Application of Normal Distribution

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ABSTRACT

This article contains main mathematical features of continuous random variable. The behavior of probability is linked to the features of the phenomenon we would predict. This link can define Probability distribution. Given the characteristics of phenomena (that we also define variables), there are defined probability distribution. For categorical (or discrete) variables, the probability can be described by a binomial or Poisson distribution in the majority of cases. Continuous probability distributions are widely used to mathematically describe random phenomena in engineering and physical sciences. In this article, we present a methodology that can be used to formalize any continuous random variable. The distribution of probability is briefly described together with some examples for their possible application. It is also known as Gaussian distribution and the bell-shaped curve.

Keywords-Random variable, Discrete, Continuous, Mean, Normal curve

I. INTRODUCTION

Random Variable- A real valued function, defined over the sample space of random experiment, is called random variable.

Random Variable is assigning a number to the experiment.

Random Variable is defined from sample space to the real numbers

$$f(X) = S \rightarrow R$$

Where, $f(X)$ is the random variable

Sample space (S) is the domain of random variable, and

Real number (R) is the range of random variable.

Random Variable is a variable whose value is determined by the possible outcome of random experiment which can be discrete or continuous.

A. Discrete Random Variable

It is random variable that can take finite number of distinct values such as 0, 1, 2 and so on.
E.g. – Heights of students, Weights of students etc.

B. Continuous Random Variable

It is random variable which take values between a range and it is variable that can have Infinite or uncountable set of values.
E.g. – Number of students who fails in a test, Number of accidents per month etc.

Probabilities of continuous random variable are defined by area underneath the curve of Probability density function. The graph of Discrete versus Continuous is as follows -

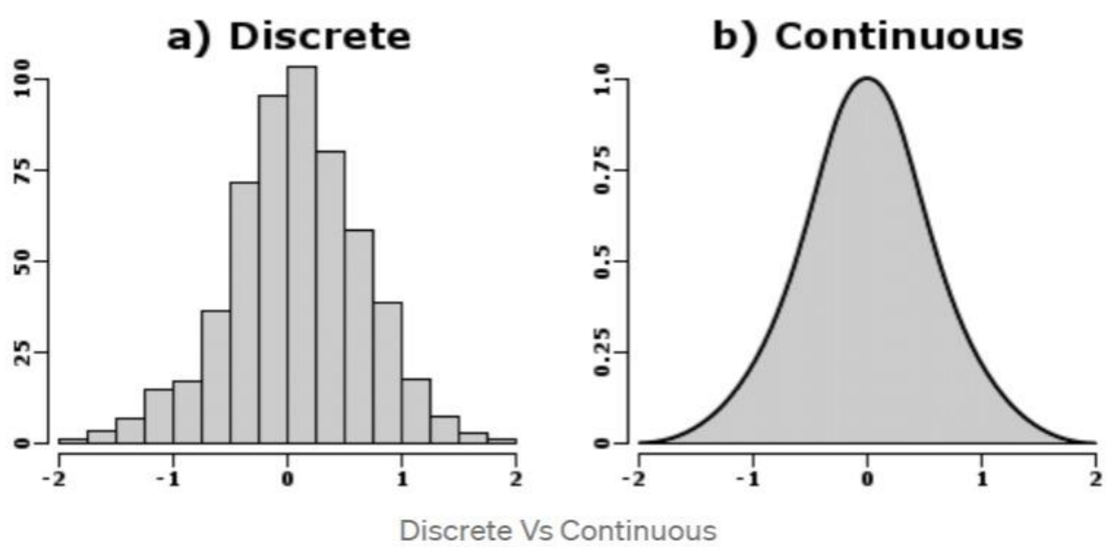

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Figure-1

II. TYPES OF DISCRETE PROBABILITY DISTRIBUTION

A. Bernoulli Distribution
B. Binomial Distribution
C. Multinomial Distribution
D. Poisson Distribution
E. Hypergeometric Distribution
F. Negative Binomial Distribution
G. Geometric Distribution

Now we will discuss about some **Discrete Probability Distribution**

- **Bernoulli Distribution-**We use Bernoulli Distribution when we perform an experiment once and it has only two possible outcomes – success and failure. The trails of this type are called Bernoulli trails.

If we perform an experiment so let ‘p’ be the probability of success and ‘1-p’ be the probability of failure.
The P.M.F is given as

$$P.M.F = \begin{Bmatrix} p\,, & Success \\ 1-p\,, & Failure \end{Bmatrix}$$

$$Mean = p$$

$$Variance = p(1-p)$$

E.g.- Guessing a simple True/False Question, Tossing a coin once, etc.

- **Binomial Distribution-** It is a sequence of identical Bernoulli Distribution. Binomial Distribution is generated for random variable with only two possible outcomes. Let 'p' denote the probability of event is a success and 'q' denote the probability of failure in any trail. It is required to find the probability of getting 'r' success in 'n' independent trails i.e., remaining 'n-r' will be failure. Then we perform the experiment repeatedly and plot the probability each time, which gives Binomial Distribution.

  The P.M.F is given as

$$P.M.F = n_{C_r} p^r q^{1-r}$$

  Where p is the probability of success,
  q is the probability of failure,
  n is the number of trails, and
  r is the number of times we obtain success.
  E.g.- Tossing a coin 'n' time and calculating the probability of getting some number of heads.

