GEOMETRIC MODELING FOR HUMAN POPULATION GROWTH

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ABSTRACT:

Simon, (1977) has studied the Economics of Population Growth. After then, Boserup (1981) has studied Population and Technological Change: A Study of Long-Term Trends. Also, Turchin, (2003) has studied Complex Population Dynamics: a Theoretical/Empirical Synthesis. And Pastor (2008) has studied Mathematical Ecology of Populations and Ecosystems. In this paper, we have studied Geometric modeling for human population growth, introducing the fundamental principles of population growth and various mechanisms that regulate population growth.

Key Words: Geometric Modeling, Population Growth, Dynamics, biological and environmental.

1. INTRODUCTION:

A Geometrical model is a explanation of a classification with Mathematical concepts, geometry along with language. The progression of mounting a geometrical model is termed as Mathematical modeling. The model may well facilitate to elucidate a structure and to study the effect of dissimilar component and to create prediction concerning manners.

Population describes a collection of individuals of some variety occupying a particular area at definite time. A few characteristic of population that is importance to biologist take in the population solidity, birth rate and death rate. If there is immigration into the population, or emigration out of it, then the immigration and emigration rate are also of interest together these population parameters or characteristic, describe how the population density changes over time. The ways in which population densities fluctuate increasing, decreasing or both over time is a subject of population dynamic. Population dynamics is a branch of life science that studies the age and size composition of population as dynamical system and the biological and environmental process driving them such as birth and death rate and by immigration and emigration.

Change in population density = (Birth + Immigration) – (Death + Emigration).

Thomas R. Malthus (1766-1834) introduced the concept of exponential population growth in Europe, suggesting that the rate of population increase outpaces food production, potentially leading to worldwide famine in the future. While he couldn't anticipate the impact of modern technological advancements on food supplies, his recognition of the rapid geometric growth of the population (1, 2, 4, 8…) rather than linear growth (1, 2, 3, 4…) underscores the swift potential for numerical escalation. So, he propounded exponential growth model for human population in the first edition of his famous book entitle "An essay on the principle of population" published in 1798.

2. FORMULATION OF THE MODEL:

During formulating the population growth model, Malthus made the following three assumptions:

(i) The population is sufficiently large.

(ii) Population is homogeneous, that is, it is evenly spread over the living space.

(iii) There is no limitation to growth i.e., no limitations of food, space and so on.

Population changes only by the occurrence of births and deaths.

Let x (t) be the size of the population at that time t which is taken to be positive integer with x (0) = x_o. Let us assume that all changes in the population result from births and deaths, therefore, there is no immigration or emigration. Let B (t) and D (t) denote respectively, the numbers of births and deaths at time t. Then per capita birth rate b and per capita death rate m are given by

$$b = \frac{1}{x(t)}\frac{dB(t)}{dt} \tag{2.1}$$

and

$$m = \frac{1}{x(t)}\frac{dD(t)}{dt} \tag{2.2}$$

Therefore per capita growth rate of the population at any time t is given by:

$$\frac{1}{x(t)}\frac{dB(t)}{dt} - \frac{1}{x(t)}\frac{dD(t)}{dt} = b - m \quad \text{or} \quad \frac{1}{x(t)}\frac{d(B-D)}{dt} = b - m \tag{2.3}$$

$$\frac{1}{x(t)}\frac{dx}{dt} = b - m = \alpha \text{ (constant), say}$$

or $$\frac{1}{x}\frac{dx}{dt} = \alpha = (Birth\, rate - death\, rate)$$

or $$\frac{dx}{dt} = \alpha x; \quad \text{where } x(0) = x_0 \tag{2.4}$$

The difference between the per capita birth and death rates $\alpha = b - m$, plays a particularly important role and is known as the intrinsic rate of growth or net growth rate. Separating the variables of (2.4), we get

$$\frac{dx}{x} = \alpha\, dt$$

On integrating both side, we get

$$\log x = \alpha t + C, \text{ where C is a constant.}$$

Initially, when $t = 0,\ x = x_0$, so that $C = \log x_0$

Therefore, $\log x = \alpha t + \log x_0$

or $\log (x/x_0) = \alpha t$

or $$x(t) = x_0\, e^{\alpha t} \tag{2.5}$$

Equation (2.5) gives the population size at any time.

Also, let us assume that, increase $\alpha > 0$, at time T_1 population will become double of its initial population i.e., $x(T_1) = 2x_0$, then from (2.5), we have

$$2x_0 = x_0\, e^{\alpha T_1} \quad or \quad e^{\alpha T_1} = 2$$

or $$T_1 = \frac{1}{\alpha} \log 2 \tag{2.6}$$

This time T_1 is called doubling period of population and it is independent from initial population. It depends on net growth rate. Thus greater the value of α_1, smaller the doubling period.

