On Some typical kind of Continuity in Soft Topological Space

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ABSTRACT

In this paper we have made an attempt to make results on some typical kind of continuous functions of Soft J Open and Soft J Closed sets in Soft Topological spaces. This study also describes the characterization of continuity with reference to our Soft J Open sets in Soft Topological Spaces.

Keywords—Soft J Open set, Soft J Continuous Functions, Soft Strongly J Continuous Functions and Soft Perfectly J Continuous Functions.

#  INTRODUCTION

 *Soft* set hypothesis was proposed by Molodtsov [4] in 1999 to manage vulnerability in a parametric way. A *Soft* set is a defined group of sets, instinctively *Soft* on the grounds that the limit of the set relies upon the boundaries. One idea of a set is the idea of dubiousness. Molodtsov [6] proposed *Soft* set as a totally nonexclusive numerical instrument for displaying vulnerabilities. There is no restricted condition to the depiction of articles. One of the critical benefits of soft topological spaces lies in their capacity to deal with complex frameworks with deficient or problematic information. They can display questionable conditions, rough thinking, and manage fractional data in a more regular and natural way contrasted with customary topological spaces.

# PRELIMINARIES

## In this section, we discuss the basic definitions and results of Soft set theory that can be considered from previous studies. Throughout this work, $X$ refers to an initial universe, $P(X)$ is the power set of $X$, $E,K,R$ denote the set of parameters and $(X,τ,E),(Y,σ,K),(Z,η,R)$ denote Soft topological spaces where no Soft separation axioms are assumed until it is explicated.

**Definition 2.1:[6]** A *Soft* set $F\_{A}$ on the universe $X$ is deﬁned by the set of ordered pairs $F\_{A}=\{(x,f\_{a}(x)):x\in E and f\_{a}(x)\in P(X)\}$, where $f\_{a}:A\rightarrow P(X)$ such that $f\_{a}\left(x\right)=ϕ$ for all $x\notin A$. Hence $f\_{a}$ is called an approximate function of the *Soft* set $F\_{A}$. The value of $f\_{a}$ may be arbitrary, some of them may be empty, some may have non empty intersection.

**Definition 2.2: [2]**

1. A *Soft* set $(F,A)$ over $X$ is said to be Null *Soft* Set denoted by *Fϕ* or $\tilde{ϕ}$ if for all $e\in A$, $F(e)=ϕ$.
2. A *Soft* set $(F,E)$ over $X$ is said to be an Absolute *Soft* Set denoted by *FX* or $\tilde{X}$ if for all $e\in A$*,* $F(e)=X.$

**Definition 2.3:[6]** Let $τ$ be a collection of *Soft* sets over $X$ with a fixed set $E$ of parameters. Then, $τ$ is called a ***Soft* topology** on $X$ if

1. $\tilde{ϕ}, \tilde{X}$ belongs to $τ$.
2. The union of any number of *Soft* sets in $τ$ belongs to $τ$.
3. The intersection of any two *Soft* sets in $τ$ belongs to $τ$.

The triplet $(X,τ,E)$ is called ***Soft* topological Space** over $X$. The members of $τ$ are called ***Soft* *open*** sets in $X$ and complements of them are called ***Soft* *closed*** sets in $X$.

**Definition 2.4:[3]** A *Soft* set $(W,E)$ of a *Soft* topological space $(X,τ,E)$ is known as a ***Soft* J *Closed* set** if $s^{\*}Cl(W,E)\tilde{⊆}Int(V,E)$ when $(W,E)\tilde{⊆}(V,E)$ and $(V,E)$ is *Soft* $\hat{g}$-*open*. $SJC(X,τ,E)$ stands for the set of all *Soft* J *closed* sets.

**Definition 2.5.[3]** A *Soft* set $(Q,E)$ of a *Soft* topological space $(X,τ,E)$ is known as a ***Soft* J *Open* set** if its complement is a *Soft* J *closed* set. $SJO(X,τ,E)$ stands for the set of all *Soft* J *open* sets.