Important Results of Binomial Distribution-

Prove that the sum of probability mass function for Binomial distribution is 1.

Proof: -

$$P(x,r) = n_{C_r} p^r q^{n-r}$$

$$P(x,r) = \sum_{n=0}^{\infty} n_{C_r} p^r q^{n-r} \qquad ; r = 0,1,2 \dots n$$

$$P(x,r) = n_{C_0} p^0 q^n + n_{C_1} p^1 q^{n-1} + \cdots + n_{C_n} p^n q^{n-n}$$

$$= q^n + n_{C_1} p^1 q^{n-1} + \cdots + p^n$$

$$= (q+p)^n$$

$$= 1$$

Hence Proved

Mean of Binomial Distribution

Suppose $x_1, x_2, x_3, \dots x_n$ are the variate values with corresponding probabilities $P_1, P_2, P_3, \dots P_n$,then

$$\mu = E(x) = \sum_{r=0}^{n} rP(x=r)$$

$$= \sum_{r=0}^{n} r\, n_{C_r} p^r q^{n-r}$$

$$= 0 + n_{C_1} p^1 q^{n-1} + 2n_{C_2} p^2 q^{n-2} + 3n_{C_3} p^3 q^{n-3} + \cdots + n n_{C_n} p^n q^0$$

$$= npq^{n-1} + n(n-1)p^2 q^{n-2} + \frac{n(n-1)(n-2)}{2!} p^3 q^{n-3} + \cdots + np^n$$

$$= np\left[q^{n-1} + n_{C_1} pq^{n-2} + n - 1_{C_2} p^2 q^{n-3} + \cdots + p^{n-1}\right]$$

$$= np[(p+q)^{n-1}]$$

$$\boldsymbol{\mu = np}$$

Variance of Binomial Distribution-

$$Variance(\sigma^2) = E(x^2) - (E(x))^2$$
$$= E(x^2) - (np)^2 \qquad ...(A)$$

$$E(x^2) = \sum_{r=0}^{n} r^2 n_{C_r} p^r q^{n-r}$$
$$= \sum_{r=0}^{n} [r(r-1) + r]\, n_{C_r} p^r$$
$$= \sum_{r=0}^{n} r(r-1) n_{C_r} p^r q^{n-r} + \sum_{r=0}^{n} r n_{C_r} p^r q^{n-r}$$
$$= \left[2.1\, n_{C_2} p^2 q^{n-2} + 3.2\, n_{C_3} p^3 q^{n-3} + \cdots + n(n-1)p^n\right] + np$$
$$= [n(n-1)p^2 q^{n-2} + n(n-1)(n-2)p^3 q^{n-3} + ... + n(n-1)p^n] + np$$
$$= n(n-1)p^2[q^{n-2} + (n-2)pq^{n-3} + \cdots + p^{n-2}] + np$$
$$= n(n-1)p^2(p+q)^{n-2} + np$$
$$= n(n-1)p^2 + np$$
$$= np(n-1)(p+1)$$

Put value of $E(x^2)$ in equation (A)

$$Variance = np(n-1)(p+1) - n^2p^2$$
$$= n^2p^2 - np^2 + np - n^2p^2$$
$$= np(1-p)$$
$$= npq$$

$$Standard\ Deviation(\sigma) = \sqrt{Variance}$$
$$\boldsymbol{\sigma = \sqrt{npq}}$$

- **Multinomial Distribution**

Multinomial distribution describes the random variable with many possible outcomes. Consider playing a game 'n' number of times. So Multinomial Distribution helps us to determine combined probability that Player 1 will win x_1 times, Player 2 will win x_2 times, and Player k will win x_k times.

The P.M.F is given as

$$P(X = x_1, X = x_2 \ldots X = x_k) = \frac{n!}{x_1!\, x_2! \cdots x_k!} {P_1}^{x^1} {P_2}^{x^2} \ldots {P_k}^{x^k}$$

Where 'n' is the number of trails

$P_1, P_2 \ldots P_k$ denote the probabilities of outcome $x_1, x_2 \ldots x_k$ respectively.

- **Poisson's Distribution-** It is a limiting case of Binomial distribution under the following conditions: -

When the number of trails 'n' is very large i.e., $n \to \infty$
Probability of success 'p' is very small i.e., $p \to 0$
Poisson's Distribution describes the event that occurs in a fixed interval of time or space.

