Inventory model for Imperfect Production System with and without Disruption and Rework

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Abstract

This study develops a mathematical production model for a single item of imperfect quality with and without disruption. The disruption in a production system may occurs due to labor strick, machine break down, power breakdown wheather problem and political issues etc. With this types types of problems, production system may produce imperfect quality items. In this paper, we have attempted to develop a production policy for an imperfect production system with and without disruption and we have compared both the situations with and without disruption. In this study we have consider that the rework process is started just after the regular production interval. A mathematical formulation has been derived for profit function and optimised a regular production time, rework process time and disrupted production time inerval. This study is analyzed through analytically, graphically and numerically.
Keywords: Imperfect production, inventory, rework disruption.

1 Introduction

Today's business era are competitive in all dimensions. Especially the competitive dimensions are cost, quality, delivery and flexibility. These all dimensions relate to manufacturing process and control technology, capacity, facilities, workforce planning etc. Every researchers as well as practitioners in the production industries always have to confirm economical production level, and finding the most economical order quantity. therefore the manufacturing sector is another aspect which attracts practitioners and researchers to recover overages and shortages of the items. This study tackle a problem of economical production quantity of imperfect quality items and to maintain the quality and quantity of items also rework process is considered. The disruption in a production system occurs due to machine breakdown, labor strikes, political issues, wheather problem unexpected events etc. These types of uncertainties production system may produce some imperfect items, so the problem become more complex with the disruption and imperfect production.

The classical economical order quantity model does not consider the disruption in production system and also assume all products/manufactured quantity are of perfect quantity.However, in real life production system due to disruption and other failures generate the imperfect items. The imperfect quality items, may be classified into two types, first one is imperfect quality items which may convert into perfect items through the rework process, called as reworkable items. Another one is imperfect items which can not be convert into perfect items, called as scrapped items. Harris (1913) was the first mathematician who made the first Economical manufacturing quantity model on inventory management. Kul *et al. (1995) presented an economical manufacturing quantity model in which they applied rework process on imperfect quality items at the end of regular production. They have developed a simple mathematical method to compute the optimal optimal outputs.*

Hayek et al. (2001) developed a finite production inventory model in which they studied and analyzed the effect of imperfect quality items to minimize the total inventory cost. Chiu et al. (2007) developed an optimal replenishment policy for imperfect quality items with two different approach. First one is An EMQ model has been derived with the help of linear differential calculus approach. Anothe one is An EMQ model has been derived with the help of algebraic approach and suggested a differential calculus approach is much better than the algebraic approach. Chiu et al. (2008) developed a finite production inventory model in which they assumed imperfect quality items are produced randomly. Furthemore they applied a rework process on imperfect quality items by assuming that a portion of defective items which become scrap be discarded before the rework on repairable defective items. Finally they suggested an optimal policy for economical lot-size and back ordering policy.

Haji et al. (2008) considered perfect and imperfect items both in their production inventory system and they applied rework to on imperfect quality items to convertconvert into perfect quality items. The rework rate have been assumed to be a function of the random variable. Talaizadeh et al. (2013) sugestted an EPQ model with random defective item's production rate by considering with rewokable and non reworkable items and allowing with shortage. They derived the optimal period time of back order quantity optimizing with the total expected cost. Chiu et al. (2014) introduced a study in which they optimized the total manufacturing cost, delivery cost for the EPQ by incorporating the rework process on imperfect quality items. Kumar et al. (2016) suggested a probabilistic inventory model for deteriorating items with constant deterioration rate and ramp type demand under stock dependent consumption. Sang et al. (2016) developed the production system for imperfect production under case of two-echelon supply chain consisting of a single manufacturer and single retailer. They derived the optimal selling price, replenishment cycle and a number of shipment for deteriorating items.

Kumar et al. (2017) suggested a model by considering a multiple production stages and single rework stage of EOQ model for imperfect quality items. They also derived the number of production setup and optimize production time inteval by optimising the total inventory cost. He et al. (2010) introduced a production inventory model for a deteriorating item and *the disruption in production is considered under different situations. This study helps to the production industries to reduce the losses caused by disruption production. Khedlekar et al. (2014) derived a production inventory model for deteriorating item with disruption in production system and analyzed the model under the various situations. Chiuet al. (2011) developed some special cases in EMQ model considering with rework and multiple shipments. They optimized the total quantity in terms of production rate and regular production time. Nigwal et al. (2022) developed a trade credit financing policy for three layer echelon supply chain for a supplier, a manufacturer and a retailer. The proposed model optimized production rate and selling price for the manufacturer and the retailer under an imperfect quality items.*

Khedlekar et al.(2017) developed an EMQ model for deteriorating items cosidering disruption in production system. study optimized production time before and after the system gets disrupted. Further study derived the model for optimizing the shortage of the products. Nigwal et al.(2022) suggested a retailers ordering policy for production syetem of imperfect quality item's in which they applied the learning curve effect on inspection process on each batch of imperfect quality items. Gupta et al.(2021) developed an EMQ production system for imperfect items for two situation which are production without and with disruption. They optimized minimum total cost for both the situations. They also use trade credit financing scheme on retailers business policy. Based on above literature review we motivated to develop a model for two different cases, first one is an EMQ model which depends on regular production time assuming with constant demand rate of imperfect quality items. The another one is an EMQ model with disruption in regular production which depends on production time assuming constant demand rate.In the first case, we optimized regular production time and total production cost and in the second case, we optimized disrupted production time and total production cost.