**Definition 2.6.** **[1,5]** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is said to be

1. ***Soft* *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* *open* in $(X,τ,E)$.
2. ***Soft* semi-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* semi-*open* in $(X,τ,E)$.
3. ***Soft* pre-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* pre-*open* in $(X,τ,E)$.
4. ***Soft*** $α$**-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* $α$-*open* in $(X,τ,E)$.
5. ***Soft*** $β$**-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* $β$-*open* in $(X,τ,E)$.
6. ***Soft* g *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* g *open* in $(X,τ,E)$.
7. ***Soft* sg *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* sg-*open* in $(X,τ,E)$.
8. ***Soft* gs *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* gs-*open* in $(X,τ,E)$.
9. ***Soft* gp *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* gp-*open* in $(X,τ,E)$.
10. ***Soft* gpr *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* pre-*open* in $(X,τ,E)$.
11. ***Soft*** $αg$**-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* $α$g-*open* in $(X,τ,E)$.
12. ***Soft* g**$α$**-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* g$α$-*open* in $(X,τ,E)$.
13. ***Soft*** $\hat{g}$**-*continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* $\hat{g}$-*open* in $(X,τ,E)$.

***Soft* JP *continuous*** if the inverse image of every *Soft* *open* set in $(Y,σ,K)$ is *Soft* JP *open* in $(X,τ,E)$.

**Result 2.7.[6]**

* + - 1. Each one of the Soft semi closed set remains Soft J closed.
			2. Each one of the Soft closed set remains Soft J closed.
			3. Each one of the Soft $α-closed$ set remains Soft J Closed.
			4. Each one of the Soft open set remains Soft J open.
			5. Each one of the Soft semi-open set remains Soft J open.
			6. Each one of the Soft α-open set remains Soft J open.
			7. Each one of the Soft J open set remains Soft gs-open.

# SOFT TOTALLY J CONTINUOUS FUNCTIONS

**Deﬁnition 3.1:** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is known to be **Soft totally J continuous** if the inverse-image of each one of the Soft open set in $(Y,σ,K)$ is both Soft J closed and Soft J open (i.e Soft J clopen) in $(X,τ,E)$.

**Theorem 3.2:** Each one of the Soft perfectly J continuous map is Soft totally J continuous.

**Proof:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ be Soft perfectly J continuous and $(U,K)$ be a Soft open set in $(Y,σ,K)$. Thereon $(U,K)$ is Soft J open in $(Y,σ,K)$. Since $f$ is Soft Perfectly J continuous, $f^{-1}(U,K)$ is Soft clopen in $(X,τ,E)$. By Result 2.7, $f$ is Soft totally J continuous.

**Remark 3.3:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.4:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $m :X\rightarrow Y$ and $n :E\rightarrow K$ as $m\left(x\_{1}\right)=y\_{1}, m\left(x\_{2}\right)=y\_{2}$ and $n\left(e\_{1}\right)=k\_{1}, n\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(M\_{1},E),(M\_{2},E)\}$ where $\left(M\_{1},E\right)$ and $(M\_{2},E)$ are described this way: $M\_{1}\left(e\_{1}\right)=\left\{x\_{1}\right\},M\_{1}\left(e\_{2}\right)=ϕ,M\_{2}\left(e\_{1}\right)=\left\{x\_{1}\right\},M\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},\left(N\_{1},K\right),(N\_{2},K)\}$ where $\left(N\_{1},K\right)$ and $(N\_{2},K)$ are described this way: $N\_{1}\left(k\_{1}\right)=ϕ,N\_{1}\left(k\_{2}\right)=\left\{y\_{1}\right\}$ and $N\_{2}\left(k\_{1}\right)=\left\{y\_{1}\right\},N\_{2}\left(k\_{2}\right)=\left\{y\_{1}\right\}$. Precisely the mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft totally J continuous. The Soft set $\left(H,K\right)$ defined as $H\left(k\_{1}\right)=ϕ,H\left(k\_{2}\right)=\left\{y\_{1}\right\}$ is a Soft J open set in $(Y,σ,K)$. But $f^{-1}\left(H,K\right)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{1}\right)\}$ is not Soft clopen in $(X,τ,E)$. Hence $f$ is not Soft perfectly J continuous.