DEFINATION- A discrete random variable X will follow Poisson's distribution if it has the following probability mass function (P.M.F)-

$$P(X = r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

Where $\lambda =$ Average number of times event has occurred in certain period of time,
$r =$ Desired outcome
$e =$ Euler's Number

Types of Continuous Probability Distribution

- Rectangular/Uniform Distribution
- Exponential Distribution
- T-Distribution
- Normal Distribution
- Chi-square Distribution
- Rayleigh Distribution

Now we will discuss about some **Continuous Probability Distribution**

- **Rectangular/Uniform Distribution**

A uniformly distributed Random Variable X in interval $[a, b]$ if P.D.F is given by-

$$f(x) = \begin{cases} \frac{1}{b-a} & , \quad a < x < b \\ 0 & , \quad elsewhere \end{cases}$$

$$Mean = \frac{a+b}{2}$$
$$Variance = \frac{b-a}{\sqrt{3}}$$

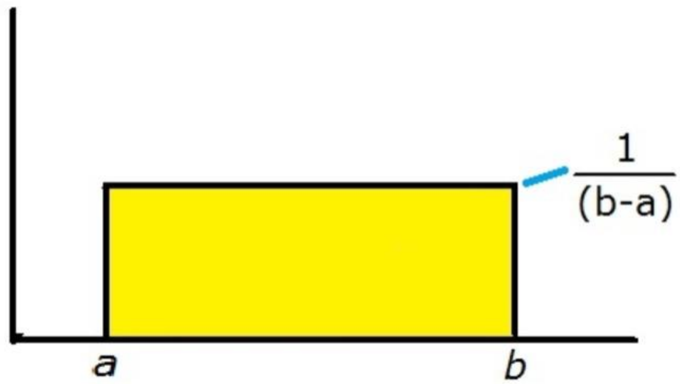

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Figure-2

- **Exponential Distribution**

A random Variable X is said to have exponential distribution of its P.D.F given by-

$$f(x) = \begin{Bmatrix} \lambda e^{-\lambda x} & ; & x \geq 0 \\ 0 & ; & elsewhere \end{Bmatrix}$$

$$Mean = \frac{1}{\lambda}$$

$$Variance = \frac{1}{\lambda^2}$$

- **T-Distribution**

This is used when Sample size is small and population variance is not known. This distribution is defined by Degree of freedom (p) and p is calculated s sample size minus 1 (n-1).

P.D.F is given by

$$f(t) = \frac{\Gamma \frac{p+1}{2}}{\sqrt{p\pi}\Gamma \frac{p}{2}} \left(1 + \frac{t^2}{p}\right)^{-\frac{p+1}{2}}$$

Where p is degree of freedom,

Γ is gamma function,

And $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ where $\bar{x}$ is sample mean, μ is population mean, s is sample variance

III. RESULTS AND DISCUSSIONS

A. NORMAL DISTRIBUTION

In Statistics and Probability Theory, the Normal Distribution is also known as Gaussian Distribution and it is the most significant Continuous Probability Distribution. It is the limiting case of Binomial Distribution where number of trails ‘n’ tends to ∞, with no restriction on p and q.

DEFINATION –

A Continuous random variable X is said to be a normal variate if it has probability density function given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \;,\; -\infty < x < \infty$$

Where x is the Variable

μ is the Mean and

σ is the Standard Deviation

Mean and Standard Deviation are the parameters of Distribution.

$$f(x) \geq 0 \;;\; -\infty < x < \infty \;,\; \sigma > 0$$

The Normal Distribution is 1 i.e., the total probability of distribution is 1.

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Important Results-

Prove that normal distribution is 1 i.e., the total probability of distribution is 1.

Proof-Let $I = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} dx$ … (C)

Let $z = \frac{x-\mu}{\sigma}$

$dz = \frac{1}{\sigma} dx$

Then equation ‘C’ becomes

$$I = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$$

$$= \frac{1}{\sqrt{2\pi}} 2 \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz \qquad \left\{ e^{-\frac{z^2}{2}} \; is\ an\ even\ function\ of\ z \right\}$$

$$= \frac{2}{\sqrt{2\pi}} * \sqrt{\frac{\pi}{2}} = 1$$

$$\left\{ \int_{0}^{\infty} e^{-\frac{z^2}{2}} dz = \sqrt{\frac{\pi}{2}} \right\}$$

Standard Normal Distribution

In Standard Normal Distribution, mean value is 0 and standard deviation is 1.

$$z = \frac{X - \mu}{\sigma}$$

To convert Normal Variate to Standard Normal Variate

X is a normal variate having following p.d.f.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \;,\; -\infty < x < \infty \qquad \dots (1)$$

For different values of μ and $\boldsymbol{\sigma}$, we get different normal curves.
To find the area under normal curves, we standardized the normal variate X by the following transformation

$$z = \frac{X-\mu}{\sigma} \qquad \dots (2)$$

From (1) and (2)

$$f(z) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \quad ; \quad -\infty < x < \infty$$

We know that

$$f(z) \geq 0 \quad ; \quad -\infty < z < \infty$$

Since $e^{-\frac{z^2}{2}}$ is an even function of z and $\int_{-\infty}^{\infty} f(z) dz = 1$

B. CENTRAL LIMIT THEOREM

Central limit theorem states that if a large number of independent random variables are drawn from any distribution, then the distribution of their sums always converge to the Normal Distribution. The larger the size of sample size, the better the approximation to the normal.
Let $X_1, X_2, X_3, \dots X_n$ are 'n' independent identically distributed random variables with $E(X_1) = \mu$ and $Var(X_1) = \sigma^2$ and if $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then the variate $Z = \frac{X-\mu}{\sigma/\sqrt{n}}$ has a distribution that approaches the standard normal distribution as $n \to \infty$ provided the moment generating function exists.

C. **APPLICATIONS OF CENTRAL LIMIT THEOREM**

1. It provides a simple method for computing approximate probabilities of sums of independent random variables.
2. It gives us the fact that empirical frequencies of so many natural "Populations" exhibit a bell-shaped curve.

