**SOLUTION OF SECOND KIND LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION VIA MOHAND DECOMPOSITION METHOD**

**\*1Sudhanshu Aggarwal and 2Shahida A.T.**

1Assistant Professor, Department of Mathematics, National Post Graduate College Barhalganj, Gorakhpur, Uttar Pradesh, India

Email: sudhanshu30187@gmail.com

2Assistant Professor, Department of Mathematics, MES Mampad College, Mampad, Kerala, India

Email: shahida@mesmampad.edu.in

\*Corresponding Author

**ABSTRACT:** This chapter presents the "Mohand decomposition method," a novel decomposition technique for figuring out the answer to the second kind linear Volterra integro-differential problem. Three numerical problems provide a detailed description and illustration of the procedure. The current approach is quite effective, according to the results, and it provides the answers without requiring laborious calculating effort.

**KEYWORDS:** Integral Transform; Mohand Transform; Inverse Mohand Transform; Convolution Theorem; Decomposition Method

**MATHEMATICS SUBJECT CLASSIFICATION:** 44A05, 44A35, 44A15, 45A05, 45J05.

**INTRODUCTION:** Integral and integro-differential equations are widely used in the development of mathematical models for the solutions of various problems, including those involving electric circuits, mechanical vibration, heat transfer, compartment problems, and bacterial growth [1-2]. In order to find the answers to issues in physics, chemical science, mathematics, mechanics, and medical science, researchers now apply a variety of integral transformations [3-22]. Utilizing Kamal, Mahgoub, Sadik, Aboodh, Mohand, Elzaki, Laplace-Carson, Laplace, Sawi, Sumudu, and Shehu transforms, researchers [23–35] were able to fully solve the first and second kinds of Volterra integro-differential equation problems. Comparative analyses of the Mohand and other transformations were conducted by Aggarwal and other researchers [36–41]. Aggarwal et al. [42-43] defined the Mohand transforms of Bessel’s and error functions in their study. Aggarwal and Gupta [44] developed the duality relations of Mohand and other various integral transforms.

This chapter's primary goal is to use the Mohand decomposition method to find the solution of the second kind linear Volterra integro-differential problem.

**DEFINITION OF MOHAND TRANSFORM**: The Mohand transform of the function for all is defined as [44]:

 where is the Mohand transform operator. Standard properties of Mohand transform and Mohand transform of useful mathematical functions are presented in Table: 1 and Table: 2 respectively (See Table: 1 & Table: 2).

**TABLE: 1 USEFUL PROPERTIES OF MOHAND TRANSFORM [42]**

|  |  |  |
| --- | --- | --- |
| S.N. | Name of Property | Mathematical Form |
| 1 | Linearity |  |
| 2 | Change of Scale |  |
| 3 | Shifting |  |
| 4 | First Derivative |  |
| 5 | Second Derivative |  |
| 6 | Third Derivative |  |
| 7 | nth Derivative |  |
| 8 | Convolution |  |

**Table: 2 MOHAND TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [9, 42-43]**

|  |  |  |
| --- | --- | --- |
| S.N. |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |

**INVERSE MOHAND TRANSFORM:** If then is called the inverse Mohand transform of .

Mathematically, it is represented as, where the operator is called the inverse Mohand transform operator. Inverse Mohand transform of useful mathematical functions are presented in Table: 3 (See Table: 3).

**TABLE: 3 INVERSE MOHAND TRANSFORMS OF USEFUL MATHEMATICAL FUNCTIONS [42-43]**

|  |  |  |
| --- | --- | --- |
| **S.N.** |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |
| 4 |  |  |
| 5 |  |  |
| 6 |  |  |
| 7 |  |  |
| 8 |  |  |
| 9 |  |  |
| 10 |  |  |
| 11 |  |  |
| 12 |  |  |
| 13 |  |  |

**MOHAND DECOMPOSITION METHOD FOR SECOND KIND LINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATION**

The second kind linear Volterra integro-differential equation is given by [25]

 (1)

where

For determining the particular solution of equation (1), it is necessary to define the initial conditions .