2 Notations & Assumptions

In this study, the following notations are used.

p_1 : *Finite Production rate per unit,*

ϕ : *Demand rate per unit time time T,*

ν_1 : *A fraction of imperfect quality items,*

θ_1 : *Production per unit time of imperfect quality items,*

c : *Production cost per unit itemes,*

c_r : *Rework cost per unit items,*

p_2 : *Rework production rate per unit time,*

k_1 : *Delivery cost per lot size,*

θ_2 : *Production rate per unit time of scrapable items during the rework process*

ν_2 : *A fraction of scrapable items after the rework process,*

h_1 : *Holding cost per unit quantity per unit time,*

c_c : *Disposal cost per unit of scraped items,*

h_2 : *Holding cost per unit of reworkable items,*

t_1 : *Regular production time interval,*

t_2 : *Time interval required for reworking on imperfect quality items,*

c_t : *Delivery cost per unit items,*

t_3 : *Time to send finished items,*

n : *Number of shipments,*

k : *Setup cost per setup,*

T : *Total time interval,*

Q : *Total quantity of items in time interval T,*

$I(t)$: *On hand inventory level of perfect quality items at time t,*

$I_d(t)$: *On hand inventory of imperfect quality items, at time t,*

$TC(t_1)$: *Total cost function (production inventory cost +delivery cost) per cycle,*

$TC_1(t_1^p)$: *Total cost function (production inventory cost+ delivery cost), per cycle,*

δP : *Reduced production rate due to occurs of disruption,*

t_d : *Regular production time when production system becomes disrupted,*

t_1^p :*Time duration when production system is disrupted,*

t_2^p :*Time duration of rework when production system is disrupted,*

$TC^*(t_d^p)$: *Total production cost and inventory delivery cost for the case of disrupted production system,*

$TC_1^*(t_d^p)$: *Total production cost and delivery cost per cycle for the case of disrupted production system, (subcase).*

As per realistic situations the used assumptions are given below.

a. *This study is based on that production system which produced imperfect quality items*

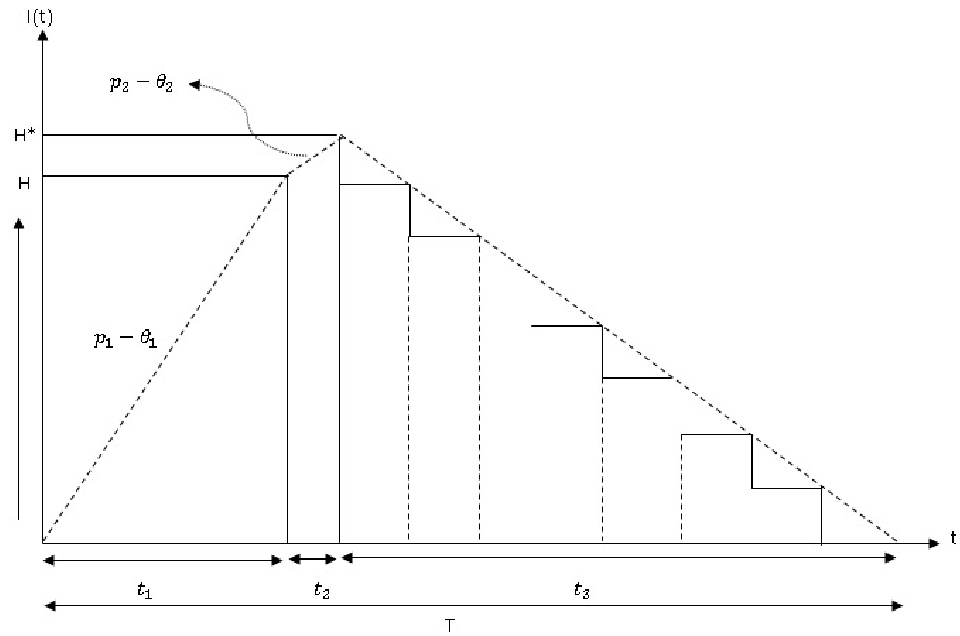

Figure 1: On Hand Inventory Structure of Perfect Items without Disruption

b. *This study considers demand rate is constant*

c. *The production system generates defective items randomly at the rate* ν_1 $(0 \leq \nu_1 \leq 1)$,

d. *If the produced defective items are* θ_1. *then we also assumed that the total defective items are found in two types, first type is reworkable and another type one is non reworkable which is called scrap items.*

e. *The rework process on defective items starts just after the end of regular production,*

f. *Let* ν_2 $(0 \leq \nu_2 \leq 1)$ *denotes the quantity of defective items, which can not be reworkable during the rework process, and becomes scrap.*

g. *During the rework process, only perfect quality items are delivered to the customers.*

Case-I:Formulation for Imperfect Production with Reworkable and Few Scrapable Items

