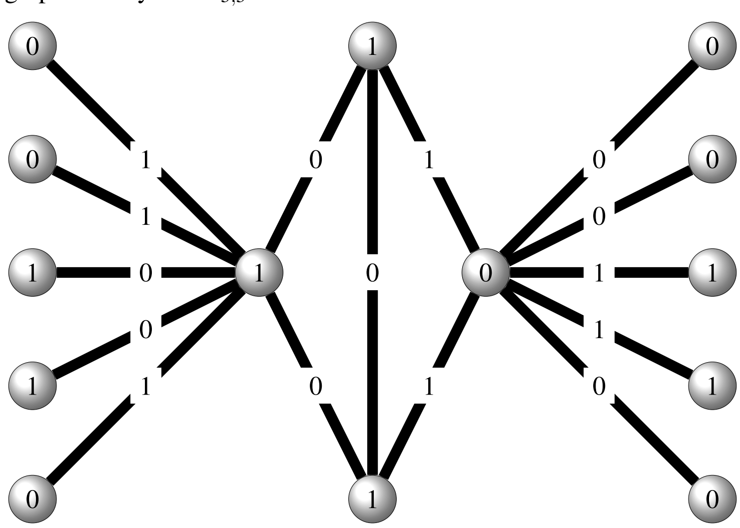
- Harmonious labeling is not unique.
- If f is a Harmonious labeling of any graph G with q edges, than $af(x) + b$ is also harmonious labeling of G. where a is invertible element of set q and b is any arbitrary element of q. (Set of integers modulo q).
- Any vertex in a harmonious graph can be assigned the label 0.
- Trees with exactly two vertices are assigned the same vertex label.
- Every tree is harmonious.
- Complete graph is harmonious if and only if $n \leq 4$.
- Let T be a harmonious labeled tree containing an edge $e = uv$ labeled as $f(u) + f(v)$ where v is a pendant vertex and $f(v)$ is the repeated vertex label. If w is any other vertex in G, we may delete edge uv and vertex v and replace them with a new vertex z and edge $e' = wz$ where z is labeled with $f(z) = f(u) + f(v) - f(y)$.
- The Peterson graph is Harmonious.
- Wheel graph $W_n = C_n + K_1$ is harmonious.
- $K_n^{(2)}$ is harmonious if $n = 4$ but not harmonious if n is odd or $n = 6$.

1.3.3 Cordial Labeling :

Cahit [9] has introduced a variation of both graceful and harmonious labeling.

Definition: 5. *A function $f: V(G) \to \{0, 1\}$ is cordial labeling of a graph G if each edge (u, v) is assigned the label $|f(u) - f(v)|$ such that the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and similarly the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.*

Exa. Cordial labeling graph of Jelly Fish $J_{5,5}$



$J_{5,5}$

Examples of standard graph having Cordial Labeling.

- Every tree is cordial[Cahit]
- The complete graph K_n is cordial if and only if $n \leq 3$
- The complete bipartite graph $K_{m,n}$ is cordial for all m and n
- The friendship graph $C_3^{(t)}$ (i.e. the one-point union of t 3-cycles) is cordial if and only if $t \not\equiv 2 \pmod 4$.
- All fan graph f_n are coordial.
- The wheel W_n is cordial if and only if $n \not\equiv 3 \pmod 4$.
- Maximal outerplanar graphs are cordial.
- An Eulerian graph is not cordial if its size is congruent to 2 (mod 4).
- Kuo, Chang, and Kwong determine all the values of m and n for which mK_n is cordial.
- Every Skolem-graceful graph is cordial.[Youssef-3242]
- A 3-regular graph of order n is cordial if and only if $n \not\equiv 4 \pmod 8$.[Liu and Zhu 1815]

Types of Cordial Labeling.