**Theorem 3.5:** Each one of the Soft totally J continuous map is Soft J continuous.

**Proof:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft totally J continuous and $(A,K)$ be Soft open set in $(Y,σ,K)$. Since $f$ is Soft perfectly J continuous, $f^{-1}(A,K)$ is Soft clopen in $(X,τ,E)$. Then by proposition 2.1.7 and 2.2.2, $f^{-1}(A,K)$ is Soft J clopen in $(X,τ,E)$. Thereupon $f$ is Soft totally J continuous.

**Remark 3.6:** It is observed from the subsequent illustration that the reverse implication of the above theorem is incorrect.

**Example 3.7:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=ϕ,H\left(k\_{2}\right)=\left\{y\_{1},y\_{2}\right\}$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a Soft mapping. Precisely $g$ is Soft J continuous but not Soft totally J continuous, because $g^{-1}\left(H,K\right)=\{\left(e\_{1},x\_{1},x\_{2}\right),(e\_{2},ϕ)\}$ is not Soft J clopen in $(X,τ,E)$.

**Theorem 3.8:** Each one of the Soft totally J continuous map is Soft JA continuous.

**Proof:** $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft J continuous. Let $(A,K)$ be a Soft closed set in $(Y,σ,K)$. Then $f^{-1}(A,K)$ is Soft J closed in $(X,τ,E)$. Also, $f^{-1}(A,K)$ is Soft JA closed in $(X,τ,E)$. Thus $f$ is Soft JA continuous.

# SOFT CONTRA J CONTINUOUS FUNCTIONS

**Deﬁnition 4.1:** A map $f:(X,τ,E)\rightarrow (Y,σ,K)$ is known to be **Soft Contra J continuous** if the inverse-image of each one of the Soft open set in $(Y,σ,K)$ is Soft J closed in $(X,τ,E)$.

**Example 4.2:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{.2},E)$ are deﬁned as $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=ϕ$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a Soft mapping. Precisely $g$ is Soft contra J continuous.

**Proposition 4.3:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra semi continuous then it is Soft contra J continuous.

**Proof:** It is verified by Result 2.7, that each one of the Soft semi closed set is Soft J closed in $(X,τ,E)$.

**Proposition 4.4:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra continuous then it is Soft contra J continuous .

**Proof:** It is verified by Result 2.7, that each one of the Soft closed set is Soft J closed in $(X,τ,E)$.

**Proposition 4.5:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra $α$-continuous then it is Soft contra J continuous .

**Proof:** It is proved by Result 2.7, that each one of the Soft $α$-closed set is Soft J closed in $(X,τ,E)$.

**Result 4.6:** It is observed from the subsequent illustration that the reverse implication of the above propositions 4.3, 4.4, 4.5 are incorrect.

**Example 4.7:** Consider the Soft open set $(H,K)$ in Example 4.2. Here, $g^{-1}\left(H,K\right)=\{\left(e\_{1},x\_{1},x\_{2}\right),(e\_{2},ϕ)\}$ is not Soft semi-closed (Soft closed, Soft $α$-closed) in $(Y,σ,K)$. Hence $g:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra J continuous but not Soft contra semi continuous (Soft contra continuous, Soft contra $α$-continuous).

**Remark 4.8:** Soft J continuity and Soft contra J continuity are independent. It is observed from the subsequent illustration.

