Bianchi Type-V Inflationary Cosmological Model with Flat Potential for Barotropic Perfect Fluid Distribution in General Relativity

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ABSTRACT

We have investigated Bianchi type-V inflationary universe with flat potential for barotropic perfect fluid distribution in general relativity. To obtain the deterministic solution of the model, we assume that the isotropic pressure p is proportional to the proper energy density ρ, which leads to $p = \gamma\rho$ and potential $V(\phi)$ as constant. The behavior of the model from physical and geometrical aspects is also discussed.

Keywords: Bianchi Type-V, Inflationary Cosmological, Barotropic Perfect fluid, Flat Potential, Cosmology.

1. INTRODUCTION

In recent years, there has been a lot of interest in cosmological models of the universe which are important in understanding the mysteries of the early stages of its evolution. In particular, inflationary models play an important role in solving a number of outstanding problems in cosmology like the homogeneity, the isotropy and the flatness of the observed universe. The standard explanation for the flatness of the universe is that it has undergone at an early stage of the evolution a period of exponential expansion named as inflation. Inflation is the rapid exponential expansion of the early universe by a factor of 10^{78} in volume driven by a negative pressure vacuum energy density.
It is well-known that self-interacting scalar fields play a vital role in the study of inflationary cosmology. Guth [15] has discussed the inflationary universe as a possible natural explanation for the observed large scale homogeneity and near critical density (flatness) of the universal expansion. A universe with only moderate anisotropy will undergo inflation and will be rapidly isotropized. Guth [15], Linde [17], Albrecht and Steinhardt [2], Abbott and Wise [1], Mijic et al. [18], Rothman and Ellis [23], Earman and Mosterin [12] and Ainsworth [3] are some of the authors who have investigated several aspects of inflationary universe in general relativity.

Several versions of inflationary scenario have been studied by number of authors viz. La and Steinhardt [16], Bali [7] discussed the significance of inflation for isotropization of universe. This inflationary scenario is also confirmed by Cosmic Microwave Background (CMB) observations (Bassett et al. [10]). In inflationary models, the universe undergoes a phase transition characterized by the evolution of Higgs field (ϕ). The inflation will take place if the potential $V(\phi)$ has flat region and in this region the ϕ field evolves slowly but the universe expands in an exponential way due to the vacuum field energy as suggested by Stein-Schabes [31]. The flat part of the potential is naturally associated with a vacuum energy which can be identified as an effective cosmological constant (Λ) and it makes the universe to enter an inflationary period. Bali and Goyal [8] investigated inflationary scenario in Bianchi Type-V space-time with variable bulk viscosity and dark energy in radiation dominated phase. Poonia et al. [19] investigated Bianchi type-VI inflationary cosmological model with massive string source in general relativity. Bali and Singh [4] have discussed Bianchi type-V inflationary universe with decaying vacuum energy (Λ). Bali and Kumari [5] have studied Bianchi type-V inflationary universe with flat potential and stiff fluid distribution in general relativity. Bali and Kumari [6] have obtained chaotic inflation in spatially homogeneous Bianchi type-V space-time. Reddy et al. [21] have discussed axially symmetric inflationary universe in general relativity. Reddy [22] have studied Bianchi Type-V inflationary universe in general relativity. Bali and Saraf [9] investigated bulk viscous creation field cosmological model with cosmological term in Bianchi type-I space-time. Sharma and Poonia [24] have obtained Bianchi Type-I inflationary cosmological model with bulk viscosity in general relativity. Poonia and Sharma [20] have discussed inflationary scenario in Bianchi type-II space with bulk viscosity in general relativity. Some more cosmological modes are also investigated by Brahma and Dewri [11], Elli et al. [13], Gron and Hervik [14], Shri Ram and Singh [25], Shri Ram et al. [26, 27], Singh et al. [28], Singh and Tiwari [29], Singh et al. [30], Tyagi and Singh [32] and Verma and Shri Ram [33, 34] to name a few.

Inspired by the above discussion, we have investigated the Bianchi type-V inflationary cosmological model with flat potential for barotropic perfect fluid distribution in general relativity. For the complete solution of the field equation, we assume that the isotropic pressure p is proportional to the proper energy density ρ and V(ϕ) is constant. The physical and geometrical aspects of the model are also discussed.