In this section we assume a production process starts with a constant production rate $p_1 > \phi$, *because of a fraction of imperfect production of defective items produced at rate* ν_1. *Consequently the quantity of total defective items are* $\theta_1 = p_1\nu_1$. *Defective items have rework process is started at a rate of* p_2 *after the end of regular production to convert into perfect items. Let the rework process randomly generate scrap items at a rate* ν_2, *then the total* *quantity of scraped items are $\theta_2 = p_2\nu_2$. The finished items of perfect quality are prepared to selling to customers in equal n parts of interval time t_3. Let regular production time is t_1, rework process time interval of a imperfect quality items is t_2, delivery time interval of the finished product is t_3. Again let on-hand inventory of perfect quality items is H and on-hand inventory of of perfect quality items after rework process is H^*, then the production cycle length T can be written as*

$$T = t_1 + t_2 + t_3 \tag{2.1}$$

let Q, be the total quantity including perfect, imperfect quality items and and scraped items. Then can be determine

$$Q = p_1 t_1 \tag{2.2}$$

Quantity of perfect quality items is

$$H = (p_1 - \theta_1)t_1 \tag{2.3}$$

Quantity of perfect and imperfect quality items

$$H^* = (1 - \nu_1\nu_2)p_1 t_1 \tag{2.4}$$

The rework process time interval is

$$t_2 = \left(\frac{\nu_1 p_1}{p_2}\right) t_1 \tag{2.5}$$

The delivery time interval of finished items is

$$t_3 = T - t_1 - t_2 = \left(\frac{p_1(1-\nu_1\nu_2)}{\theta} - \frac{\nu_1 p_1}{p_2} - 1\right) t_1 \tag{2.6}$$

Total number of defective items at the time t_1 is $\theta_1 t_1$, then

$$\theta_1 t_1 = p_1\nu_1 t_1, \qquad \text{where} \;\; \nu_1 = \frac{\theta_1}{p_1} \tag{2.7}$$

Total number of scrapped items in the whole cycle T is $\nu_2\theta_1 t_1$, then

$$\nu_2\theta_1 t_1 = p_1\nu_1\nu_1 t_1, \qquad \text{where} \;\; \nu_2 = \frac{\theta_2}{p_2} \tag{2.8}$$

Theorem 2.1 *If n is the number of batches of installment of fixed quantity of finished items, which became to deliver to the costumer in a fixed time interval. Then the holding cost of perfect quality items during the time interval t_3 is given by*

$$h_1 \left(\frac{n-1}{2n}\right) H^* t_3 \tag{2.9}$$

Proof. *The inventory level of finished items is shown fig.(1), the proof of this theorem can be understood by induction method.*
For $n = 1$, total holding costs in delivery time t_3 is zero,
For $n = 2$, total holding costs in delivery time t_3 become

$$h_1\left(\frac{H^*}{2}\times\frac{t_3}{2}\right) = h_1\left(\frac{1}{2^2}\right)H^*t_3$$

For $n = 3$, total holding costs in the delivery time iterval t_3 is

$$h_1\left(\frac{2H^*}{3}\times\frac{t_3}{3}+\frac{H^*}{3}\times\frac{t_3}{3}\right) = h_1\left(\frac{2+1}{3^2}\right)H^*t_3$$

For $n = 4$, total holding costs in delivery time iterval t_3 is

$$h_1\left(\frac{3H^*}{4}\times\frac{t_3}{4}+\frac{2H^*}{4}\times\frac{t_3}{4}+\frac{H^*}{4}\times\frac{t_3}{4}\right) = h_1\left(\frac{3+2+1}{4^2}\right)H^*t_3$$

Therefore, the general term for total holding costs the during delivery time iterval t_3 can be derived by

$$h_1\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n} i\right)H^*t_3 = h_1\left(\frac{1}{n^2}\right)\left(\frac{n(n-1)}{2}\right)H^*t_3 = h_1\left(\frac{n-1}{2n}\right)H^*t_3$$

The delivery cost can be derived as the total delivery costs for n shipments in a whole cycle is

$$n\left(k_1 + c_t\left(\frac{H^*}{n}\right)\right) = nk_1 + c_t(1-\nu_1\nu_2)p_1t_1 \tag{2.10}$$

Thus, the total production cost and the delivery cost in the time interval t_1

$$\begin{aligned}TC(t_1) =& cp_1t_1 + k + c_r\left(\nu_1p_1t_1\right) + c_s\left(\nu_1\nu_2p_1t_1\right) + nK_1 + c_t\left((1-\nu_1\nu_2)p_1t_1\right)\\ &+ h_2.\frac{p_2t_2}{2}t_2 + h_1\left(\frac{H+\theta_1t_1}{2}t_1 + \frac{H+H^*}{2}t_2\right) + h_1\left(\frac{n-1}{2n}\right)ht_3.\end{aligned}$$

Total production inventory and the delivery cost at time t_1 is

$$\begin{aligned}TC(t_1) =& (k+nk_1) + \left(c + c_r\nu_1 + c_s\nu_1\nu_2 + c_t(1-\nu_1\nu_2)\right)p_1t_1\\ &+ h_2\frac{(\nu_1p_1)^2}{2p_2}{t_1}^2 + h_1\left(\frac{p_1}{2}+\frac{\nu_1{p_1}^2}{2p_2}(1-\nu_1\nu 2)\right){t_1}^2 + h_1\left(\frac{n-1}{2n}\right)\\ &\left(\frac{{p_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)p_1 - \frac{\nu_1(1-\nu_1\nu_2){p_1}^2}{p_2}\right){t_1}^2\end{aligned} \tag{2.11}$$

