**Neutrosophic chaotic b-closed set in neutrosophic chaotic topological space**

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**Abstract:** This article focuses on devising the novel idea of framing a b-open set in neutrosophic chaotic topological space. We further devote this article to the study of the properties posed by this newly developed set suitable examples are provided as and when required.

**Keywords:** neutrosophic chaotic set**,** neutrosophic chaotic topological space, neutrosophic chaotic open sets, neutrosophic chaotic b-closed set.

**1. Introduction**

Zadeh [12] introduced fuzzy sets in 1965, allowing elements to have varying degrees of membership in the set. The real unit interval [0, 1] is where the membership degrees are found. Developed by Atnassov [1] in 1983, the intuitionistic fuzzy set (IFS) permits both membership and non-membership to the elements. In 1998, Smarandache [9] introduced the neutrosophic set by adding a single extra component to the IFS set. The truth, indeterminacy, and falsity membership functions are the three components of the neutrosophic set, in that order. This neutrosophic set aids in the efficient handling of the ambiguous and inconsistent data. P.ishwarya and K. Bagherathi introduced the concept of neutrosophic semi-open sets in neutrosophic topological space in 2016[4]. The concept of neutrosophic pre-closed in neutrosophic topological spaces and neutrosophic pre-open sets was introduced in 2017 by V. Venkateswara Rao and Y. Srinivasa Rao [11]. In 2018 P. Evanzalin Ebenanjar et al[3] introduced the concept of neutrosophic b-open sets in neutrosophic topological space The concept of chaotic function in general metric space was introduced by R.L.Devaney[2]. It has many applications in trafficforecasting, animation, computer graphics, medical field, image processing, etc. T.Thrivikraman and P.B. Vinod Kumar[10] defined chaos and fractals in general topological spaces. The idea of the fuzzy chaotic set was introduced by R.Malathi and M.K. Uma[5] in 2018. In [6] we introduced the concept of neutrosophic chaotic continuous functions. In this we extend the neutrosophic b-closed set in neutosophic chaotic topological space.

2. Preliminaries

**2.1 Definition[9]** Let X be a universe. A Neutrosophic set ($NS$) A on X can be defined as follows:

$$A= \{<x,T\_{A}(x),I\_{A}(x),F\_{A}(x)>:x\in X\}$$

Where $T\_{A },I\_{A},F\_{A}:U\rightarrow \left[0,1\right]$ and $ 0\leq T\_{A}\left(x\right)+I\_{A}\left(x\right)+F\_{A }\left(x\right)\leq 3$

Here,$T\_{A}(x)$ is the degree of membership, $I\_{A}(x)$ is the degree of inderminancy and $F\_{A}(x)$ is the degree of non-membership.

**2.2 Deﬁnition [9]** Let X be a non empty set, M = <x,MT,MI,MF> and V = <x,VT,VI,VF> be neutrosophic sets on X, and let {Ai : i∈ J} be an arbitrary family of neutrosophic sets in X, where Mi = <x,MT, MI, MF>

1. M V if and only if MT VT, MI VI and MF VF

1. M = V if and only if M V and V M.

1. $\overbar{M}$= <x,MF,1-MI,MT>
2. M∩V=<x,MTVT,MIVI,MFVF>

1. M∪V=<x,MTVT,MIVI,MFVF>

1. ∪Mi = <x,MiT,MiI,MiF>

1. ∩Mi = <x,MiT,MiI,MiF>

1. M − V = M $\overbar{V}$.

1. 0N = <x,0,1,1>; 1N = <x,1,0,0>.

**2.3 Deﬁnition [8]** A neutrosophic topology ($NT$ for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) 0N, 1N ∈ τ.

(ii) G1$∩$G2 ∈ τ for any G1,G2∈ τ.

(iii) $∪$Gi∈ τ for any arbitrary family {Gi :i∈J} ⊆ τ.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set(NOS for short) in X. The complement A of a neutrosophic open set A is called a neutrosophic closed set (NCS for short) in X.

**2.4 Deﬁnition [8]** Let (X, τ) be a neutrosophic topological space and A = <X,AT,AI,AF> be a set in X. Then the closure and interior of A are deﬁned by

Ncl(A) = $∩${K : K is a neutrosophic closed set in X and A K},

Nint(A) = $∪${G : G is a neutrosophic open set in X and G A}.