**Example 4.9:**

1. Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $u :X\rightarrow Y$ and $v :E\rightarrow K$ as $u\left(x\_{1}\right)=y\_{1}, u\left(x\_{2}\right)=y\_{2}$ and $v\left(e\_{1}\right)=k\_{1}, v\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},\left(F\_{1},E\right),\left(F\_{2},E\right)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=x\_{2},F\_{1}\left(e\_{2}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{1},x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H\_{1},K),(H\_{2},K),(H\_{3},K)\}$ where $(H\_{1},K),(H\_{2},K)$ and $(H\_{3},K)$ are described this way: $H\_{1}\left(k\_{1}\right)=\left\{y\_{1}\right\},H\_{1}\left(k\_{2}\right)=ϕ,H\_{2}\left(k\_{1}\right)=ϕ,H\_{2}\left(k\_{2}\right)=\left\{y\_{2}\right\},H\_{3}\left(k\_{1}\right)=\left\{y\_{1}\right\},H\_{3}\left(k\_{2}\right)=\left\{y\_{2}\right\}$. Precisely the mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft J continuous. The inverse-image of the Soft open set $(H\_{2},K)$, $f^{-1}(H\_{2},K)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{2}\right)\}$ is not a Soft J closed set in $(X,τ,E)$. Hence $f$ is not Soft contra J continuous.
2. Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\}.$ Define $p :X\rightarrow Y$ and $q :E\rightarrow K$ as $p\left(x\_{1}\right)=y\_{1}, p\left(x\_{2}\right)=y\_{2}$ and $q\left(e\_{1}\right)=k\_{1}, q\left(e\_{2}\right)=k\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},\left(M\_{1},E\right),\left(M\_{2},E\right)\}$ where $\left(M\_{1},E\right)$ and $(M\_{2},E)$ are deﬁned as $M\_{1}\left(e\_{1}\right)=x\_{2},M\_{1}\left(e\_{2}\right)=\left\{x\_{2}\right\},M\_{2}\left(e\_{1}\right)=\left\{x\_{1},x\_{2}\right\},M\_{2}\left(e\_{2}\right)=\left\{x\_{2}\right\}$ and $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=y\_{1}$. Let $g:(X,τ,E)\rightarrow (Y,σ,K)$ be a Soft mapping. Precisely $g$ is Soft contra J continuous. The Soft set $(R,K)$ defined as $R\left(k\_{1}\right)=ϕ,R\left(k\_{2}\right)=y\_{2}$ is Soft closed in $\left(Y,σ,K\right)$ but its inverse-image $g^{-1}\left(R,K\right)=\{\left(e\_{1},ϕ\right),\left(e\_{2},x\_{2}\right)\}$ is not Soft J closed in $(X,τ,E)$. Hence $g$ is not Soft J continuous.

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**Lemma 4.10:** The following properties hold for the Soft subsets $(A,E), (B,E)$ of a space $(X,τ,E)$.

1. $(A,E)\tilde{⊂}SKer(A,E)$ and $(A,E)=SKer(A,E)$ if $(A,E)$ is Soft open in $(X,τ,E)$.

2. $(A,E)\tilde{⊂}(B,E)$ then $SKer(A,E)\tilde{⊂}SKer(B,E)$.

**Proof:** The proof is obvious.

**Theorem 4.11:** For a Soft mapping $f:(X,τ,E)\rightarrow (Y,σ,K)$ the subsequent properties are equivalent. Assume that $SJO(X,τ,E)$ is closed under any union and $SJC(X,τ,E)$ is closed under any intersection.

1. $f$ is Soft contra J continuous.

2. The inverse-image of a Soft closed set $(B,K)$ of $(Y,σ,K)$ is Soft J open.

3. $f(SJCl(A,E))\tilde{⊂}SKer f((A,E))$ for each one of the Soft subset of $(A,E)$ of $(X,τ,E)$.

4. $SJCl(f^{-1}(C,K))\tilde{⊂}f^{-1}(SKer(C,K))$ for each one of the subset $(C,K)$ of $(Y,σ,K)$.

**Proof:**

$(1)⇒(2):$ It is evident.

$(2)⇒(3):$ Let $(A,E)$ be any Soft subset of $(X,τ,E)$. Suppose $(y,e)\tilde{\notin }SKer (f(A,E))$. Then by lemma 4.10, there exists $(C,K)$ such that $f(A,E)\tilde{∩}(C,K)=\tilde{ϕ}$. Thus $(A,E)\tilde{∩}f^{-1}(C,K)=\tilde{ϕ}$ and $SJCl(A,E)\tilde{∩}f^{-1}(C,K)=\tilde{ϕ}$. Therefore, $f(SJCl(A,E))\tilde{∩}(C,K)=\tilde{ϕ}$ and $(y,e)\tilde{\notin }f(SJCl(A,E)$. Thereon $f(SJCl(A,E))\tilde{⊂}SKer f((A,E))$ for every Soft subset of $(A,E)$ of $(X,τ,E)$.