Area under Standard Probability Curve

Since $\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx = 1 = P(-\infty < x < \infty)$

Clearly for Standard normal variate

$$\int_{-\infty}^{\infty} f(z)dz = 1 = P(-\infty < x < \infty)$$

So $P(-\infty < x < 0) = 0.5$
and $P(0 < x < \infty) = 0.5$

Working Procedure to find $P(x_1 < X < x_2)$

- $\boldsymbol{P(\mu_X - \sigma < X < \mu_X + \sigma)}$

Here $z = \frac{X-\mu}{\sigma}$
At $x = \mu - \sigma$, $z = \frac{\mu-\sigma-\mu}{\sigma}$; $z = -1$, and
At $x = \mu + \sigma$, $z = \frac{\mu+\sigma-\mu}{\sigma}$; $z = 1$
So, $P(-1 < z < 1) = 2P(0 \le z \le 1)$
$= 2 * 0.34135$ [By normal table $P(0 \le z \le 1) = 0.34135$]
$= 0.6827$

Here 68% area lies within $\mu \pm \sigma$

- $\boldsymbol{P(\mu_X - 2\sigma < X < \mu_X + 2\sigma)}$

At $x = \mu - 2\sigma$, $z = -2$
At $x = \mu + 2\sigma$, $z = 2$
So, $P(-2 < z < 2) = 2P(0 \le z < 2)$
$= 2 * 0.4774$ [By normal table $P(0 \le z < 2) = 0.4774$]
$= 0.9545$

Here 95% of area lies within $\mu \pm 2\sigma$

- $\boldsymbol{P(\mu_X - 3\sigma < X < \mu_X + 3\sigma)}$

At $x = \mu - 3\sigma$, $z = -3$
At $x = \mu + 3\sigma$, $z = 3$
So,$P(-3 < z < 3) = 2P(0 \le z \le 3)$
$= 2 * 0.4986$ [By normal table $P(0 \le z \le 3) = 0.4986$]
$= 0.9973$

Here 99% of area lies within $\mu \pm 3\sigma$
Now the graph for area within $\mu \pm \sigma$, $\mu \pm 2\sigma$, $\mu \pm 3\sigma$ is as follows-

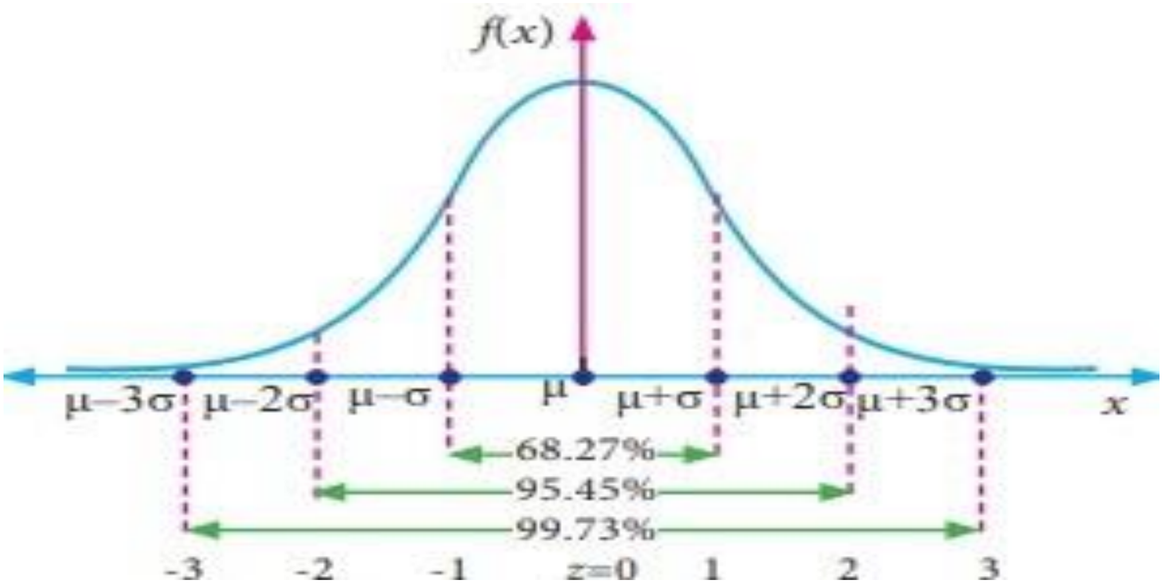

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Figure-4

D. Curve of Normal Distribution

We know that Mean helps to determine line of symmetry of graph, whereas with help of Standard Deviation we know how far the data are spread out.

If Standard Deviation is smaller, the data are close to each other and if Standard Deviation is larger, the data are more dispersed and graph becomes wider.

Mean is the location parameter and Standard Deviation is the scale parameter.

- Mean determines peak of curve is centered. Increasing mean moves curve right while decreasing mean moves the curve left.