**2.5 Deﬁnition[7]** Let X be a nonempty set and f : X → X be any mapping. Let $α$ be any neutrosophic set in X. The neutrosophic orbit Of($α$) of $α$ under the mapping f is defined as OfT($α$) = {$ α$,f1($α$),f2($α$),...fn($α$)}, OfI($α$) = {$ α$,f1($α$),f2($α$),...fn($α$)} , OfF($α$) = {$ α$,f1($α$),f2($α$),...fn($α$)} for $α$ ∈ X and n ∈ Z+.

**2.6 Definition[7]** Let X be a nonempty set and let f : X→ X be any mapping. The neutrosophic orbit set of $α$ under the mapping f is defined as NOf($α$) = <$ α$,OfT($α$),OfI($α$),OfF($α$)> for $α$ ∈ X, where OfT($α$)= {$ α∧$f1($α$)$ ∧$f2($α$)$ ∧$...$ ∧$fn($α$)}, OfI($α$)= {$ α∨$f1($α$)$ ∨$f2($α$)$ ∨$...$ ∨$fn($α$)}, OfF($α$)= {$ α∨$f1($α$)$ ∨$f2($α$)$ ∨$...$ ∨$fn($α$)}.

**2.7 Definition[6]** Let X be a nonempty set and let f : X → X be any mapping. Then a neutrosophic set of X is called neutrosophic periodic set with respect to f if fn(𝛶) = 𝛶, for some n ∈ Z+. smallest of these n is called neutrosophic periodic of X.

**2.8 Definition [6]** Let (X, T) be a neutrosophic topological space. Let f : X → X be any mapping. The neutrosophic periodic set with respect to f which is in neutrosophic topology τ is called neutrosophic periodic open set with respect to f. Its complement is called a neutrosophic periodic closed set with respect to f.

**2.9 Notation** P = $∩${neutrosophic periodic open sets with respect to f }.

**2.10 Definition [6]** Let (X, τ) be a neutrosophic topological space and 𝜆 ∈ NF(X) (Where NF(X) is a collection of all nonempty neutrosophic compact subsets of X). Let f : X → X be any mapping. Then f is neutrosophic chaotic with respect to 𝜆 if

(i) cl NOf (𝜆) = 1,

(ii) P is neutrosophic dense.

**2.11 Example** Let X = {a, b, c}. Define $τ$ = {0, 1, 𝜇1, 𝜇2, 𝜇3, 𝜇4} where 𝜇1, 𝜇2, 𝜇3, 𝜇4 : X → [0,1] are defined as 𝜇1 (a) =<a,0.4,0.3,0.6>, 𝜇1 (b) =<b,0.8,0.7,0.2>, 𝜇1 (c) = <c,0.4,0.3,0.6>, 𝜇2 (a) = <a,0.4,0.3,0.6>, 𝜇2(b) =<b, 0.8,0.2,0.2>, 𝜇2 (c) = <c,0.5,0.2,0.5> 𝜇3 (a)=<a,0.8,0.2,0.2>, 𝜇3 (b)=<b,0.8,0.7,0.2>, 𝜇3 (c)=<c,0.6,0.2,0.4>, 𝜇4 (a)=<a,0.9,0.2,0.1>, 𝜇4 (b)=<a,0.8,0.7,0.2>, 𝜇4 (c)=<a,0.9,0.2,0.1>,

Let 𝜆: X → I be defined as 𝜆 (a) =<a,0.3,0.2,0.7> (b) = <b,0.6,0.5,0.4> 𝜆(c) =<c, 0.3,0.2,0.7>. Define f : X → X as f(a) = b, f(b) = c, f(c) = a. The neutrosophic orbit set of 𝜆 under the mapping f is defined as NOf (𝜆) = 𝜆 $∩$f(𝜆) $∩$ f2(𝜆) $∩$…$⇒$ NOf (𝜆)(a) = <a,0.3,0.2,0.7>, NOf (𝜆)(b) = <b,0.6,0.5,0.4>, NOf (𝜆)(c) =<c, 0.3,0.2,0.7>. Therefore cl(NOf (𝜆)) = 1. Here P(a) = <a,0.4,0.3,0.6>, P(b) =<b, 0.8,0.7,0.2> ,P(c) = <c,0.4,0.3,0.6> and cl (P) is neutrosophic dense. Hence f is neutrosophic chaotic with respect to 𝜆.