$(3)⇒(4):$ Let $(C,K)$ be any Soft subset of $(Y,σ,K)$. Then by (3) and Lemma 4.10, $f(SJCl(f^{-1}(C,K)))\tilde{⊂}SKer(f(f^{-1}(C,K)))\tilde{⊂}SKer(C,K)$ and then $SJCl(f^{-1}(C,K))\tilde{⊂}f^{-1}(SKer(C,K))$.

$(4)⇒(1):$ Let $(V,K)$ be any Soft open subset of $(Y,σ,K)$. Therefore, by hypothesis and by Lemma 4.11, $SJCl(f^{-1}(V,K))\tilde{⊂}f^{-1}(SKer(V,K))= f^{-1}(V,K)$. So, $SJCl(f^{-1}(V,K)=f^{-1}(V,K)$. This reveals that $f^{-1}(V,K)$ is Soft J closed in $(X,τ,E)$. Hence $f$ is Soft contra J continuous.

**Result 4.12:** The composition of two Soft contra J continuous functions need not be Soft contra J continuous and it is observed from the subsequent illustration.

**Example 4.13:** Let $X = \left\{x\_{1},x\_{2}\right\},Y = \left\{y\_{1},y\_{2}\right\},Z= \left\{z\_{1},z\_{2}\right\},E=\left\{e\_{1},e\_{2}\right\}, K = \left\{k\_{1},k\_{2}\right\} $and$ R= \left\{r\_{1},r\_{2}\right\}.$ Define $s :X\rightarrow Y,t :Y\rightarrow Z$ and $p:E\rightarrow K,q :K\rightarrow R$ as $s\left(x\_{1}\right)=y\_{1}, s\left(x\_{2}\right)=y\_{2},t\left(y\_{1}\right)=z\_{1}, t\left(y\_{2}\right)=z\_{2}$ and $p\left(e\_{1}\right)=k\_{1}, p\left(e\_{2}\right)=k\_{2},q\left(k\_{1}\right)=r\_{1}, q\left(k\_{2}\right)=r\_{2}$. Consider the Soft topologies $τ=\{\tilde{ϕ}, \tilde{X},(F\_{1},E),(F\_{2},E)\}$ where $\left(F\_{1},E\right)$ and $(F\_{2},E)$ are described this way: $F\_{1}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{1}\left(e\_{2}\right)=\left\{x\_{1}\right\},F\_{2}\left(e\_{1}\right)=\left\{x\_{2}\right\},F\_{2}\left(e\_{2}\right)=\left\{x\_{1},x\_{2}\right\}$, $σ=\{\tilde{ϕ}, \tilde{Y},(H,K)\}$ where $\left(H,K\right)$ is described this way: $H\left(k\_{1}\right)=\left\{y\_{1},y\_{2}\right\},H\left(k\_{2}\right)=ϕ$ and $η=\{\tilde{ϕ}, \tilde{Z},\left(M\_{1},R\right),\left(M\_{2},R\right)\}$ where $\left(M\_{1},R\right)$ and $(M\_{2},R)$ are described this way: $M\_{1}\left(r\_{1}\right)=z\_{2},M\_{1}\left(r\_{2}\right)=\left\{z\_{2}\right\},M\_{2}\left(r\_{1}\right)=\left\{z\_{1},z\_{2}\right\},M\_{2}\left(r\_{2}\right)=\left\{z\_{2}\right\}$. Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be two Soft mappings. Precisely $f$ and $g$ are Soft contra J continuous but their composition is not Soft contra J continuous because $\left(g∘f\right)^{-1}\left(H,R\right)=\{(e\_{1},x\_{2}),(e\_{2},x\_{2})\}$ is not Soft J closed in $(X,τ,E)$.

**Proposition 4.14:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft J irresolute and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft contra J continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous.

**Proof:** Let $(Q,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft contra J continuous $g^{-1}(Q,R)$ is Soft J open in $(Y,σ,K)$. Because $f$ is Soft J irresolute, $f^{-1}(g^{-1}(Q,R))=(g∘f)^{-1}(Q,R)$ is Soft J open in $(X,τ,E)$. So $g∘f$ is Soft contra J continuous.

**Proposition 4.15:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra J continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous.