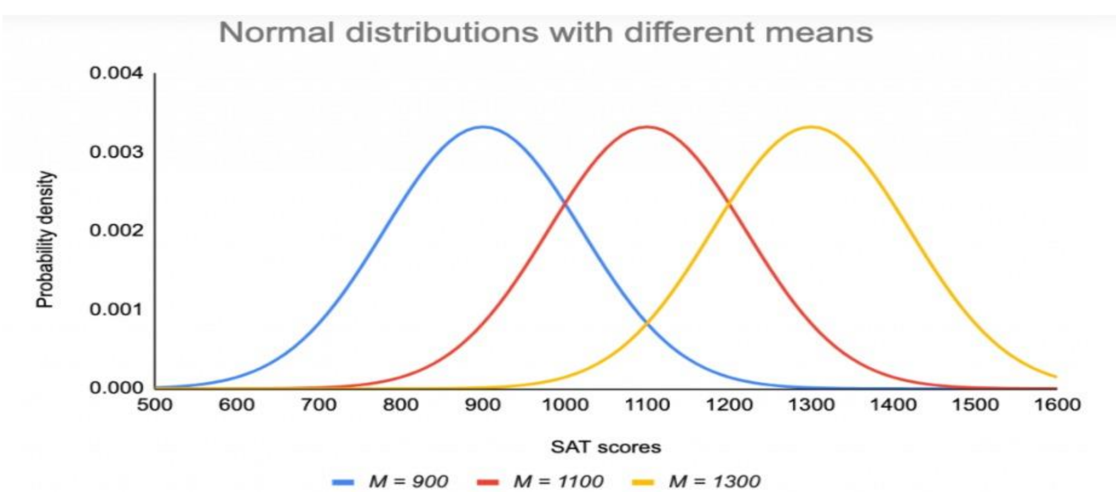

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Figure-5

- Standard Deviation stretches or squeezes the curve. A small standard deviation results in narrow curve while a large standard deviation leads to wide curve.

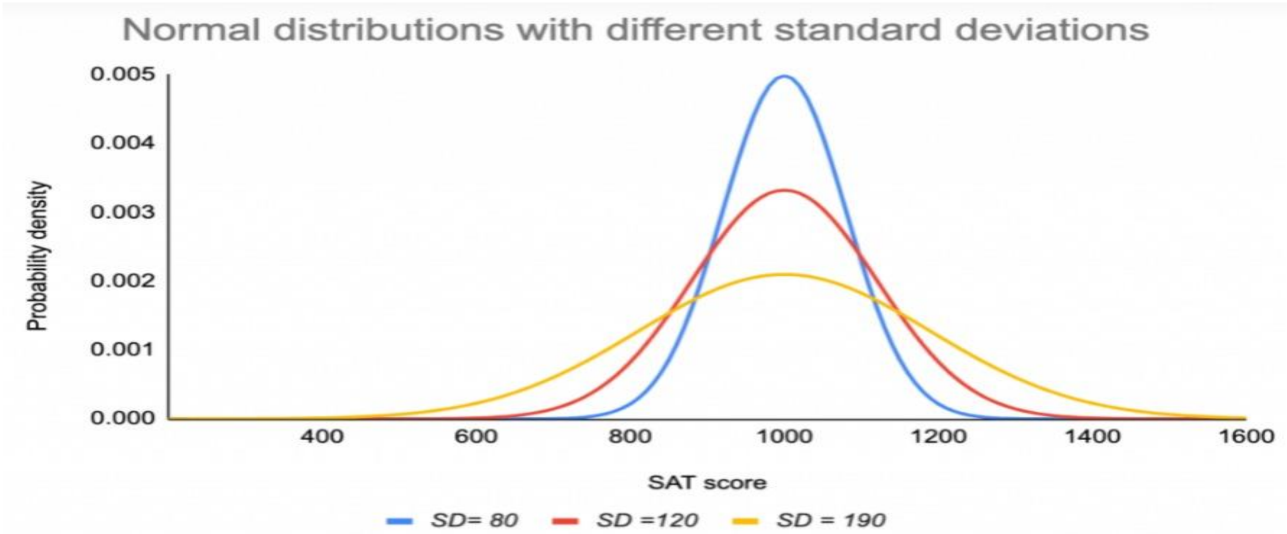

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Figure-6

E. Empirical Rule

Empirical formula is also known as 68-95-99.7 rule. It tells us where most of values lies in normal Distribution.

- Around 68% of data falls within one standard deviation of means.
- Around 95% of data falls within two standard deviations of mean.
- Around 99.7% of data lies within three standard deviations of mean.