In this work, we will assume that the kernel of equation (1) is a difference kernel that can be expressed as . Putting this in equation (1), it becomes

 (2)

Consider the initial condition as

 (3)

Applying Mohand transform on both sides of equation (2), we get

 (4)

Using equation (3) in equation (4), we have

 (5)

Using convolution theorem of Mohand transform in equation (5), we get

 (6)

Operating inverse Mohand transform on both sides of equation (6), we get

 (7)

The Mohand decomposition method assumes the solution into infinite series as

 (8)

Using equation (8) into equation (7), we have

In general, the recursive relation for the required solution is given by

**NUMERICAL PROBLEMS:** In this section, some numerical problems are considered and solved completely using Mohand transform.

**Problem: 1** Consider the following second kind linear Volterra integro-differential equation

 (9)

with (10)

**Solution:** Applying Mohand transform on both sides of equation (9), we get

 (11)

Using equation (10) in equation (11), we have

 (12)

Using convolution theorem of Mohand transform in equation (12), we get

 (13)

Operating inverse Mohand transform on both sides of equation (13), we get

 (14)

The Mohand decomposition method assumes the solution into infinite series as

 (15)

Using equation (15) into equation (14), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

 (16)

 (17)

Using equation (15), the required solution of equation (9) with equation (10) is given by

that converges to the exact solution .

**Problem: 2** Consider the following second kind linear Volterra integro-differential equation

 (18)

with (19)

**Solution:** Applying Mohand transform on both sides of equation (18), we get

 (20)

Using equation (19) in equation (20), we have

 (21)

Using convolution theorem of Mohand transform in equation (21), we get

 (22)

Operating inverse Mohand transform on both sides of equation (22), we get

 (23)

The Mohand decomposition method assumes the solution into infinite series as

 (24)

Using equation (24) into equation (23), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

 (25)

 (26)

Using equation (24), the required solution of equation (18) with equation (19) is given by

that converges to the exact solution .

**Problem: 3** Consider the following second kind linear Volterra integro-differential equation

 (27)

with (28)

**Solution:** Applying Mohand transform on both sides of equation (27), we get

 (29)

Using equation (28) in equation (29), we have

 (30)

Using convolution theorem of Mohand transform in equation (30), we get

 (31)

Operating inverse Mohand transform on both sides of equation (31), we get

 (32)

The Mohand decomposition method assumes the solution into infinite series as

 (33)

Using equation (33) into equation (32), we have

From above equation, the recursive relation for the required solution is given by

Using above recursive relation, the first few components of are given as

 (34)

 (35)

Using equation (33), the required solution of equation (27) with equation (28) is given by

that converges to the exact solution .

**CONCLUSION**

This chapter effectively determines the solution to the second kind linear Volterra integro-differential problem by using the Mohand decomposition method. The solutions to the issues under consideration show that the Mohand decomposition approach can solve a second-kind linear Volterra integro-differential equation quickly and with little computational effort. In the future, the system of simultaneous linear Volterra integro-differential equations can be solved using the current method.