**Proof:** Let $(P,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft continuous $g^{-1}(P,R)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft contra J continuous $f^{-1}(g^{-1}(P,R))=(g∘f)^{-1}(P,R)$ is Soft J open in $(X,τ,E)$. Thus $g∘f$ is Soft contra J continuous.

**Proposition 4.16:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra semi continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous.

**Proof:** Let $(F,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft continuous $g^{-1}(F,R)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft contra semi continuous $f^{-1}(g^{-1}(F,R))=(g∘f)^{-1}(F,R)$ is Soft semi open in $(X,τ,E)$. By Result 2.7, $(g∘f)^{-1}(F,R)$ is Soft J open in $(X,τ,E)$. Thus $g∘f$ is Soft contra J continuous.

**Proposition 4.17:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra $α$-continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous.

**Proof:** Let $(S,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft continuous $g^{-1}(P,R)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft contra $α$-continuous, $f^{-1}(g^{-1}(P,R))=(g∘f)^{-1}(P,R)$ is Soft $α$-open in $(X,τ,E)$. By Result 2.7, $(g∘f)^{-1}(P,R)$ is Soft J open in $(X,τ,E)$. Thus $g∘f$ is Soft contra J continuous.

**Theorem 4.18:** Let $\{(X\_{i},τ,E)∕i\in I\}$ be any family of Soft topological spaces. If $f:(X,τ,E)\rightarrow Q(X\_{i},τ,E)$ is Soft contra J continuous then $Q\_{i}f:(X,τ,E)\rightarrow (X\_{i},τ,E)$ is Soft contra J continuous for each $i\in I$, where $Q\_{i}$ is the Soft projection of $Q(X\_{i},τ,E)$ onto $(X\_{i},τ,E)$.

**Proof:** It has been verified by the combination of facts that Soft projection is continuous.

**Theorem 4.19:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is a Soft surjective J open map and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is a map such that their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous, then $g$ is Soft contra J continuous.

**Proof:** Let $(H,R)$ be a Soft closed set in $(Z,η,R)$. Since $g∘f$ is a Soft contra J continuous, $(g∘ f)^{-1}(H,R)=f^{-1}(g^{-1}(H,R))$ is Soft J open in $(X,τ,E)$. Because $f$ is Soft surjective and Soft J open, $f((g∘f)^{-1}(H,R))=(g^{-1}(H,R))$ is Soft J open in $(Y,σ,K)$. Thereon $g$ is Soft contra J continuous.

**Theorem 4.20:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft contra J continuous and $(Y,σ,K)$ is Soft regular then $f$ is Soft J continuous.

**Proof:** Let $(x,e)$ be an arbitrary Soft point of $\tilde{X}$ and $(F,K)$ be a Soft open set of $(Y,σ,K)$ containing $f((x,e))$. Since $(Y,σ,K)$ is Soft regular, there exists a Soft open set $(V,K)$ in $(Y,σ,K)$ containing $f((x,e))$ such that $Cl(V,K)\tilde{⊂}(F,K)$. Now, $Cl(V,K)$ is a Soft closed set in $(Y,σ,K)$ containing $f((x,e))$ and $f$ is Soft contra J continuous. Therefore, by theorem 4.11 there exists $(U,E)\tilde{\in }SJPO(X,τ,E)$ such that $f(V,K)\tilde{⊂}Cl(V,K)$. Then $f(U,E)\tilde{⊂}(F,K)$. Hence $f$ is Soft J continuous.

**Proposition 4.21:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is contra Soft semi continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft contra continuous $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft J continuous.

**Proof:** Let $(H,R)$ be a any Soft closed set in $(Z,η,R)$. Since $g$ is Soft contra continuous, $g^{-1}(H,R)$ is Soft open in $(Y,σ,K)$. Since $f$ is contra Soft semi continuous $f^{-1}(g^{-1}\left(H,R\right))$ is Soft semi closed set in $(X,τ,E)$. Because each one of the Soft semi closed set is Soft J closed, $f^{-1}(g^{-1}\left(H,R\right))$ is Soft J closed set in $(X,τ,E)$. Thus $g∘f$ is Soft J continuous.