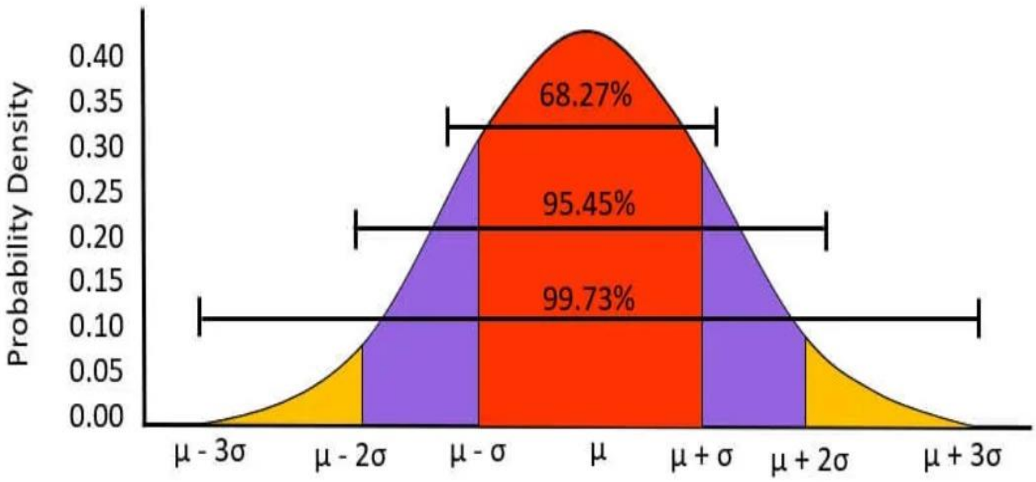

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Figure-7

Properties of Normal Probability Curve

- It is a bell-shaped curve
- It is symmetric about $z = 0$ i.e., $x = \mu$.
- In this distribution Mean = Mode = Median.
- Area lying under the normal probability curve is 1 because $\int_{-\infty}^{\infty} f(x)dx = 1$.
- There are exactly half values which are to the left of center and exactly half of the values which are to the right of center.
- Normal curve must have only one peak.
- As x increases numerically, $f(x)$ decreases rapidly. The maximum probability attains its maximum value at $x = \mu$ and given by $P_{max} = \frac{1}{\sigma\sqrt{2\pi}}$.
- Since f(x) being the probability, can never be negative, no portion of curve lies below x-axis.
- x-axis is an asymptote of normal probability curve.
- The Points of inflexion of the curve are given by $\mu \pm \sigma$.

Mean of Normal Distribution

By definition of mean

$$\mu = \bar{X} = E(X) = \int_{-\infty}^{\infty} x f(x)dx$$

In normal distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$; $-\infty < x < \infty$

$$Mean = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$Mean = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{(x-\mu_X)^2}{2\sigma^2}} dx$$

Let $z = \frac{x-\mu_X}{\sigma}$

$$dz = \frac{1}{\sigma} dx$$

$$\sigma dz = dx$$

Put $x = \sigma z + \mu_X$ in Mean formula

$$Mean = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu_X + \sigma z) e^{-\frac{z^2}{2}} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu_X + \sigma z) e^{-\frac{z^2}{2}} dz$$

$$= \frac{2}{\sqrt{2\pi}} \left[\int_0^{\infty} \mu_X e^{-\frac{z^2}{2}} dz + \int_0^{\infty} \sigma z e^{-\frac{z^2}{2}} dz \right]$$

$$= \frac{2}{\sqrt{2\pi}} \left[\mu_X \int_0^{\infty} e^{-\frac{z^2}{2}} dz + \sigma \int_0^{\infty} z e^{-\frac{z^2}{2}} dz \right]$$

$$= \frac{2}{\sqrt{2\pi}} \left[\mu^X \sqrt{\frac{\pi}{2}} + \sigma \int_0^{\infty} z e^{-\frac{z^2}{2}} dz \right]$$

$$= \frac{2\mu_X}{\sqrt{2\pi}} * \sqrt{\frac{\pi}{2}} + \sigma(0) \qquad \left\{ z e^{-\frac{z^2}{2}} \text{ is an odd function} \right\}$$

$$\mathbf{Mean = \mu_X}$$

Variance of Normal Distribution

σ^2 = Second Moment about Mean

By definition, we have $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

Let $z = \frac{x-\mu^X}{\sigma} \Rightarrow x = \mu_X + \sigma_z$; $dx = \sigma dz$

$$\sigma^2 = \int_{-\infty}^{\infty} \sigma^2 z^2 . \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} \sigma dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-\frac{z^2}{2}} dz \qquad \left(Since\ z^2 e^{-\frac{z^2}{2}}\ is\ an\ even\ function \right)$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} . 2 \int_{0}^{\infty} z^2 e^{-\frac{z^2}{2}} dz$$

Let $\frac{z^2}{2} = y \Rightarrow\!\!> z^2 = 2y$

$z = \sqrt{2y}$

$2zdz = 2dy\ ; 2dz = dy$

$$= \frac{\sigma^2}{\sqrt{2\pi}} . 2\sqrt{2} \int_{0}^{\infty} e^{-y} . y^{\frac{1}{2}} dy$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} . 2\sqrt{2} \Gamma \frac{3}{2} \qquad \left\{ By\ Gamma\ function\ \int_{0}^{\infty} \mu^{n-1} e^{-\mu} du = \Gamma n \right\}$$

Variance=σ^2

IV. APPLICATIONS OF NORMAL DISTRIBUTION IN DEFENSE RECRUITMENT PROCESS-

Following is the data for defense recruitment process-

Heights in Inches (X)	Number of Candidates (f)
60	0
61	4
62	20
63	23
64	75
65	114

66	186
67	212
68	252
69	218
70	175
71	149
72	46
73	18
74	8
75	0

Table-1

Now, as we know,
The probability density function of Normal distribution is given by-

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

And $z = \frac{x-\mu}{\sigma}$

Now we will make a table of above data and find mean and standard deviation of the data.