By incorporating all the cost componets, the total cost function can be written as

$$\begin{aligned}TC(t_1) = \frac{\phi}{p_1(1-\nu_1\nu_2)}&\left[\frac{(k+nk_1)}{t_1} + \Big(c + c_r\nu_1 + c_s\nu_1\nu_2 + c_t(1-\nu_1\nu_2)\Big)p_1\right.\\ &+ h_2\frac{(\nu_1p_1)^2}{2p_2}t_1 + h_1\left(\frac{p_1}{2}+\frac{\nu_1{p_1}^2}{2p_2}(1-\nu_1\nu_2)\right)t_1\\ &\left.+ h_1\left(\frac{n-1}{2n}\right)\left(\frac{{p_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)p_1 - \frac{\nu_1(1-\nu_1\nu_2){p_1}^2}{p_2}\right)t_1\right]\end{aligned} \tag{2.12}$$

Theorem 2.2 *If the production rate per unit time is p_1 and reguler production time interval is t_1, then the optimal replenishment lot size is $Q = pt_1$, where t_1 is given by the equation* (2.15)

Proof. *the reguler production time interval t_1 can be obtained by minimizing condition of cost function at time t_1, so differentiate* (2.12) *with respect to t_1, we get*

$$\begin{aligned}\frac{TC(t_1)}{dt_1} = \frac{\phi}{p_1(1-\nu_1\nu_2)}\Bigg[&\frac{-(k+nk_1)}{{t_1}^2} + h_2\frac{(\nu_1 p_1)^2}{2p_2} + h_1\left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2}(1-\nu_1\nu_2)\right)\\ &+ h_1\left(\frac{n-1}{2n}\right)\left(\frac{{p_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)p_1 - \frac{\nu_1(1-\nu_1\nu_2){p_1}^2}{p_2}\right)\Bigg]\end{aligned} \tag{2.13}$$

and the second order derivative is

$$\frac{d^2TC(t_1)}{d{t_1}^2} = \frac{2\phi(k+nk_1)}{p(1-\nu_1\nu_2){t_1}^3} \geq 0. \quad \textit{where } (1-\nu_1\nu_2) \geq 0. \tag{2.14}$$

At the optimal time ${t_1}^*$, the equation (2.13) *become zero i.e.*

$$\begin{aligned}\frac{(k+nk_1)}{{t_1}^2} \quad &- \quad h_2\frac{(\nu_1 p_1)^2}{2p_2} - h_1\left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2}(1-\nu_1\nu_2)\right)\\ &- \quad h_1\left(\frac{n-1}{2n}\right)\left(\frac{{p_1}^2(1-\nu_1\nu_2)}{\phi} - (1-\nu_1\nu_2)p - \frac{\nu_1(1-\nu_1\nu_2){p_1}^2}{p_2}\right) = 0.\end{aligned}$$

So, the optimal time ${t_1}^*$ is

$${t_1}^* = \sqrt{\frac{(k+nk_1)}{h_2\frac{(\nu_1 p_1)^2}{2p_2} + h_1\left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2\xi}{2p_2}\right) + h_1\left(\frac{n-1}{2n}\right)\left(\frac{{p_1}^2\xi}{\phi} - \xi p - \frac{\nu_1\xi {p_1}^2}{p_2}\right)}} \tag{2.15}$$

where, $\xi = (1-\nu_1\nu_2)$

corollary 2.1 *If $\nu_2 = 0$, therefore all the imperfect quality items are reworkable.*

2.1 Subcase: when $\nu_2 = 0$

When $\nu_2 = 0$, then total production inventory and the delivery cost per cycle at the time t_1 is

$$\begin{aligned}TC_1(t_1) =& cp_1t_1 + k + c_r(\nu_1 p_1 t_1) + nk_1 + c_t p_1 t_1 + h_2\frac{p_2}{2}.{t_2}^2\\ &+ h_1\left(\frac{H+\theta_1 t_1}{2}t_1 + \frac{H+H^*}{2}t_2\right) + h_1\left(\frac{n-1}{2n}\right)ht_3\end{aligned}$$

$$
\begin{aligned}
TC_1(t_1) &= (k+nk_1) + (c + c_r\nu_1 + c_t)\, p_1 t_1 + h_2 \frac{(\nu_1 p_1)^2}{2p_2} {t_1}^2 \\
&\quad + h_1 \left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right) {t_1}^2 + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{p_1^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right) {t_1}^2
\end{aligned} \tag{2.16}
$$

By using the substitutions in terms of t_1 we can write the production inventory and delivery cost at time t_1 as

$$
\begin{aligned}
TC_1(t_1) = \frac{\phi}{p_1} \Bigg[\frac{(k+nk_1)}{t_1} &+ \Big(c + c_r\nu_1 + c_t \Big) p_1 + h_2 \frac{(\nu_1 p_1)^2}{2p_2} t_1 + h_1 \left[\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right] t_1 \\
&+ h_1 \left(\frac{n-1}{2n} \right) \left[\frac{{p_1}^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right] t_1 \Bigg]
\end{aligned} \tag{2.17}
$$

Theorem 2.3 *If $\nu_1 \geq 0$, then the total production inventory and the delivery cost $TC_1(t_1)$ is convex function with respect to t_1, and optimal regular production production time t_1^* is given by the equation*

$$
{t_1}^* = \sqrt{\frac{(k+nk_1)}{h_2 \frac{(\nu_1 p_1)^2}{2p_2} + h_1 \left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right) + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{p_1}^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right)}} \tag{2.18}
$$

proof. *the reguler production time interval t_1 can be obtained by minimizing condition of cost function at time t_1, so differentiate (2.17) with respect to t_1, we get*

$$
\begin{aligned}
\frac{dTC_1(t_1)}{dt_1} = \frac{\phi}{p_1} \Bigg[&\frac{-(k+nk_1)}{{t_1}^2} + h_2 \frac{(\nu_1 p_1)^2}{2p_2} + h_1 \left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right) \\
&+ h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{p_1}^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right) \Bigg]
\end{aligned} \tag{2.19}
$$