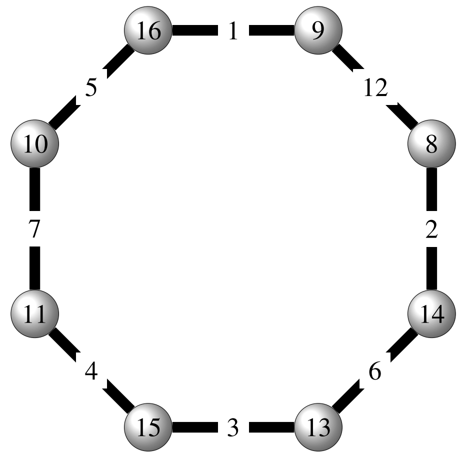
- Difference cordial labeling graph.
- Edge product cordial labeling graph.
- Prime cordial labeling graph.
- Planar grid cordial labeling graph.
- Context of Duplication cordial graphs.
- Second order cordial labeling graph.
- SET cordial labeling graph.
- Integer cordial labeling graph.
- Signed product cordial labeling graph.
- Mean cordial labeling graph.
- Geometric Mean cordial labeling graph.
- Harmonic Mean cordial labeling graph.

1.3.4 Magic labeling of a graph:

Using the concept of magic squares in number theory, magic labeling had been introduced by Sedlacek in 1963 [41].
a)Vertex magic labeling of a graph:

Definition: 6. *A (p,q) connected graph $G = (V,E)$ is said to be vertex magic if there exists a bijection $f : V(G) \bigcup E(G) \to \{1,2,...,p+q\}$, such that for all vertices of G, the sum of label on a vertex and the labels of its incident edges is constant such a bijection is called vertex magic labeling of G. A graph that admits vertex magic labeling is called a vertex magic graph.*

Exa.Vertex magic labeling of cycle graph C_8 with magic constant K=22.

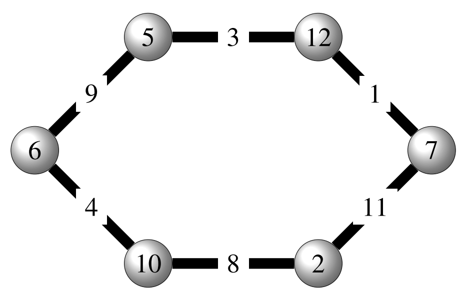


Cycle C_8

b) Edge magic labeling of a graph

Definition: 7. *A (p,q) connected graph $G=(V,E)$ is said to be edge magic if there exists a bijection $f: V(G)\bigcup E(G) \rightarrow \{1,2,...,p+q\}$, such that for all edges uv of G, $f(u)+f(v)+f(uv)$ is constant such a bijection is called magic labeling of G.This is also called edge magic labeling of G. A graph that admits edge magic labeling is called an edge magic graph. s a connected graph semi-magic if there is a labeling of edges with integers such that for each vertex v the sum of all edges incident with v is the same for all v.*

Edge magic labeling of cycle graph C_6 with magic constant K=20.



Cycle C_6

Examples of a graph having Magic Labeling.

- The complete graph K_n is magic for $n=2$ and $n \geq 5$
- The complete bipartite graph $K_{n,n}$ is magic for all $n \geq 3$
- The friendship graph $C_3^{(t)}$ (i.e. the one-point union of t 3-cycles) is cordial if and only if $t \not\equiv 2 \pmod 4$.
- The fan graph f_n are magic if and only if n is odd and $n \geq 3$.
- The wheel graph W_n are magic for $n \geq 4$ and W_n with one spoke deleted is magic for n=4 and $n \geq 6$.
- A connected graph with p vertices and q edges other than P_2 exits and is magic if and only if $5p/4 < q \leq p(p-1)/2$.

Types of Magic Labeling.