Heigts in Inches (x)	Number of Candidates (f)	${x_i}^2$	$x_i f_i$	$f_i {x_i}^2$
60	0	3600	0	0
61	4	3721	244	14884

62	20	3844	1240	76880
63	23	3969	1449	91287
64	75	4096	4800	307200
65	114	4225	7410	481650
66	186	4356	12276	810216
67	212	4489	14204	951668
68	152	4624	17136	1165248
69	218	4761	15042	1037898
70	175	4900	12250	857500
71	149	5041	10579	751109
72	46	5184	3312	238464
73	18	5329	1314	959322
74	8	5476	595	43808
75	0	5625	0	0
	$\sum f_i = 1500$		$\sum f_i x_i = 101848$	$\sum f_i {x_i}^2 = 6923734$

Table-2

From above table 2-

$$Mean, \mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$= \frac{101848}{1500}$$

$$\boldsymbol{\mu = 67.8986}$$

$$Standard\ Deviation, \sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$$

$$= \sqrt{\frac{6923734}{1500} - (67.898)^2}$$

$$= \sqrt{4615.8226 - 4610.2289}$$

$$= \sqrt{5.6836}$$

$$\boldsymbol{\sigma = 2.365}$$

Now we draw a table and find z and probability density function of normal distribution for all data ‘x’ i.e., the heights of the candidates.

Heights in Inches (x)	$z = \frac{x - \mu}{\sigma}$	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{z^2}{2}}$
60	-3.3397	0.000633
61	-2.8934	0.00254
62	-2.4739	0.00769
63	-2.0545	0.02
64	-1.6350	0.0439
65	-1.2156	0.0799
66	-0.7961	0.1218
67	-0.3766	0.1558
68	0.0427	0.16719
69	0.4622	0.1503
70	0.8817	0.1134
71	1.30117	0.0717
71	1.7206	0.0385
73	2.1401	0.0169
74	2.5595	0.006308

75	3.0027	0.00184

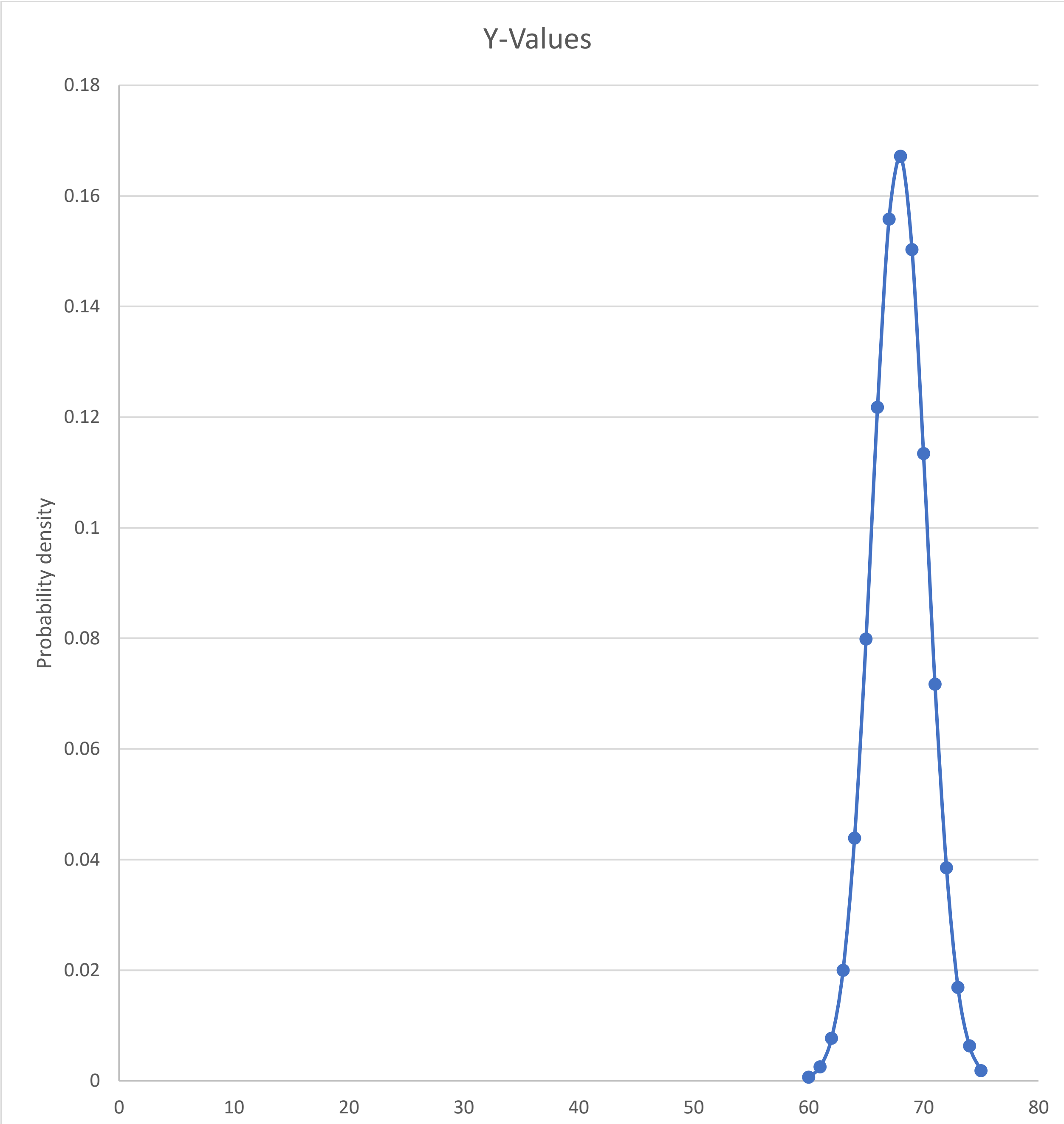
Table-3

Now from above table 3, we get the values of z and the probability density function of Normal distribution i.e., $f(x)$