**2.12 Notation** (i) NC (𝜆) = {f: X → X **/** f is neutrosophic chaotic with respect to 𝜆}.

(ii) NCH(𝜆) = {𝜆∈ NF(X) **/** NC(𝜆) ≠ 𝜙}.

**2.13 Definition** A neutrosophic topological space (X, τ) is called a neutrosophic chaos space if NCH (𝜆) ≠ 𝜙. If (X, τ) is neutrosophic chaos space then the element of the NCH(X) are called chaotic sets in X.

**3. Neutrosophic chaotic b-closed set in neutrosophic chaotic topological space**

**3.1 Definition:** Let (X, τ) be a neutrosophic (neu-) chaos space. Let $C$ be the collection of neu- chaotic sets in X satisfying the following conditions:

1. 0NC,1NC $\in C ,$
2. If A1, A2 $\in C$ ,then A1$∩$A2$\in C$
3. If {Aj:j$\in $J}$⊂C$, then $\bigcup\_{j\in J}^{}A\_{j}\in C.$

Then $C$ is called the neu- chaotic topological space in X. The triple (X, τ,$ C$) is called a neu- chaotic topological space. The element of $C$ are called neu- chaotic open sets. The complement of neu- chaotic open set is called neu- chaotic closed set.

**3.2 Example:** Let X = {a, b, c}. Define $τ$ = {0, 1, 𝜇1, 𝜇2, 𝜇3, 𝜇4} where 𝜇1, 𝜇2, 𝜇3, 𝜇4 : X → [0,1] are such that 𝜇1 (a) =<a,0.4,0.3,0.6>, 𝜇1 (b) =<b,0.8,0.7,0.2>, 𝜇1 (c) = <c,0.4,0.3,0.6>, 𝜇2 (a) = <a,0.4,0.3,0.6>, 𝜇2(b) =<b, 0.8,0.2,0.2>, 𝜇2 (c) = <c,0.5,0.2,0.5> 𝜇3 (a)=<a,0.8,0.2,0.2>, 𝜇3 (b)=<b,0.8,0.7,0.2>, 𝜇3 (c)=<c,0.6,0.2,0.4>, 𝜇4 (a)=<a,0.9,0.2,0.1>, 𝜇4 (b)=<a,0.8,0.7,0.2>, 𝜇4 (c)=<a,0.9,0.2,0.1>.Let $C=\{0, 1, $𝜇1, 𝜇2, 𝜇3}.Clearly (X,T,$ C$) is called neu- chaotic topological space.

**3.3 Definition:** Let(X,T,$ C$) be neu- chaotic topological space and A=<x, TA,IA,FA> be neu- chaotic set in X. then the neu- chaotic interior and neu- chaotic closure are defined by

1. intNC(A)= $\bigcup\_{}^{}\left\{{M}/{M}is a NCOS in X and M⊆A\right\},$
2. clNC(A)= $\bigcap\_{}^{}\left\{{N}/{N}is a NCCS in X and A⊆N\right\}.$

Note that for any neu- chaotic set A in (X,T,$ C$) , we have clNC(Ac)=(intNC(A))c and intNC(Ac)=(clNC (A))c.

It can be also shown that clNC(A) is NCCS and intNC(A) is NVOS in X.

1. A is NCCS in X if and only if clNC(A)=A.
2. A is NCOS in X if and only if intNC(A)=A.

**3.4 Proposition:** Let A be any neu- chaotic set in X. Then

1. intNC(1NC-A) = 1NC-(clNC(A)) and
2. clNC(1NC-A) = 1NC-(intNC(A))

Proof: (i) By definition clNC(A) = $\bigcap\_{}^{}\left\{{N}/{N}is a NCCS in X and A⊆N\right\}.$

1NC-(clNC(A)) = 1NC-$\bigcap\_{}^{}\left\{{N}/{N}is a NCCS in X and A⊆N\right\}$

 = $\bigcup\_{}^{}\left\{{1\_{NC}-N}/{N}is a NCCS in X and A⊆N\right\}$

 = $\bigcup\_{}^{}\left\{{M}/{M}is a NCOS in X and M⊆1\_{NC}-A\right\}$

 = intNC(1NC-A)

(ii) The proof is similar to (i).