- Semi-Magic Labeling
- Super-Magic Labeling
- Anti-magic labeling
- Prime-magic labeling

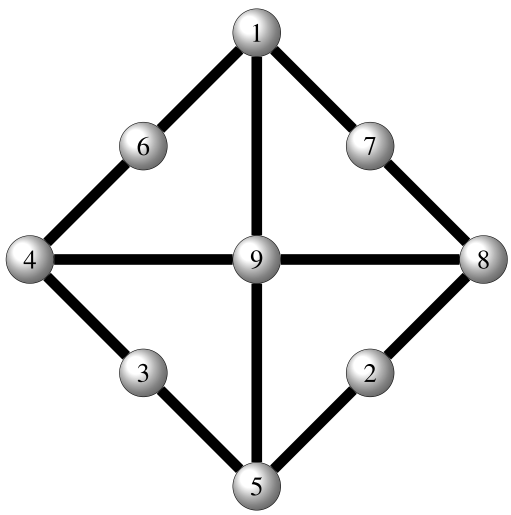
- H-magic labeling
- Magic labeling of type (a,b,c)
- Sigma labeling/ Distance-magic labeling

1.3.5 Radio labeling:

In 2001 Chartrand, Erwin, Zhang, and Harary [15] were motivated by regulations for channel assignments of FM radio stations to introduce radio labeling of graphs.

Definition: 8. *A radio labeling of a connected graph G is an injection c from the vertices of G to the natural numbers such that $d(u,v)+|c(u)-c(v)| \geq 1+diam(G)$ for every two distinct vertices u and v of G.*

The radio number of c, $rn(c)$, is the maximum number assigned to any vertex of G. The radio number of G, $rn(G)$, is the minimum value of $rn(c)$ taken over all radio labelings c of G.
Exa. Radio mean labeling of a graph



Cycle C_8

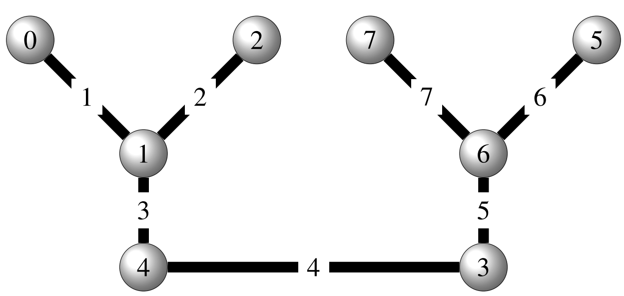
Examples of Radio Labeling graph.

- The radio mean number of the sunflower graph SF_n is its order.
- The radio mean number of a Helm H_n is $2n+1$.
- The radio mean number of a gear graph G_n is $2n+1$.

1.3.6 Mean labeling of graphs:

The notion of mean labeling of graphs has been introduced by Somasundaram and Ponraj [40].

Definition: 9. *A graph G with p vertices and q edges is called a mean graph if there is an injective function f from the vertices of G to $\{0,1,2,-----,q\}$ such that when each edge (u,v) is labeled with $(f(u)+f(v))/2$ if $f(u)+f(v)$ is even and $(f(u)+f(v)+1)/2$ if $f(u)+f(v)$ is odd, then the resulting edge labels are distinct.*



$F_{2,4}$

Examples of Mean Labeling graph.

- The path graph P_n,the cycle graph C_n, the bipartite graph $K_{2,n}$, triangular snakes, quadrilateral snakes etc. all are mean graphs..
- K_n if and only if $n < 3$
- $K_{1,n}$ if and only if $n < 3$
- The friendship graph $C_3^{(t)}$ if and only if $t < 2$
- Bistars $B_{m,n}(m > n)$ if and only if $m < n+2$

Types of Mean Labeling graph.

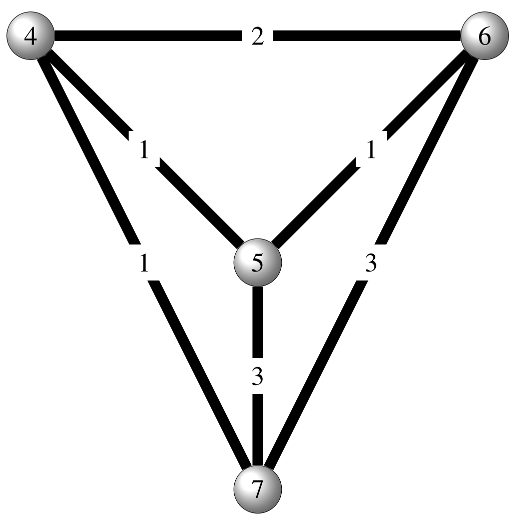
- Vertex even and odd mean labeling graph.
- Super Mean labeling graph.