**REFERENCES**

1. Wazwaz, A.M., Linear and Nonlinear Integral Equations: Methods and Applications, Higher Education Press, Beijing, 2011.
2. Jerri, A., Introduction to Integral Equations with Applications, Wiley, New York, 1999.
3. Aggarwal, S., Gupta, A.R., Singh, D.P., Asthana, N. and Kumar, N., Application of Laplace transform for solving population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(9), 141-145, 2018.
4. Sharma, N. and Aggarwal, S., Laplace transform for the solution of Abel’s integral equation, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 8-15, 2019.
5. Aggarwal, S. and Sharma, S.D., Application of Kamal transform for solving Abel’s integral equation, Global Journal of Engineering Science and Researches, 6(3), 82-90, 2019.
6. Aggarwal, S., Gupta, A.R., Asthana, N. and Singh, D.P., Application of Kamal transform for solving population growth and decay problems, Global Journal of Engineering Science and Researches, 5(9), 254-260, 2018.
7. Aggarwal, S., Pandey, M., Asthana, N., Singh, D.P. and Kumar, A., Application of Mahgoub transform for solving population growth and decay problems, Journal of Computer and Mathematical Sciences, 9(10), 1490-1496, 2018.
8. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of population growth and decay problems by using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3277-3282, 2018.
9. Aggarwal, S., Sharma, S.D. and Gupta, A.R., A new application of Mohand transform for handling Abel’s integral equation, Journal of Emerging Technologies and Innovative Research, 6(3), 600-608, 2019.
10. Aggarwal, S., Asthana, N. and Singh, D.P., Solution of population growth and decay problems by using Aboodh transform method, International Journal of Research in Advent Technology, 6(10), 2706-2710, 2018.
11. Aggarwal, S. and Sharma, S.D., Solution of Abel’s integral equation by Aboodh transform method, Journal of Emerging Technologies and Innovative Research, 6(4), 317-325, 2019.
12. Aggarwal, S., Singh, D.P., Asthana, N. and Gupta, A.R., Application of Elzaki transform for solving population growth and decay problems, Journal of Emerging Technologies and Innovative Research, 5(9), 281-284, 2018.
13. Aggarwal, S. and Gupta, A.R., Sumudu transform for the solution of Abel’s integral equation, Journal of Emerging Technologies and Innovative Research, 6(4), 423-431, 2019.
14. Aggarwal, S., Sharma, S.D. and Gupta, A.R., Application of Shehu transform for handling growth and decay problems, Global Journal of Engineering Science and Researches, 6(4), 190-198, 2019.
15. Aggarwal, S. and Gupta, A.R., Shehu transform for solving Abel’s integral equation, Journal of Emerging Technologies and Innovative Research, 6(5), 101-110, 2019.
16. Aggarwal, S. and Bhatnagar, K., Sadik transform for handling population growth and decay problems, Journal of Applied Science and Computations, 6(6), 1212-1221, June 2019.
17. Aggarwal, S. and Bhatnagar, K., Solution of Abel’s integral equation using Sadik transform, Asian Resonance, Vol. 8 No. 2, (Part-1), 57-63, April 2019.
18. Singh, G.P. and Aggarwal, S., Sawi transform for population growth and decay problems, International Journal of Latest Technology in Engineering, Management & Applied Science, 8(8), 157-162, August 2019.
19. Higazy, M., Aggarwal S. and Hamed, Y. S., Determination of number of infected cells and concentration of viral particles in plasma during HIV-1 infections using Shehu transform, Journal of Mathematics, 2020, 1-13, 2020. <https://doi.org/10.1155/2020/6624794>
20. Priyanka and Aggarwal, S., Solution of the model of the bacteria growth via Rishi transform, Journal of Advanced Research in Applied Mathematics and Statistics, 7(1&2), 5-11, 2022. <https://doi.org/10.24321/2455.7021.202202>
21. Aggarwal, S., Kumar, R. and Chandel, J., Rishi transform for determining concentrations of the chemical compounds in first order successive chemical reaction, Journal of Advanced Research in Applied Mathematics and Statistics, 8(1 & 2), 10-17, 2023. <https://doi.org/10.24321/2455.7021.202303>
22. Aggarwal, S., Sharma, S.D., Kumar, N. and Vyas, A., Solutions of population growth and decay problems using Sumudu transform, International Journal of Research and Innovation in Applied Science, 5(7), 21-26, 2020.