At the optimal time ${t_1}^*$ the equation (2.19), becomes zero i.e.

$$
\frac{(k+nk_1)}{{t_1}^2} - h_2 \frac{(\nu_1 p_1)^2}{2p_2} - h_1 \left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right) - h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{p_1}^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right) = 0.
$$

So, the optimal regular production time can be derived as

$$
{t_1}^* = \sqrt{\frac{(k+nk_1)}{h_2 \frac{(\nu_1 p_1)^2}{2p_2} + h_1 \left(\frac{p_1}{2} + \frac{\nu_1 {p_1}^2}{2p_2} \right) + h_1 \left(\frac{n-1}{2n} \right) \left(\frac{{p_1}^2}{\phi} - p_1 - \frac{\nu_1 {p_1}^2}{p_2} \right)}} \tag{2.20}
$$

the second order derivative

$$
\frac{d^2 TC_1(t_1)}{d{t_1}^2} = \frac{2\phi(k+nk_1)}{p_1 {t_1}^3} > 0, \tag{2.21}
$$

fulfill the condition of cost minimization at ${t_1}^*$. Therefore the optimal repleshment lot size is

$$
Q = pt_1^*
$$

when $\nu = 0$ i.e all the imperfect items are becomes perfect items.

3 Formulation for Imperfect Production with Rework and Few Scrapable Items and Disruption

In this section we assume a production process starts with constant production at a rate $p_1 > \phi$, becuase of a fraction of imperfect production of defective items produced at rate ν. Let t_d be the regular production time and after some time the production system is get disrupted at time $t_1^{p_1}$,the production reduced at a rate δp_1. After time $t_1^{p_2}$ the rework process starts with rate p_2, during the rework process. The finished items of perfect quality are prepared to selling to customers in equal n parts of interval time t_3. Let regular production time iterval is t_1, disrupted production time is $t_1^{p_1}$, rework process time interval of imperfect quality items is $t_2^{P_1}$, delivery time period of finished product is t_3, on hand inventory levels H_1, H_2 and H_3 respectably, then the length of production cycle time T can be written as.

$$T = t_d + t_1^{p_1} + t_2^{p_1} + t_3 \tag{3.1}$$

let Q, be the total quantity including perfect items, imperfect items and scrap items. Then Q can be written as

$$Q = Pt_d + (p_1 + \delta p_1)t_1^{p_1} \tag{3.2}$$

Inventory level before disruption is

$$H_1 = (p_1 - \theta_1)t_d \tag{3.3}$$

Inventory level after disruption is

$$H_2 = Q - \theta(t_d + t_1^{P_1}) \tag{3.4}$$

Inventory level after rework process is

$$H_3 = \left(1 + (p_2 - \theta_1)\frac{\nu_1}{p_2}\right) Q - \theta_1(t_d + t_1^{p_1}) \tag{3.5}$$

Time duration when production system is get disrupted is

$$t_2^P = \frac{\nu_1}{p_2}Q \tag{3.6}$$

Delivery time duration to send the finished items is

$$t_3 = T - t_d - t_1^p - \frac{\nu_1}{p_2}Q \tag{3.7}$$

Theorem 3.1 *Suppose H^* is the total production inventory level without disruption and let H_3 be a total inventory level while system get disrupted. Then the production time t_1^p is*

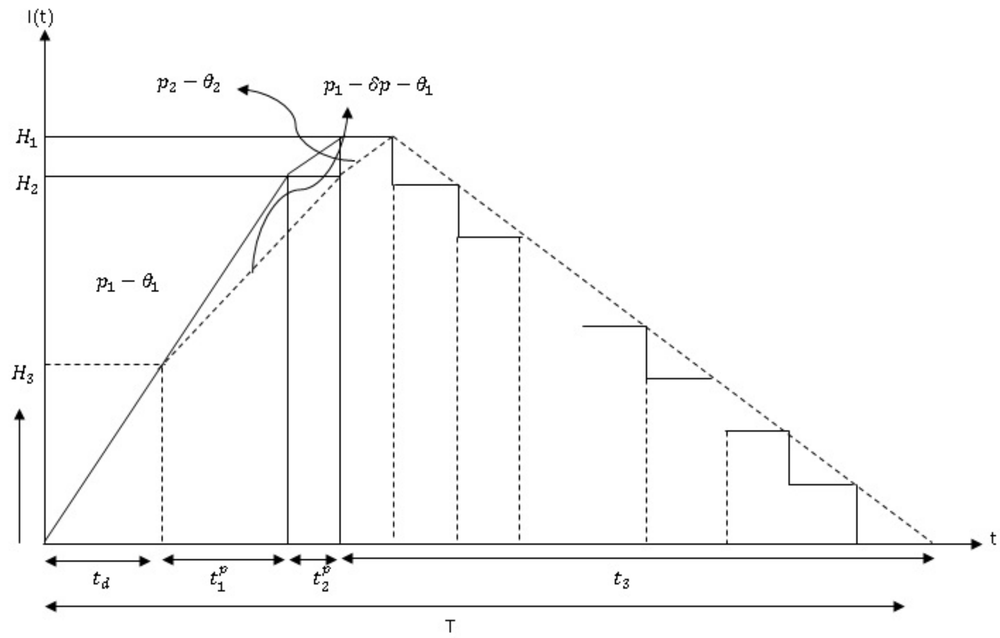

Figure 2: On Hand Inventory Structure of Perfect Items with Disruption

$$t_1^p = \frac{(1-\nu_1\nu_2)(t_1-t_d)p_1}{(1+(1-\nu_2)\nu_1)(p_1+\delta p_1)-\nu_1 p_1} \tag{3.8}$$