2. THE METRIC AND FIELD EQUATIONS

We consider Bianchi type-V line element in orthogonal form as:

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}(B^2dy^2 + C^2dz^2) \tag{1}$$

in which A(t), B(t) and C(t) are cosmic scale functions.

We assume the co-ordinate to be co-moving so that

$$v^1 = 0 = v^2 = v^3,\ v^4 = 1$$

In case of gravity minimally coupled to a scalar field with potential V(ϕ), is given by Stein-Schabes [31], we have

$$S = \int \sqrt{-g}\left[R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right]\mathrm{d}^4x \tag{2}$$

The Einstein's field equation (in gravitational units $c = 8\pi G = 1$), in the case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{3}$$

with

$$T_{ij} = (\rho + p)v_iv_j + pg_{ij} + \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_r\phi\partial^r\phi + V(\phi)\right]g_{ij} \tag{4}$$

Here ρ is the energy density, p the isotropic pressure, ϕ is Higgs field, V the potential and v_i is the unit time like vector.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}\partial^{\mu}\phi\right)=-\frac{dV}{d\phi} \tag{5}$$

The Einstein's field equation (3) for the line-element (1) leads to non-linear differential equations as follows

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{6}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{7}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-p-\frac{1}{2}\phi_4^2+V(\phi) \tag{8}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{3}{A^2}=\rho+\frac{1}{2}\phi_4^2+V(\phi) \tag{9}$$

$$\frac{2A_4}{A}-\frac{B_4}{B}-\frac{C_4}{C}=0 \tag{10}$$

Equation (10) leads to

$$A^2=mBC \tag{11}$$

where *m* is constant of integration.

The equation (5) for scalar field (ϕ) leads to

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=-\frac{dV}{d\phi} \tag{12}$$

where suffix '4' indicates derivative with respect to time t.

3. SOLUTION OFFIELD EQUATIONS

The field equations (6) to (10) represent a system of five independent equations with unknown parameters A, B, C, ρ, p, ϕ. To obtain the deterministic solution, we assume the following conditions:

(i) $V(\phi)$ is constant

i.e. $V(\phi) = K$ (13)

(ii) The isotropic presence (p) is proportional to the proper energy density (ρ)

i.e. $p = \gamma\rho,\ 0 \le \gamma \le 1$ and $\theta = 3H,\ \rho = 3H^2$ (14)

Equations (12) and (13) lead to

$$\phi_4 = \frac{E}{ABC} \tag{15}$$

where E is constant of integration.

The scale factor R for line-element (1) is given by

$$R^3 = A^3,\ m = 1 \tag{16}$$

From equations (7) and (8), we get

$$C^2 \left(\frac{B}{C}\right)_4 = \frac{F}{\sqrt{BC}} \tag{17}$$

where F is constant of integration.

We assume $BC = \mu$ and $\frac{B}{C} = \nu$ in equation (17), we get

$$\frac{\nu_4}{\nu} = \frac{F}{\mu^{\frac{3}{2}}} \tag{18}$$

Equations (6) and (9), lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{1}{2}\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 - \frac{4}{A^2} = \rho - p + 2V(\phi) \tag{19}$$

From equation (19), we get

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + a\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 = 2K + \frac{4}{A^2} \tag{20}$$

where $a = \frac{3\gamma - 1}{4}$.

From equation (20), we get

$$\mu_{44} + \frac{a}{\mu}\mu_4^2 = 2K\mu + 4 \tag{21}$$

Let us consider $\mu_4 = f$ and $\mu_{44} = ff^{'}$, where $f^{'} = \frac{df}{d\mu}$, in equation (21) which leads to

$$\frac{df^2}{d\mu} + \frac{2a}{\mu} f^2 = 4K\mu + 8 \tag{22}$$

After integrating equation (22) we get

$$f = \frac{d\mu}{dt} = \sqrt{b\mu^2 + h\mu + J\mu^{-2a}} \tag{23}$$

where $b = \left(\frac{2K}{a+1}\right)$, $h = \left(\frac{8}{2a+1}\right)$ and J is constant of integration.