In case $\alpha < 0$, let us assume that at time T_2, the population will become half its initial value x_0 i.e. $x(T_2) = \frac{1}{2}x_0$, then from (2.5), we have

$$\frac{1}{2}x_0 = x_0\, e^{\alpha T_2} \qquad \text{or} \qquad e^{\alpha T_2} = \frac{1}{2}$$

$$\text{or} \qquad T_2 = \frac{1}{\alpha} log\left(\frac{1}{2}\right) \qquad \text{or} \qquad T_2 = -\frac{1}{\alpha} log\ 2$$

$$\text{or} \qquad T_2 = \frac{1}{|\alpha|} log\ 2 \qquad (\text{since } \alpha < 0 \text{ so } |\alpha| = -\alpha)$$

This time T_2 is also important from initial population and also $T_2 > 0$. This time T_2 is called half-life period of the population and greater the value of α, smaller the value of t_2 ,

(i) Now if we plot $\frac{1}{x}\frac{dx}{dt}$ against *x*, we get a straight line parallel to $t - axis$

(ii) If we plot $\frac{dx}{dt}$ against x, we get straight line through the origin.

EFFECTS OF IMMIGRATION AND EMIGRATION ON POPULATION:

First of all we define Immigration and Emigration. The process, in which some individual are added from outside to the population is known as Immigration. The process, in which some individual went out of the population, is known as Emigration. If immigration into the population from outside is at a rate proportional to the population size, the effect is equivalent to increasing the birth rate. Similarly, if emigration from the population is at rate proportional to the population size, the effect is equivalent to increasing in the death rate.

If emigration and immigration take place constant rate I and e respectively, then the rate of change in population is given by

$$\frac{dx}{dt} = \alpha x + i - e = \alpha x + \beta \qquad \text{where } i - e = \beta(constant), say$$

$$\text{or} \qquad \frac{dx}{\alpha x + \beta} = dt$$

$$\text{or} \qquad \frac{1}{\alpha} Log\ (\alpha x + \beta) = t + D, \qquad \text{where } D \text{ being a constant}$$

Initially, when $t = 0$, $x = x_0$, so that

$$D = \frac{1}{\alpha} \log (\alpha x_0 + \beta)$$

Therefore $\left(\frac{1}{\alpha}\right) \log (\alpha x + \beta) = t + \left(\frac{1}{\alpha}\right) \log (\alpha x_0 + \beta)$

or $\left(\frac{1}{\alpha}\right) \log \left(\frac{\alpha x + \beta}{\alpha x_0 + \beta}\right) = t$

or $\log \left(\frac{\alpha x + \beta}{\alpha x_0 + \beta}\right) = \alpha t$ or $\left(\frac{\alpha x + \beta}{\alpha x_0 + \beta}\right) = e^{\alpha t}$

$$\alpha x + \beta = (\alpha x_0 + \beta)\, e^{\alpha t}$$

or $x = \frac{1}{\alpha} [\, (\alpha x_0 + \beta)\, e^{\alpha t} - \beta \,] = (x_0 + \frac{\beta}{\alpha})\, e^{\alpha t} - \frac{\beta}{\alpha}$

or $x(t) = x_0\, e^{\alpha t} + \frac{\beta}{\alpha} (e^{\alpha t} - 1)$

This equation gives the population at time t in the presence of immigration and emigration. We see that the growth the of the population depend upon two terms: first the exponential function $x_0\, e^{\alpha t}$, which depend upon α alone, and second $\frac{\beta}{\alpha}(e^{\alpha t} - 1)$, which is also rapidly increasing but depends on both α and β and the contribution of due to net immigration.

APPLICATIONS OF THE MODEL:

The applications of this geometric model possibly will facilitate to elucidate an organization and nature phenomena likely, Microbiology (growth of bacteria)**,** Conservation biology (restoration of disturbed population)**,** Insects rearing (Prediction of yield), Plant or insect quarantine (population growth of introduced species), Fishery (prediction of fish dynamics). Some other applications are military campaigns, resource allocation for water distribution, dispatch of distributed generators, lab experiments, transport problems, communication problems, among others.

LIMITATIONS OF THE MODEL:

In an ideal scenario where the availability of space, food, and resources doesn't limit growth, biological populations typically exhibit initial exponential growth. However, as the population size increases significantly, overcrowding begins to impede growth. Consequently, the net growth rate is not constant; instead, it varies depending on the population's size.

PROPOSITIONS OF THIS MODEL:

The principles outlined by Malthus can be concisely captured in the ensuing statements:

1. "Food is an indispensable requirement for human survival, and consequently, it serves as a significant constraint on population growth. In simpler terms, the population is inherently restricted by the availability of sustenance (i.e., food)."
2. "The rate of population growth surpasses that of food production. While the population expands exponentially, food production grows at a linear rate."
3. "Population tends to rise when the resources for sustenance increase, unless it is restrained by significant checks".

CONCLUSIONS:

The Malthusian theory suggests that, during the agricultural stage of development, social surpluses beyond the maintenance of subsistence consumption was channeled primarily into population growth with negligible lengthy sprint ejects on income per capita. As such, at any point in time, population density in a given region would have largely reflected its carrying capacity, determined by the effective resource constraints that were binding at that point in time. The Malthusian theory predicts that provincial deviation in population solidity in the lengthy sprint would eventually reproduce variations in earth production and biogeographic attributes.

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