proof. *As per inventory system depicted in the fig.2, the total production inventory level must be same for the both of the cases, then*

$$H_3 = H^*$$

Using the values of H_3, H^* *respectively from the equations* (2.4) *and* (3.5)

$$(1+(p_2-\theta_1)\frac{\nu_1}{p_2})Q - \theta_1(t_d+t_1{}^p) = (1-\nu_1\nu_2)p_1t_1$$

$$(1+(1-\nu_2)\nu_1)\,p_1t_d + (1+(1-\nu_2)\nu_1)\,(p+\delta p)t_1^P - \nu_1p_1t_d - \nu_1p_1t_1^p = (1-\nu_1\nu_2)p_1t_1$$

$$((1+(1-\nu_2)\nu_1)\,(p_1+\delta p_1) - \nu_1p_1)\,t_1^p = (1-\nu_1\nu_2)p_1t_1 - ((1+(1-\nu_2)\nu_1)-\nu_1)p_1t_d$$

therefore, the production time after disruption

$$t_1^p = \frac{(1-\nu_1\nu_2)(t_1-t_d)p_1}{((1+(1-\nu_2)\nu_1)\,(p_1+\delta p_1)-\nu_1 p_1)} \tag{3.9}$$

from equation (3.6) *the rework production time after disruption* t_2^P *is*

$$t_2^p = \frac{\nu_1}{p_2}p_1t_1^p$$

$$t_2^p = \frac{(1-\nu_1\nu_2)(t_1-t_d)\nu_1{p_1}^2}{p_2\,((1+(1-\nu_2)\nu_1)\,(p_1+\delta p_1)-\nu_1 p_1)} \tag{3.10}$$

The total production inventory and delivery cost at the time t_d^p is

$$TC^*(t_1^p) = \frac{\phi}{(1-\nu_1\nu_2)}\left(\frac{\tau}{(t_d+t_1^p)} + (\pi+\kappa+\eta)p_1(t_d+t_1^P)\right) \tag{3.11}$$

where

$$\tau = \frac{k+nk_1}{p_1}$$

$$\pi = h_2\frac{{\nu_1}^2 p_1}{2p_2} + h_1\left(\frac{1}{2} + \nu_1(1-\nu_1\nu_2)\frac{p_1}{2p_2}\right)$$

$$\kappa = h_1\frac{n-1}{n}\left(\frac{p_1(1-\nu_1\nu_2)^2}{\phi} - (1-\nu_1\nu_2) - \frac{(1-\nu_1\nu_2)\nu_1 p_1}{p_2}\right)$$

$$\eta = (c + c_r\nu_1 + c_s\nu_1\nu_2 + c_t(1-\nu_1\nu_2))$$

corollary 3.1 *If the scrap items rate is $\nu_2 = 0$, i.e. all imperfect items are reworkable.*

3.1 Subcase:when $\nu_2 = 0$

Theorem 3.2 *Suppose H^* is the total production inventory level without disruption and let H_3 be a total inventory level while system get disrupted. Then the production time t_1^p is*

$$t_1^p = \frac{(t_1 - t_d)p_1}{(1+\nu_1)(p_1+\delta p_1) - \nu_1 p_1} \tag{3.12}$$

proof. *As per inventory system depicted in the fig.2, the total production inventory level must be same for the both of the cases, then*

$$H_3 = H^*$$

Using the values of H_3, H^* respectively from the equations (2.4) *and* (**??**)

$$(1+\nu_1)P_1t_d + (1+\nu_1)(P_1+\delta P_1)t_1^P - \nu_1 P_1 t_d + \nu_1 P_1 t_1^P = P_1 t_1 \tag{3.13}$$

$$(1+\nu_1)p_1t_d + (1+\nu_1)(p_1+\delta p_1)t_1^p - \nu_1 p_1 t_d + \nu_1 p_1 t_1^p = p_1 t_1$$

$$p_1 t_d + ((1+\nu_1)(p_1+\delta p_1) - \nu_1 p_1)t_1^p = p_1 t_1$$

therefore, the production time after disruption

$$t_1^p = \frac{(t_1 - t_d)p_1}{(1+\nu_1)(p_1+\delta p_1) - \nu_1 p_1} \tag{3.14}$$

from equation (3.6) *the rework production time after disruption t_2^P is*

$$t_2^p = \frac{\nu_1 p_1}{p_2} t_1^p \tag{3.15}$$

$$t_2^p = \frac{(t_1 - t_d){p_1}^2}{((1+\nu_1)(p_1 + deltap_1) - \nu_1 p_1)p_2} \tag{3.16}$$

The total production inventory and delivery cost at the time t_d^p *is*

$$TC^*(t_1^p) = \phi\left(\frac{\tau}{(t_d + t_1^p)} + (\pi + \kappa + \eta)p_1(t_d + t_1^p)\right) \tag{3.17}$$

where

$$\tau = \frac{k + nk_1}{p_1}, \quad \pi = h_2\frac{{\nu_1}^2 p_1}{2p_2} + h_1\left(\frac{1}{2} + \frac{\nu_1 p_1}{2p_2}\right)$$

$$\kappa = h_1\frac{n-1}{n}\left(\frac{p_1}{\phi} - 1 - \frac{\nu_1 p_1}{p_2}\right), \quad \eta = (c + c_r\nu_1 + c_t)$$