From equation (23), we get

$$t = \int \frac{\mu^a d\mu}{\sqrt{b\mu^{2a+2} + h\mu^{2a+1} + J}} + M \tag{24}$$

where M is constant of integration.

From equation (18) and (23), we get

$$\nu = e^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}} dT + N} \tag{25}$$

where $\mu = T$ and N are constant of integration.

Therefore,

$$A^2 = T, \tag{26}$$

$$B^2 = Te^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}} dT + N} \tag{27}$$

$$C^2 = Te^{-\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}} dT + N} \tag{28}$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -\left(\frac{T^{2a}}{bT^{2a+2}+hT^{2a+1}+J}\right)dT^2 + TdX^2 + e^{2X}T\left\{e^{\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}} dT + N} dY^2 + e^{-\int \frac{FT^{a-\frac{3}{2}}}{\sqrt{bT^{2a+2}+hT^{2a+1}+J}} dT + N} dZ^2\right\} \tag{29}$$

Where $x = X$, $y = Y$ and $z = Z$

4. PHYSICAL AND GEOMETRICAL ASPECTS

For the model (29), the rate of Higgs field

$$\phi = E\int \frac{1}{\sqrt{\frac{8K}{3(\gamma+1)}T^5 + \frac{16}{3\gamma+1}T^4 + JT^{\frac{7-3\gamma}{2}}}}\,dT + P \tag{30}$$

where P is constant of integration.

For the model (29), pressure (p), Energy density (ρ), the spatial volume (R^3), the expansion (θ), shear (σ), decelerating parameter (q) and Hubble parameter (H) are given by

$$p = K + \frac{3J(1-2\gamma)}{4T^{\frac{3(\gamma+1)}{2}}} + \frac{1}{T} - \frac{12}{(3\gamma+1)T} - \frac{2E^2+F^2}{4T^3} - \frac{14K}{3(\gamma+1)} \tag{31}$$

$$\rho = \frac{1}{\gamma}\left[K + \frac{3J(1-2\gamma)}{4T^{\frac{3(\gamma+1)}{2}}} + \frac{1}{T} - \frac{12}{(3\gamma+1)T} - \frac{2E^2+F^2}{4T^3} - \frac{14K}{3(\gamma+1)}\right] \tag{32}$$

$$R^3 = T^{\frac{3}{2}} \tag{33}$$

$$\theta = \left(\frac{3}{2}\right)\left(\sqrt{\frac{8K}{3(\gamma+1)} + \frac{16}{(3\gamma+1)T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}}\right) \tag{34}$$

$$\sigma = \frac{F}{2T^{\frac{3}{2}}} \tag{35}$$

$$q = -\left[\frac{b - \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}}{b + \frac{h}{T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}}\right] \tag{36}$$

$$q < 0 \text{ if } b > \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}$$

$$q > 0 \text{ if } b < \frac{J(3\gamma+1)}{2T^{\frac{3(\gamma+1)}{2}}}$$

and

$$H = \frac{1}{2}\sqrt{\frac{8K}{3(\gamma+1)} + \frac{16}{(3\gamma+1)T} + \frac{J}{T^{\frac{3(\gamma+1)}{2}}}} \tag{37}$$

From equations (34) and (35), we get

$$\frac{\sigma}{\theta} = \frac{F}{3\sqrt{\frac{8K}{3(\gamma+1)}T^5 + \frac{16}{3\gamma+1}T^4 + JT^{\frac{7-3\gamma}{2}}}} \tag{38}$$

5. CONCLUSION

The model (29) starts expanding with Big-bang at $T = 0$. The expansion (θ) decreases as time increases and when $T \to \infty$ universe expand constantly.

The Spatial Volume (R^3) increases as time increases. It represents inflationary scenario of universe containing massless scalar field with flat potential.

Since $\lim_{T\to\infty} \frac{\sigma}{\theta} = 0$, for large values of time T, the model is not anisotropic.

The Hubble parameter (H) is initially large but decreases as time increases and when $T \to \infty$ it is constant. The energy density and pressure of the model is initially large.

When $T \to \infty$ then deceleration parameter tends to -1, so the model represent accelerating phase of universe.

The rate of Higgs field (ϕ) is initially large, but decreases as time increases and constant for $T \to \infty$. The model has point type singularity at $T = 0$.

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