**3.5 Proposition:** Let(X, τ,$ C$) beneu- chaotic topological space and A, B be neu-chaotic sets in X. Then the following properties hold:

1. intNC(A) $⊆$ A
2. A $⊆$ clNC(A)
3. A $⊆$ B $⇒$ intNC(A) $⊆$ intNC(B)
4. A $⊆$ B $⇒$ clNC(A) $⊆$ clNC(B)
5. intNC(intNC(A))=intNC(A)
6. clNC(clNC(A))=clNC(A)
7. intNC(A$ ∩$ B) = intNC(A)$ ∩$ intNC(B)
8. clNC(A$ ∪$ B) = clNC(A)$ ∪$ clNC(B)
9. intNC(1NC) = 1NC
10. clNC(0NC) = 0NC

 Proof:

(a),(c) and (i) are obvious, (e) follows from (a)

g) From intNC(A$∩$B) $⊆$ (A) and intNC(A$∩$B) $⊆$ (B) we obtain intNC(A$ ∩$ B) $⊆$ intNC(A)$ ∩$ intNC(B). On the other hand, from the facts intNC(A) $⊆$ A and intNC(B) $⊆$ B $⇒$ intNC(A)$ ∩$ intNC(B) $⊆$ A$ ∩$ B and intNC(A)$ ∩$ intNC(B)$ \in C$ we see that intNC(A)$ ∩$ intNC(B)$ ⊆$ intNC(A$∩$B), for which we obtain the required result.

(a)-(j) They can be easily deduced from (a)-(i).

**3.6 Definition.** Let A be a neu- chaotic set of a neu- chaotic topological space. Then A is said to be neu- chaotic pre open [NCPO]set of X if there exists a neu- chaotic open set NCO such that NCOANCO(clNC(A)).

**3.7 Definition:** A neu- chaotic set M=<x, TM,IM,FM> in neutrosophic chaotic topological space (X, τ,$ C$) is said to be

1. A neu- chaotic pre- open set if MintNC(clNC(M)) and neu- chaotic pre-closed set if clNC(intNC(M))M.
2. A neu- chaotic α-open set if MintNC(clNC(Nint(M))) and neu- chaotic α-closed set if clNC(intNC(clNC(M)))M.
3. neu- chaotic semi-open set MclNC(intNC(M)) and neu- chaotic semi-closed set if intNC(clNC(M))M.
4. neu- chaotic b-open set if MintNC(clNC(M))clNC(intNC(M)) and neu- chaotic b-closed set intNC(clNC(M))clNC(intNC(M))M.
5. a neu- chaotic $β$-open set, if MclNC(intNC(clNC(M))) and neu- chaotic $β$-closed set if intNC(clNC(intNC(M)))M.
6. neu- chaotic regular open set if M =intNC(clNC(AM)) and neu- chaotic regular closed set, if M = clNC(intNC(M)).

**3.8 Definition** Let (X, τ,$ C$) be a neu- chaotic topological space and M=<x, TM,IM,FM> be a NCS in X. The neu- chaotic b interior of A and denoted by bintNC(M) is defined to be the union of all neu- chaotic b-open sets of X which are contained in M. The intersection of all neu- chaotic b-closed sets containing M is called the neu- b-closure of M and is denoted by bclNC(M).

1. bintNC(M) = $\bigcup\_{}^{}\left\{{U}/{U}is a NCbOS in X and U⊆M\right\},$
2. bclNC(M) = $\bigcap\_{}^{}\left\{{N}/{N}is a NCbCS in X and M⊆N\right\}.$

**3.9 Theorem** In a neu- chaotic topological space X

1. An arbitrary union of neu- chaotic b-open sets is a neu- chaotic b-open set.
2. An arbitrary intersection of neu- chaotic b-cosed sets is a neu- chaotic b-closed set.