4 Numerical Examples

In this section we have given two separated numerical example to verify the both above cases

4.1 for Case I: Without Disruption

We consider that a manufacturer produces items at the rate of 100 units per year and has a constant demand rate is 50 units per year. Let during the reguler production time defective items produced at a rate $\theta_1 = 0.1$, *and during the rework process defective items become scrap at a rate* $\theta_2 = 0.1$. *Let production cost is* $c = 4$ *unit per items, setup cost is* $k = 20000$ *per production run, the fixed delivery cost is* $k_1 = 10000$ *per shipment, the delivery cost from shipment to customers is* $c_t = 10$ *per items, per unit rework cost* $c_r = 4$, *for each rework items, disposal cost* $c_s = 2$, *for each scraped items, holding cost* $h_1 = 0.10$ *per items per year, rework holding cost* $h_2 = 0.02$ *per item per year.*

The optimal regular production time period $t_1 = 15.1092$, *and the rework production time period* $t_2 = 0.3021$ *are obtained. The production inventory delivery cost* $TC(t_1) = 270761.9$ *is calculated by using equation* (2.12).

4.2 for Case II With Disruption

We consider that a manufacturer produces items at the rate of 100 units per year and has a constant demand rate is 50 units per year. Let during the reguler production time defective items produced at a rate $\theta_1 = 0.1$, *and during the rework process defective items become scrap at a rate* $\theta_2 = 0.1$. *Let production cost is* $c = 4$ *unit per items, setup cost is* $k = 20000$ *per production run, the fixed delivery cost is* $k_1 = 10000$ *per shipment, the delivery cost from shipment to customers is* $c_t = 10$ *per items, per unit rework cost* $c_r = 4$, *for each rework*

items, disposal cost c_s = 2, for each scraped items, holding cost h_1 = 0.10 per items per year, rework holding cost h_2 =0.02 per item per year.
Let production gets disrupted at the time t_d= 6, and let δp = -10 unit per year due to disruption, then by using equations (3.9), (3.10) *and* (3.11)*, then we obtained an optimal disrupted regular production time is ${t_1}^p$ = 10.13, and the disrupted rework production time is ${t_2}^p$= 2.23. The production inventory delivery cost $TC^*({t_d}^p)$ = 75434.39/-. and the production inventory delivery cost is $TC_1^*(t_d^p)$ = Rs. 75031.33/-.*

4.3 Sensitivity Analysis

As per data analysis of table 1, if the production rate of defective items increases then the regular production time without disruption t_1 and with disruption t_d^p are decreases and rework production time without disruption t_2 and with disruption t_2^p are increases more sharply. Moreover high production rate od defective rate leads to more production time for both with and without disrupted system. Consequently the total cost of production system with and without disruption increses.

Table 1: Effect of (θ_1) on optimal results with and without disruption, when $\theta_2 > 0$.

θ_1	t_1	t_2	t_d^p	t_2^p	$TC(t_1)$	$TC^*(t_d^p)$
10%	13.6712	2.7342	11.1487	2.2297	65562.36	75434.39
11%	13.0402	2.8688	10.37073	2.2815	68386.87	78603.60
12%	12.4881	2.9971	9.69050	2.3257	71101.11	81639.06
13%	11.9999	3.1199	9.08940	2.3632	73718.90	84557.53

If the scraped rate of defective items during the rework process θ_2 increases, then the regular production time without disruption t_1 and with disruption t_d^p increases as well as the rework production time without disruption t_2 and with disruption t_2^p also increase marginally. Furthermore, if the production rate of scrap items becomes leads to increase production time for both with and without disruption. Consequantly the total cost increases accordingly as shown in table 2. *Analsis of data table 3 show that the regular production time period*

Table 2: Effect of θ_2 on optimal results with and without disruption, when $\theta_1 > 0$.

θ_2	t_1	t_2	t_d^P	t_2^P	$TC(t_1)$	$TC^*(t_d^P)$
10%	13.6712	2.7342	11.1487	2.2297	65562.11	75434.39
11%	13.6781	2.7356	11.1576	2.2315	65595.14	75475.05
12%	13.6850	2.7370	11.1664	2.3232	65627.97	75515.78
13%	13.6919	2.7383	11.1664	2.2350	65660.85	75556.58