**Theorem 4.22:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft J irresolute if both $f$ and $g$ are Soft J irresolute.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft J continuous if $f$ is Soft J irresolute and $g$ is Soft J continuous.

**Proof:**

1. Let $(V,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft J irresolute, $g^{-1}(V,R)$ is Soft J closed in $(Y,σ,K)$. Because $f$ is Soft J irresolute $f^{-1}\left(g^{-1}\left(V,R\right)\right)=\left(g∘f\right)^{-1}(V,R)$ is Soft J closed in $(X,τ,E)$. So, $g∘f$ is Soft J irresolute.
2. Let $(V,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft J continuous $g^{-1}(V,R)$ is Soft J closed in $(Y,σ,K)$. Since $f$ is Soft J irresolute $f^{-1}\left(g^{-1}\left(V,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft J closed in $(X,τ,E)$. So, $g∘f$ is Soft J continuous.

**Theorem 4.23:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two Soft maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft continuous if $f$ is Soft strongly J continuous and $g$ are Soft J continuous.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft strongly J continuous if both $f$ and $g$ are Soft strongly J continuous.
3. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft strongly J continuous if $f$ is Soft continuous and $g$ is Soft strongly J continuous.
4. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft continuous if $f$ is Soft strongly J continuous and $g$ is Soft continuous.
5. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft J irresolute if $f$ is Soft J continuous and $g$ is Soft strongly J continuous.

**Proof:**

1. Let $(H,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft J continuous $g^{-1}(H,R)$ is Soft J closed in $(Y,σ,K)$. Since $f$ is Soft strongly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft closed in $(X,τ,E)$. So, $g∘f$ is Soft continuous.
2. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft strongly J continuous $g^{-1}(H,R)$ is Soft closed in $(Y,σ,K)$. By Result 2.7,$ g^{-1}(H,R)$ is Soft J closed set in $(Y,σ,K)$. Because $f$ is also Soft strongly J continuous, $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft closed in $(X,τ,E)$. Thus $g∘f$ is Soft strongly J continuous.
3. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft strongly J continuous $g^{-1}(H,R)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft closed in $(X,τ,E)$. Thus $g∘f$ is Soft strongly J continuous.
4. Let $(H,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft continuous $g^{-1}(H,R)$ is Soft closed in $(Y,σ,K)$. By Result 2.7, $g^{-1}(H,R)$ is Soft J closed set in $(Y,σ,K)$. As $f$ is Soft strongly J continuous, $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft closed in $(X,τ,E)$. So, $g∘f$ is Soft continuous.
5. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Because $g$ is Soft strongly J continuous $g^{-1}\left(H,R\right)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft J closed in $(X,τ,E)$. Thus $g∘f$ is Soft J irresolute.

**Theorem 4.24:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two Soft maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft perfectly continuous if $f$ is Soft strongly continuous and $g$ are Soft perfectly continuous.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft perfectly J continuous if both $f$ and $g$ are Soft perfectly J continuous.
3. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft perfectly J continuous if $f$ is Soft perfectly J continuous and $g$ is Soft J irresolute.

**Proof:**

1. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft perfectly J continuous $g^{-1}\left(H,R\right)$ is Soft clopen in $(Y,σ,K)$. Since $f$ is Soft strongly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$. Thus $g∘f$ is Soft perfectly continuous.
2. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft perfectly J continuous $g^{-1}\left(H,R\right)$ is Soft clopen in $(Y,σ,K)$. By Result 2.7, $g^{-1}\left(H,R\right)$ is Soft J closed set in $(Y,σ,K)$. Now, $f$ is also Soft perfectly J continuous, then $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$. Thus$ g∘f$ is Soft perfectly J continuous.
3. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft J irresolute, $g^{-1}\left(H,R\right)$ is Soft J closed set in $(Y,σ,K)$. Since $f$ is Soft perfectly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$. So, $g∘f$ is Soft perfectly J continuous.

**Theorem 4.25:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two Soft maps then

1. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft perfectly continuous if $f$ is Soft perfectly J continuous and $g$ are Soft strongly J continuous.
2. $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft strongly J continuous if $g$ is Soft perfectly J continuous and $f$ is Soft continuous.