Now we will draw a graph between heights of candidates and probability density function i.e., $f(x)$

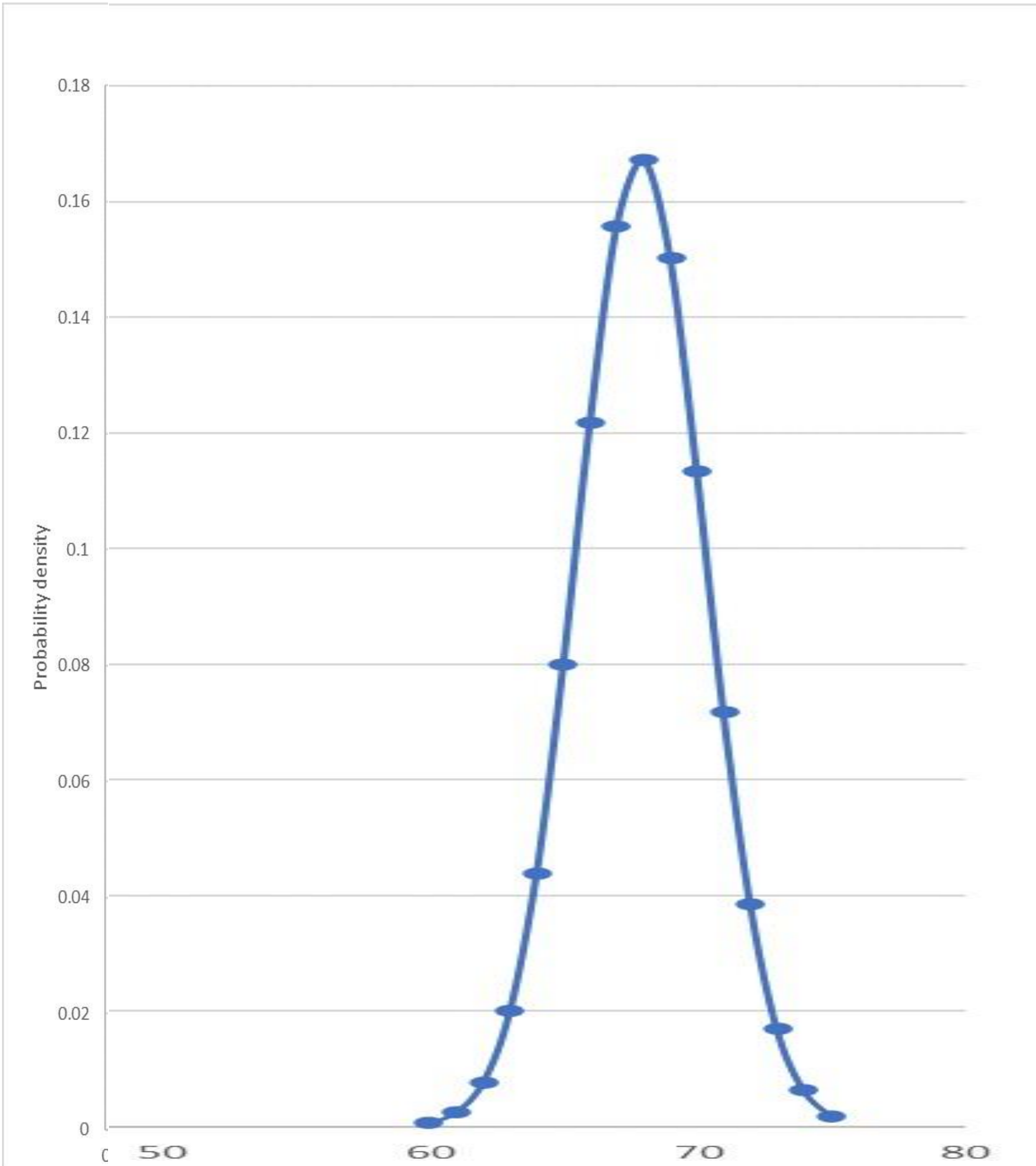
x-axis represents the heights of candidates, and

y-axis represents the Probability density function



Graph-1

Now an enlarged picture of graph of normal distribution of above data-



Graph-2

CONCLUSION

The probability distributions are a common way to describe, and possibly predict, the probability of an event. The main point is to define the character of the variables whose behavior we are trying to describe, through probability (discrete or continuous). The identification of the right category will allow a proper application of a model (for instance, the standardized normal distribution) that would easily predict the probability of a given event. In this dissertation we explained how we can apply normal distribution on defense recruitment process.

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