Proof: (i) Let {$M\_{α}$} be a collection of neu- chaotic b-cosed sets. Then for each $α$, $M\_{α}⊆cl\_{NC}(int\_{NC}\left(M\_{α}\right))∪int\_{NC}\left(cl\_{NC}\left(M\_{α}\right)\right).$Now$ ∪M\_{α}⊆∪(cl\_{NC}(int\_{NC}\left(M\_{α}\right))∪$ $int\_{NC}\left(cl\_{NC}\left(M\_{α}\right)\right))⊆cl\_{NC}(int\_{NC}\left(∪M\_{α}\right))∪$ $int\_{NC}\left(cl\_{NC}\left(∪M\_{α}\right)\right)$. Thus $∪M\_{α}$ is a neu- chaotic b-open set.

(ii) Similarly by taking complements.

**3.10 Theorem.**

1. Every neu- chaotic open set in the neu- chaotic topological space in X is neu- chaotic pre-open set in X.
2. Every neu- chaotic pre-open set in the neu- chaotic topological spaces (X, τ,$ C$) is neu- chaotic b-open set in (X, τ,$ C$).
3. Every neu- chaotic semi-open set in the neu- chaotic topological spaces (X, τ,$ C$) is neu- chaotic b-open set in (X, τ,$ C$).
4. Every neu- chaotic -open set in the neu- chaotic topological spaces (X, τ,$ C$) is neu- chaotic b-open set in (X, τ,$ C$).
5. Every neu- chaotic regular-open set in the neu- chaotic topological spaces (X, τ,$ C$) is neu- chaotic b-open set in (X, τ,$ C$).
6. Every neu- chaotic -open set in the neu- chaotic topological spaces (X, τ,$ C$) is neu- chaotic b-open set in (X, τ,$ C$).

Proof: (i) Consider M be neu- chaotic open set in neu- chaotic topological space. Then M=intNC(M).Clearly MclNC(M) taking interior on both sides we get intNC(M)intNC(clNC(M)). Since M=intNC(M), MintNC(clNC(M)). A is a neu- chaotic pre-open set in X.

(ii) Assune M be neu- chaotic pre-open set in a neu- chaotic topological space. Then MintNC(clNC(M)) which implies MintNC(clNC(M))intNC(M) intNC (clNC(M))clNC(intNC M). Hence M is a neu- chaotic b-closed sets.

(iii) Consider M be neu- chaotic semi-open set in a neu- chaotic topological space. Then MclNC(intNC(M)) which implies MclNC(intNC(M))intNC(M) clNC(intNC(M))intNC(clNC(M)). Hence M is a neu- chaotic b-closed sets.

(iv) (v) and (vi) Proof is obvious from above Definition.

**3.11 Remark.** The converse of above theorem need not be true as shown by the following examples

**3.12 Example**  Let X={x1,x2}. Define $f:X\rightarrow X$ as f(x1)=x2, f(x2)=x1. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. Here 𝜇1 (x) =<(x1,0.5,0.6,0.4)( x2,0.3,0.2,0.5)>, 𝜇2 (x) = <(x1,0.5,0.6,0.4)( x2,0.3,0.2,0.5)> . Define A=<(x1,0.5,0.4,0.3)( x2,0.2,0.1,0.5)> Then the set A is neu- chaotic b-open set but not neu- chaotic regular open set. Since A = clNC(intNC(A))=1NC$\ne $A.

**3.13 Example.** Let X={x}. Define $f:X\rightarrow X$ as f(x)=x. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. Here 𝜇1 (x) =<x,0.3,0.5,0.8>, 𝜇2 (x) = <x,0.4,0.6,0.7>. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. A={<x,0.1, 0.3, 0.5>}. Then the set A is neu- chaotic b- open set AclNC(intNC(A))intNC(clNC(A)) 1Nc. but not neu- chaotic semi- open set. Since A⊈ clNC(intNC(A))⊈0NC.