**Proof:**

1. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft strongly J continuous $g^{-1}\left(H,R\right)$ is Soft closed set in $(Y,σ,K)$. By Result 2.7, $g^{-1}\left(H,R\right)$ is Soft JP closed set in $(Y,σ,K)$. Since $f$ is Soft perfectly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$. Thus $g∘f$ is Soft perfectly continuous.
2. Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft perfectly J continuous $g^{-1}\left(H,R\right)$ is Soft clopen in $(Y,σ,K)$. Since $f$ is Soft continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$.Thus $g∘f$ is Soft perfectly J continuous.

**Theorem 4.26:** Let $f:(X,τ,E)\rightarrow (Y,σ,K)$ and $g:(Y,σ,K)\rightarrow (Z,η,R)$ be any two maps then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft perfectly J continuous if $g$ is Soft strongly continuous and $g$ is Soft perfectly J continuous.

**Proof:** Let $(H,R)$ be a Soft J closed set in $(Z,η,R)$. Since $g$ is Soft strongly continuous $g^{-1}\left(H,R\right)$ is Soft closed in $(Y,σ,K)$. Since $f$ is Soft perfectly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft J clopen in $(X,τ,E)$. Thus $g∘f$ is Soft perfectly J continuous.

**Theorem 4.27:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft strongly J continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft contra J continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra continuous.

**Proof:** Let $(H,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft contra J continuous $g^{-1}\left(H,R\right)$ is Soft J open in $(Y,σ,K)$. Since $f$ is Soft strongly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft open in $(X,τ,E)$. Thus $g∘f$ is Soft contra continuous.

**Theorem 4.28:** If $f:(X,τ,E)\rightarrow (Y,σ,K)$ is Soft perfectly J continuous and $g:(Y,σ,K)\rightarrow (Z,η,R)$ is Soft contra J continuous then their composition $g∘f:(X,τ,E)\rightarrow (Z,η,R)$ is Soft contra J continuous.

**Proof:** Let $(H,R)$ be a Soft closed set in $(Z,η,R)$. Since $g$ is Soft contra J continuous $g^{-1}\left(H,R\right)$ is Soft J open in $(Y,σ,K)$. Since $f$ is Soft perfectly J continuous $f^{-1}\left(g^{-1}\left(H,R\right)\right)=\left(g∘f\right)^{-1}(H,R)$ is Soft clopen in $(X,τ,E)$. Then by Result 2.7, $\left(g∘f\right)^{-1}(H,R)$ is Soft J open in $(X,τ,E)$. Thus $g∘f$ is Soft contra J continuous.­

##### REFERENCES

[1] Arockia Rani, I & Selvi, A 2015, ‘On *Soft* Contra $πg$ *Continuous* Functions in *Soft* Topological Spaces’, International Journal of Mathematical Trends & Technology, vol. 19, no. 1, pp. 80-90.

[2] Jackson, S & Chitra, S 2021, ‘The New Class of *Closed* and *Open* Sets in *Soft* Topological Spaces’, Journal of the Maharaja Sayajirao University of Baroda, vol. 55, pp. 78-85.

[3] Jackson S and Jenifa N, ‘A New Class of *Soft* Generalized *Closed* Sets in *Soft* Topological Spaces’, Studies in Indian Place Names, vol. 40(70),pp.3380-3385, 2020.

[4] Jun, YB, Kim, HS & Park, CH 2011, ‘Positive Implicative Ideals of BCK-algebras Based on A *Soft* Set Theory’, Bull. Malays. Math. Sci. Soc., vol. 34, no. 2, pp. 345-354.

[5] Maji, PK, Roy, AR & Biswas 2002, ‘An Application of *Soft* Sets in A Decision Making Problem’, Computers and Mathematics with applications, vol. 44, no. 8/9, pp. 1077-1083.

[6] Molodtsov, D 1999, ‘Soft Set Theory First Results’, Computers and Mathematics with Applications, vol. 37, pp. 19-31.

[7] Nakaoka, F & Oda, N 2003, ‘Some Properties of Maximal *Open* Sets,’ International Journal of Mathematics and Mathematical Sciences, vol. 21, pp. 1331-1340.