1.3.7 Irregular labeling of graph:

In 1988 Chartrand, Jacobson, Lehel, Oellermann, Ruiz, and Saba [14] defined an irregular labeling of a graph.

Definition: 10. *An irregular labeling of a graph G with no isolated vertices as an assignment of positive integer weights to the edges of G in such a way that the sums of the weights of the edges at each vertex are distinct.*

The minimum of the largest weight of an edge over all irregular labeling is called the irregularity strength $s(G)$ of G. If no such weight exists, $s(G) = \infty$.
Chartrand et al. gave a lower bound for $s(mKn)$.
Faudree, Jacobson, and Lehel [784] gave an upper bound for$s(mK_n)$ when $n \geq 5$ and proved that for graphs G with $\delta(G) \geq n-2 \geq 1, s(G) \leq 3$, where $\delta(G)$ is the minimum degree of G .
They also proved that if G has order n and $\delta(G) = n-t$ and $1 \leq t \leq \sqrt{n/18}), s(G) \leq 3$.
Aigner and Triesch proved $s(G) \leq n+1$ for any graph G with $n \geq 4$ vertices for which $s(G)$ is finite.

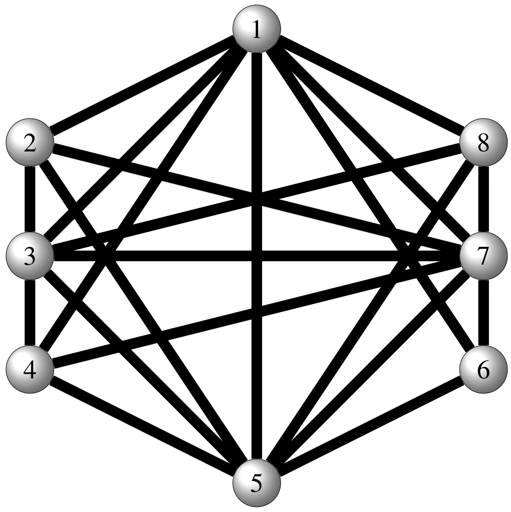


An irregular 3-labeling of the wheel W_3

1.3.8 Prime labeling of graph:

Prime labeling originated with Entringer and was introduced by Tout, Dabboucy, and Howalla.[20]

Definition: 11. *Let G be a graph with p vertices and q edges. A bijection $f: V(G) \rightarrow \{1,2,...,p\}$ is called as prime labeling of graph G if for each edge(u,v), gcd $(f(u), f(v)) = 1$.*



R_8

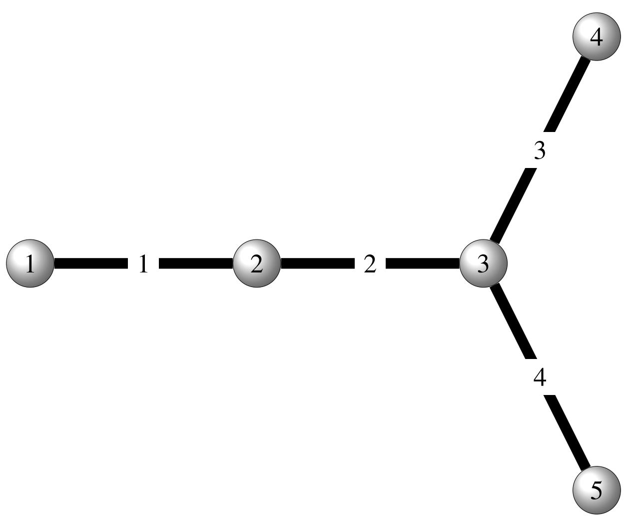
Examples of Prime Labeling of graph.