**3.14 Example.** Let X={x}. Define $f:X\rightarrow X$ as f(x)=x. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. Here 𝜇1 (x) =<x,0.5,0.6,0.5>, 𝜇2 (x) = <x,0.4,0.7,0.8>. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. A={<x,0.4, 0.4, 0.5>}.Then the set A is neu- chaotic b- open set but not neu- pre- open set. Since A⊈intNC(N clNC (A))⊈<0.5, 0.6, 0.5>

**3.15 Example.** Let X={x}. Define $f:X\rightarrow X$ as f(x)=x. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. Here 𝜇1 (x) =<x,0.5,0.6,0.5>, 𝜇2 (x) = <x,0.4,0.7,0.8>. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. A={<x,0.4, 0.4, 0.5>}.Then the set A is neu- chaotic b- open set but not neu- $α$- open set. Since A⊈intNC(N clNC (intNC (A)))⊈<0.5, 0.6, 0.5>

**3.12 Example** Let X={x1,x2}. Define $f:X\rightarrow X$ as f(x1)=x2, f(x2)=x1. Let $C=\{0\_{NC}, 1\_{NC}, $𝜇1, 𝜇2} be neu- chaotic topology on X. Here 𝜇1 (x) =<(x1,0.5,0.6,0.5)( x2,0.3,0.2,0.5)>, 𝜇2 (x) = <(x1,0.4,0.8,0.5)( x2,0.2,0.3,0.6)> . Define A=<(x1,0.5,0.4,0.6)( x2,0.5,0.8,0.9)> Then the set A is neu- chaotic $β$-open set but not neu- chaotic b-open set. Since A ⊈ clNC(intNC(A))intNC(clNC(A)) ⊈ <(x1,0.4,0.7,0.8)( x2,0.4,0.5,0.6)>.

**3.11 Remark.** The diagrammatic representation of above theorem.

**3.15 Theorem:** Let A be a neu- chaotic set in neu- chaotic topological space .then

1. sclNC(A) = A∪ intNC (clNC(A)) and

sintNC(A) = A∩ clNC (intNC (A))

1. pclNC(A) = A ∪ clNC (intNC (A)) and

pintNC(A) =A ∩ intNC (clNC(A)).

Proof: (i) sclNC(A)$⊇$ intNC (clNC(sclNC(A))$ ⊇$ intNC (clNC(A)).

A $∪ $sclNC(A) = sclNC(A) $⊇$ A $∪$ intNC (clNC(A)).

So A $∪$ intNC (clNC(A))$⊆$ sclNC(A) -----------------(1)

Also A$⊆$ sclNC(A)

intNC (clNC(A))$⊆$ intNC (clNC(sclNC(A))$ ⊆$ sclNC(A).

A $∪$ intNC (clNC(A))$ ⊆$ sclNC(A)$ ∪ $A$⊆$ sclNC(A)------------------(2)

From (1) and (2), sclNC(A) = A∪ intNC (clNC(A)).

sintNC(A) = A $∩$ clNC (intNC(A)) can be proved by taking the complement of sclNC(A) = A∪ intNC (clNC(A)). This proves (i).

The proof for (ii) is analogous.

**3.16 Theorem:** Let A be a neu- chaotic set in neu-chaotic topological space .then

1. bclNC(A) = sclNC(A) $∩$ pclNC(A)
2. bintNC(A) = sintNC(A) $∩$ pintNC(A)

Proof: Since bclNC(A) is a neu- chaotic b-closed set.

We have bclNC(A) $⊇$ intNC(clNC(bclNC(A))$ ∩$ clNC(intNC(bclNC(A))$ ⊇$ intNC(clNC(A))$ ∩$ clNC(intNC(A)) and also bclNC(A) $⊇$ A$∪$intNC(clNC(A))$ ∩$ clNC(intNC(A))= sclNC(A) $∩$ pclNC(A). The reverse inclusion is clear. Therefore bintNC(A) = sintNC(A) $∩$ pintNC(A).

Analogously (ii) can be proved.

**3.17 Theorem:** Let A be a neu- chaotic set in neu- chaotic topological space. Then

(i) sclNC(sintNC(A)) = sintNC(A) ∪ intNC(clNC(intNC(A)))

(ii) sintNC(sclNC(A)) = sclNC(A) $∩$ clNC(intNC(clNC(A)))

Proof: We have sclNC(sintNC(A)) = sintNC(A) ∪ intNC(clNC(sintNC(A))) = sintNC(A) ∪ intNC (clNC[A $∩$clNC (intNC(A)]) $⊆$ sintNC(A) $∪$ intNC [clNC(A)$ ∩$ clNC (clNC (intNC(A)))] = sintNC(A) ∪ intNC [clNC (intNC (A))]

To establish the opposite inclusion we observe that,

sclNC (sintNC (A)) = sintNC(A)$ ∪$ intNC(clNC(sintNC(A)) $⊇$ sintNC(A) ∪ intNC(clNC(intNC (A))).