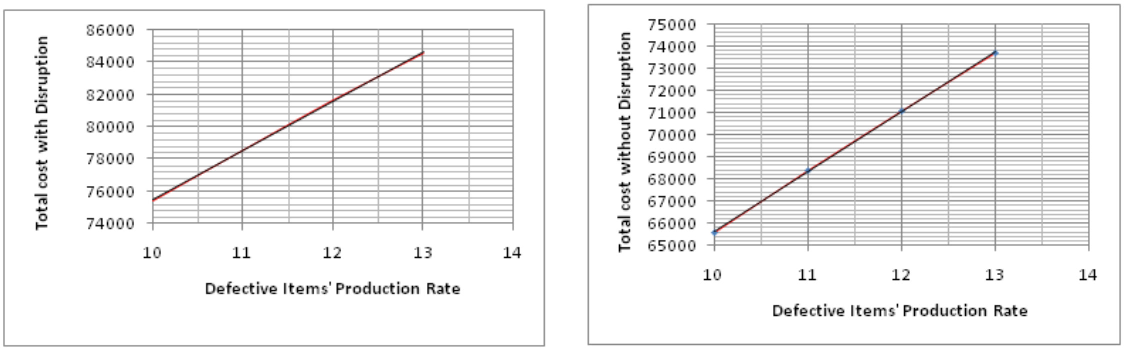

Figure 3: Total cost with respect to regular time

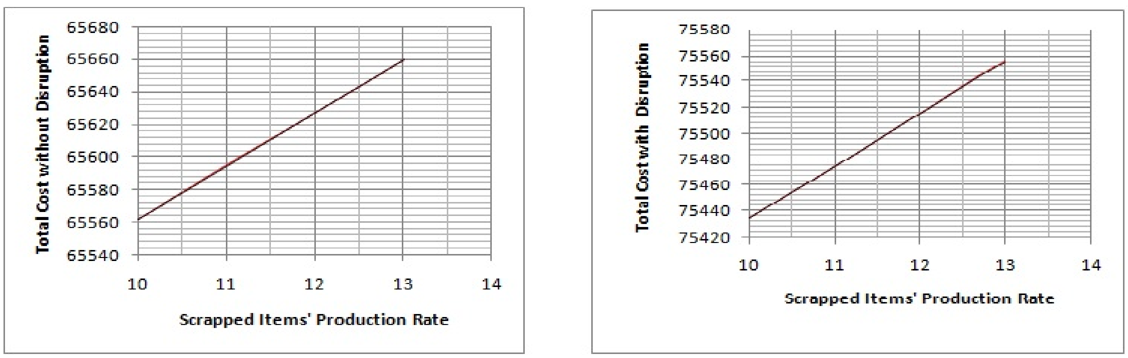

Figure 4: Total cost with respect to scrapped items

with t_d^p and without disruption t_1 and rework production time period with t_2^p and without disruption t_2 increases as increases the production rate of defective items, when production rate of scrapped items become zero. Moreover, high defective rate leads to more production time period for both the cases with and without disrupted system. Consequently, the total cost of production increases accordingly as shown in table 3.

Table 3: Effect of (θ_1) on optimal results with and without disruption for ($\theta_2 = 0$.)

θ_1	t_1	t_2	t_d^P	t_2^P	$TC(t_1)$	$TC^*(t_d^P)$
10%	13.6030	2.7206	11.0610	2.2122	75031.33	65237.47
11%	12.9688	2.8531	10.2786	2.2613	78137.62	68012.53
12%	12.4138	2.9793	9.59430	2.3026	81106.71	70920.35
13%	11.9226	3.0999	8.98910	2.3371	83955.43	73238.35

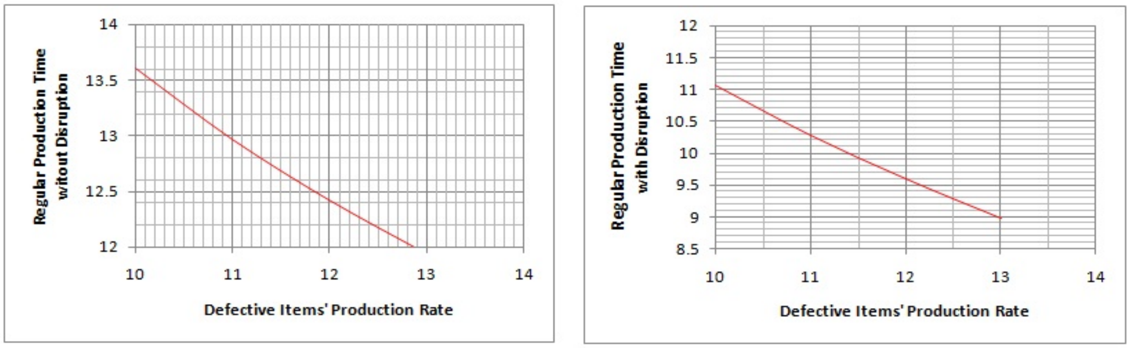

Figure 5: Regular Production Time with respect to defective items' production rate

5 Conclusion and Suggestions

This study presents an economical production quantity model for production of imperfect quality items and incrporates a rework process on imperfect quality items. Furthermore, the study is developed for two strategic situations like as (i)economical production quantity model without disruption (ii) economical production quantity model with disruption. For both of the strategic situation we derived the total production inventory cost and delivery cost functions for two sub cases, in the first one we assumed defective and scrap both items exists in the system and another one we assumed only defective items exist in the system. In the first case we optimized the regular production time period, rework production time period and total production costs for each subcases

In the second case, we optimized the production inventory cost and delivery cost functions for EMQ model with disruption for above two subcases. In the first subcase we considered defective and scrapped items exists in the inventory system, and in the another second subcases only defective items exists in the inventory system. We have optimized the disrupted production time period, rework production time period and total production cost. The sensitivity analysis shows that the production cost without disrupted system is less than the production cost with disrupted system. Consequently it is suggestion for inventory manager that, to remove the disruptions in the production system for reducing the production cost and to earning the more profit. One can extended this model by incorporating variable production rate and one can extended this model by incorporating price sensitive demand. Also it is also extended by appling learning cuve effects on imperfect quality items to seperate perfect and imperfect items.

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