- all caterpillars with maximum degree at most 5 are prime
- Paths, stars, complete binary trees, spiders etc.

1.3.9 Power Mean Labeling of graph:

Mercy P.[37]

Definition: 12. *A graph $G = (V,E)$ with p vertices and q edges is said to be a Power Mean Graph if it is possible to label the vertices x/inV with distinct labels $f(x)$ from $1,2,3,...,q+1$ is such a way that when each edge $e = uv$ is labeled with*
$f(e = uv) = \lfloor (f(u)^{f(v)} * f(v)^{f(u)})^{\frac{1}{f(u)+f(v)}} \rfloor$
$f(e = uv) = \lceil (f(u)^{f(v)} * f(v)^{f(u)})^{\frac{1}{f(u)+f(v)}} \rceil$
then the resulting edge labels are distinct and are from $\{1,2,3,...,q\}$ .In this case f in called Power mean labeling of G .



Examples of Combination Labeling of a graph.

- Tadpoles $T(n,k)$ is a Power mean graph.
- C_n is a combination graph for $n > 3$.
- $K_{n,n}$ is a combination graph if and only if $n \leq 2$.

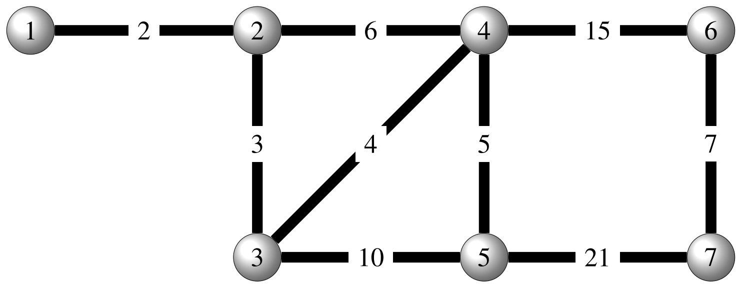
Result:

- The graph obtained by joining any two cycles C_m and C_n by a pathP_n is a Power mean graph
- For $m \geq> 2$, $T(P_m)$ is a Power mean graph.
- For $n \geq> 2$ Subdivision of any path is a Power mean graph.
- For $n \geq> 3$, a subdivision of any cycle C_n is a Power mean graph.

1.3.10 Combination and Permutation labeling of graph:

Hegde and Shetty [20] **a)Combination labeling of graph:**

Definition: 13. *A $f : V(G) \to \{1, 2, ..., p\}$ is called as combination labeling of graph G if each edge (u, v) is assigned the label $(f(u))!/[f(u) - f(v)]!(f(v))!$ where $f(u) > f(v)$, which are all distinct.*

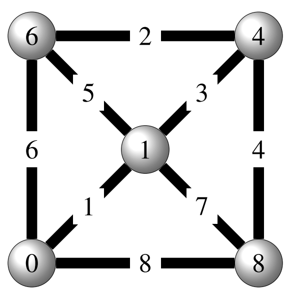


Examples of Combination Labeling of a graph.

- K_n is a combination graph if and only if $n \leq 5$
- C_n is a combination graph for $n > 3$.
- $K_{n,n}$ is a combination graph if and only if $n \leq 2$.

2. Let G = (V; E) be a caterpillar with extended central path P consisting of p vertices. If G has at least 3p 6 vertices, then T is a combination **b) Permutation Labeling of graph:**

Definition: 14. *A bijection $f : V(G) \to \{1, 2, ..., p\}$ is called as as a permutation labeling of graph G if each edge (u, v) is assigned the label $(f(u))!/[f(u) - f(v)]!$ Where $f(u) > f(v)$, which are all distinct.*



grap **Examples of Permutation Labeling of a graph.**

- K_n is a permutation graph if and only if $n \leq 5$

Result:

- If G is a permutation graph then $G - e$(= the graphobtained from G by deleting the edge e) is also a permutation graph.
- The graph k-wheel admits a permutation labeling for every integer $n \geq> 2$.
- The graph k-fan $F_{n,k}$ admits a permutation labeling.
- The gear graph G_{2n} has a permutation labeling.