23. Aggarwal, S. and Gupta, A.R., Solution of linear Volterra integro-differential equations of second kind using Kamal transform, Journal of Emerging Technologies and Innovative Research, 6(1), 741-747, 2019.
24. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integro-differential equations of second kind using Mahgoub transform, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(5), 173-176, 2018.
25. Aggarwal, S., Gupta, A.R. and Sharma, S.D., Application of Sadik transform for handling linear Volterra integro-differential equations of second kind, Universal Review, 10(7), 177-187, July 2019.
26. Aggarwal, S., Sharma, S.D. and Vyas, A., Aboodh transform for solving first kind faltung type Volterra integro-differential equation, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 71-77, 2020.
27. Aggarwal, S., Vyas, A. and Sharma, S.D., Mohand transform for handling convolution type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 78-84, 2020.
28. Aggarwal, S., Sharma, S.D. and Vyas, A., Solution of convolution type Volterra integro-differential equation of first kind using Kamal transformation, International Journal of Research and Innovation in Applied Science, 5 (7), 137-143, 2020.
29. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of faltung type Volterra integro-differential equation of first kind using Elzaki transform, International Journal of Research and Innovation in Applied Science, 5(7), 144-150, 2020.
30. Aggarwal, S., Sharma, S.D. and Vyas, A., Laplace-Carson transform for the primitive of convolution type Volterra integro-differential equation of first kind, International Journal of Research and Innovation in Applied Science, 5(7), 30-36, 2020.
31. Aggarwal, S., Sharma, S.D. and Vyas, A., Laplace transformation for the solution of faltung type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(7), 5-11, 2020.
32. Aggarwal, S., Vyas, A. and Sharma, S.D., Analytical solution of first kind Volterra integro-differential equation using Sadik transform, International Journal of Research and Innovation in Applied Science, 5(8), 73-80, 2020.
33. Aggarwal, S., Sharma, S.D. and Vyas, A., Application of Sawi transform for solving convolution type Volterra integro-differential equation of first kind, International Journal of Latest Technology in Engineering, Management & Applied Science, 9(8), 13-19, 2020.
34. Aggarwal, S., Sharma, S.D. and Vyas, A., Application of Sumudu transform for handling Volterra integro-differential equation of first kind with convolution kernel, International Journal of Research and Innovation in Applied Science, 5(8), 24-30, 2020.
35. Aggarwal, S., Vyas, A. and Sharma, S.D., Primitive of first kind Volterra integro-differential equation using Shehu transform, International Journal of Research and Innovation in Applied Science, 5(8), 31-38, 2020.
36. Aggarwal, S. and Chaudhary, R., A comparative study of Mohand and Laplace transforms, Journal of Emerging Technologies and Innovative Research, 6(2), 230-240, 2019.
37. Aggarwal, S., Sharma, N., Chaudhary, R. and Gupta, A.R., A comparative study of Mohand and Kamal transforms, Global Journal of Engineering Science and Researches, 6(2), 113-123, 2019.
38. Aggarwal, S., Mishra, R. and Kumar, A., A comparative study of Mohand and Elzaki transforms, Global Journal of Engineering Science and Researches, 6(2), 203-213, 2019.
39. Aggarwal, S. and Chauhan, R., A comparative study of Mohand and Aboodh transforms, International Journal of Research in Advent Technology, 7(1), 520-529, 2019.
40. Aggarwal, S. and Sharma, S.D., A comparative study of Mohand and Sumudu transforms, Journal of Emerging Technologies and Innovative Research, 6(3), 145-153, 2019.
41. Aggarwal, S., A comparative study of Mohand and Mahgoub transforms, Journal of Advanced Research in Applied Mathematics and Statistics, 4(1), 1-7, 2019.
42. Aggarwal, S., Gupta, A.R. and Kumar, D., Mohand transform of error function, International Journal of Research in Advent Technology, 7(5), 224-231, 2019.
43. Aggarwal, S., Chauhan, R. and Sharma, N., Mohand transform of Bessel’s functions, International Journal of Research in Advent Technology, 6(11), 3034-3038, 2018.
44. Aggarwal, S. and Gupta, A.R., Dualities between Mohand transform and some useful integral transforms, International Journal of Recent Technology and Engineering, 8(3), 843-847, September 2019.