Therefore we have sclNC(sintNC(A)) = sintNC(A) ∪ intNC(clNC(intNC(A))).

This proves (i).

The proof for (ii) is analogous.

**3.18 Theorem**  Let A be a neu- chaotic set in neu-chaotic topological space. Then

(i) pclNC(pintNC(A)) = pintNC(A) ∪ clNC(intNC(A))

(ii) pintNC(pclNC(A)) = pclNC(A) $∩$ intNC(clNC(A))

Proof: We have pclNC(pintNC(A)) = pintNC(A) ∪ clNC(intNC(pintNC(A))) = pintNC (A) ∪ clNC(intNC [A $∩$ intNC(clNC (A))] = pintNC(A) ∪ clNC[intNC(A) $∩$ intNC(intNC(clNC(A)))] = pintNC(A) ∪ clNC (intNC(A))

To establish the opposite inclusion we observe that,

pclNC(pintNC(A)) = pintNC(A) ∪ intNC(clNC(pintNC(A))$ ⊇$ pintNC(A) ∪ intNC(clNC(intNC(A))).

Therefore we have pclNC(pintNC(A)) = pintNC(A) ∪ clNC(intNC(A))

This proves (i).

Analogously (ii) can be proved.

**3.19 Theorem**  Let (X, τ,$ C$) be a neu- chaotic topological space. If A is a neu- chaotic open set and B is a neu- chaotic b- open set in X. Then A$∩$B is a neu- chaotic b- open set in X.

Proof: Let A be a neu- chaotic open set and B is a neu- chaotic b- open set.

Now, M= A$∩$B = intNC(A) $∩$ bintNC(A)$ ⊆ $bintNC(A) $∩$ bintNC(A)= bintNC(A $∩$ B)= bintNC(M)

(i.e) M$⊆$ bintNC(M). But bintNC(M)$ ⊆$ M. Hence, M= bintNC(M). (i.e) M= A$∩$B is a neu- chaotic b- open set.

**3.20 Theorem**  Let (X, τ,$ C$) be a neu-chaotic topological space. If A is a neu- chaotic $α$- open set and B is a neu- chaotic b- open set in X. Then A$∩$B is a neu- chaotic b- open set in X.

Proof: Let A be a neu- chaotic $C$- open set and B is a neu- chaotic b- open set.

Now, M= A$∩$B = $α$intNC(A) $∩$ bintNC(A)$ ⊆ $bintNC(A) $∩$ bintNC(A)= bintNC(A $∩$ B)= bintNC(M)

(i.e) M$⊆$ bintNC(M). But bintNC(M)$ ⊆$ M. Hence, M= bintNC(M). (i.e) M= A$∩$B is a neu- chaotic b- open set.

**3.21 Theorem** If A be a subset of a space (X, τ,$ C$), then bintNC(bclNC(A))= bclNC(bintNC(A)).

Proof: LetA be a subset of a space (X, τ,$ C $), Now, bintNC(bclNC(A))=sintNC(bclNC(A))$∪$ pintNC(bclNC(A)) = bclNC(sintNC(A))$∪$ pintNC(bclNC(A)) = sclNC(sintNC(A))$∪$ pintNC(pclNC(A))--(1)

And bclNC(bintNC(A))=bclNC(sintNC (A) $∪$ pintNC(A)) = bclNC(sintNC(A))$∪$ bclNC(pintNC(A)) = sclNC(sintNC(A))$∪$ pintNC(pclNC(A))--------------(2)

Hence from (1) and (2) we get bintNC(bclNC(A))= bclNC(bintNC(A)).

Hence the theorem.

**4. Conclusion**

By introducing the aforementioned definitions into topological spaces that are chaotic and neutrosophic. By making this new idea more general, we can increase its reach. This would create new research opportunities within the current neutrosophic topological framework.

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