If G is a permutation graph then G - e(= the graphobtained from G by deleting the edge e) is also a permutation graph.

1.3.11 Triangular sum labeling of graph:

Hegade and Shankaran[36]

Definition: 15. *A labeling of the graph with q edges is called a triangular sum labeling if the vertices can be assigned distinct non-negative integers in such a way that, when an edge whose vertices are labeled i and j is labeled with the value $i+j$, the edges labels are $k(k+1)/2 : k = 1,2,---q$.*

P_6

Examples of Triangular sum Labeling of graph.

- Paths, stars, complete n-ary trees, and trees obtained from a star by replacing each edge of the star by a path all have triangular sum labeling.
- K_n has a triangular sum labeling if and only if n is 1 or 2

References

[1] M. E. Abdel-Aal, New classes of odd graceful graphs, Internat. J. Appl. Graph Theory in Wireless ad hoc Networks and Sensor Networks, 5(2) June 2013.

[2] J. Abrham and A. Kotzig, Extensions of graceful valuations of 2-regular graphs consisting of 4-gons, Ars Combin., 32 (1991) 257-262.

[3] B. D. Acharya, K. A. Germina, K. Abhishek, and P. J. Slater, Some new results on set-graceful and set-sequential graphs, Combin. System Sci., 37(2-4) (2012) 229-239

[4] B. D. Acharya, S. B. Rao, T. Singh, and V. Parameswaran, Neighborhood magic graphs, 2004, unpublished.

[5] B. D. Acharya and S. M. Hegde, Strongly indexable graphs, Discrete Math., 93 (1991) 123-129.

[6] V. Ajitha, S.Arumugum and K.A. Gremina, On square sum graphs, AKCE International Journal of graphs and Combinatorics, Volume 6(1) (2009),1-10.

[7] B. Beavers, Golomb rulers and graceful graphs, http://webcourse.cs. technion.ac.il/236801/Spring2009/ho/WCFiles/Golomb_Rulers Gracefu_Graphs.pdf

[8] L. W. Beineke and S. M. Hegde, Strongly multiplicative graphs, Discuss. Math. Graph Theory, 21 (2001) 63-75.

[9] I. Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin., 23 (1987) 201-207.

[10] ahit, On harmonious tree labellings, Ars Combin., 41 (1995) 311-317.

[11] . Cattell, Graceful labellings of paths, Discrete Math., 307 (2007) 3161-3176.

[12] . J. Chang, D. F. Hsu, and D. G. Rogers, Additive variations on a graceful theme: some results on harmonious and other related graphs, Congr. Numer., 32 (1981) 181-197.

[13] G. Chartrand, S. M. Lee, and P. Zhang, Uniformly cordial graphs, Discrete Math., 306 (2006) 726-737.

[14] G. Chartrand, M. Jacobson, J. Lehel, O. Oellermann, S. Ruiz, and F. Saba, Irregular networks, Congr. Numer., 64 (1988) 187-192.

[15] G. Chartrand, D. Erwin, and P. Zhang, A graph labeling problem suggested by FM channel restrictions, Bull. Inst. Combin. Appl., 43 (2005) 43-57.

[16] G. Chartrand, D. Erwin, P. Zhang, and F. Harary, Radio labelings of graphs, Bull. Inst. Combin. Appl., 33 (2001) 77-85.

[17] A. Durai Baskar, S. Arockiaraj, and B. Rajendran, F-geometric mean labeling of some chain graphs and thorn graphs, Kragujevac J. Math. 37(1) (2013) 163-186.

[18] H. L. Fu and K. C. Huang, On prime labelling, Discrete Math., 127 (1994) 181-186.

[19] Y. Fukuchi, Edge-magic labelings of wheel graphs, Tokyo J. Math., 24 (2001) 153-167.

[20] Gallian, JA 2010, ‘A dynamic survey of graph labeling’, The Electronic Journal of Combinatorics, vol.17 (DS6).

[21] J. A. Gallian, Labeling prisms and prism related graphs, Congr. Numer., 59 (1989) 89-100.

[22] J. A. Gallian, A survey: recent results, conjectures and open problems on labelinggraphs, J. Graph Theory, 13 (1989) 491-504.

[23] J. A. Gallian, Open problems in grid labeling, Amer. Math. Monthly, 97 (1990)133-135.

[24] J. A. Gallian, A guide to the graph labeling zoo, Discrete Appl. Math., 49 (1994)213-229.

[25] J. A. Gallian and D. S. Jungreis, Labeling books, Scientia, 1 (1988) 53-57.

[26] J. A. Gallian, J. Prout, and S. Winters, Graceful and harmonious labelings of prisms and related graphs, Ars Combin., 34 (1992) 213-222.

[27] J. A. Gallian and L. A. Schoenhard, Even harmonious graphs, AKCE J. Graphs Combin., 11(1) (2014) 27-49.

[28] J. A. Gallian and D. Stewart, Properly even harmonious labelings of disconnected graphs, AKCE J. Graphs Combin., 12(2-3) (2015) 193-203.

[29] J. A. Gallian and D. Stewart, Even harmonious labelings of disjoint graphs with a small component, AKCE J. Graphs Combin., 12(2-3) (2015) 204-215.

[30] J. A. Gallian and D. Stewart, Even harmonious labelings of disjoint unions with even sequential graphs, J. Graph Labeling, 1(1) (2015) 1-10.

[31] T. Grace, Graceful, Harmonious, and Sequential Graphs, Ph. D. Thesis, University Illinois at Chicago Circle, 1982.

[32] T. Grace, On sequential labelings of graphs, J. Graph Theory, 7 (1983) 195-201

[33] R. L. Graham and N. J. A. Sloane, On additive bases and harmonious graphs, SIAM J. Alg. Discrete Methods, 1 (1980) 382-404.

[34] Harary, F 1988, ‘Graph Theory’, Narosa publishing House Reading, New Delhi.

[35] N. Hartsfield and G. Ringel, Pearls in Graph Theory, Academic Press, San Diego, 1990.

[36] S. M. Hegde and P. Shankaran, On triangular sum labelings of graphs, in Labeling of Discrete Structures and Applications, Narosa Publishing House, New Delhi,2008, 109-115.

[37] Mercy P., Power mean series, Ph.D. Thesis.

[38] M. Mollard, C. Payan, and S. Shixin, Graceful problems, Seventh Hungarian Colloquium on Finite and Infinite.

[39] M. A. Razzaq, S. T. Rizvi, and K. Ali, H-groupmagic total labelings of families of fan graphs, Util. Math., 108 (2018) 213219

[40] Rosa, A 1967, ‘On certain valuations of the vertices of a graphs’, Theory of graphs, (Internet, symposium, Rome, July 1966), Gorden and Breach, N.Y.and Dunod Paris, pp.349–355.

[41] S. S. Sandhya, S. Somasundaram, R. Ponraj, Some results on harmonic mean graphs, Int. J. Contemp. Math. Sci., 7 (2012) 197-208.

[42] J. Sedlacek, Problem 27, in Theory of Graphs and its Applications, Proc. Symposium Smolenice, June, (1963) 163.

[43] S. K. Vaidya and C. M. Barasara, Product cordial labeling of line graph of some graphs, Kragujevac J. Math., 40(2) (2016) 290-297.

[44] S. Somasundaram and R. Ponraj, Mean labelings of graphs, Natl. Acad, Sci. Let., 26 (2003) 210-213.

[45] S. Somasundaram, P. Vidhyarani and R. Ponraj, Geometric mean labelings of graphs, Bull. Pure Appl. Sci,, 30E(2) (2011) 153-160.