Pentapartitioned Neutrosophic Pythagorean Generalized Pre-closed Sets

**3.1 Definition**

An PNPS A is said to be an Pentapartitioned Neutrosophic Pythagorean generalized pre-closed set (PNPGPCS in short) in (X, τ) if PNPPCl(A) ⊆ U whenever A ⊆ U and U is a PNPOS in X. The family of all PNPGPCSs of an PNPTS (X, τ) is denoted by PNPGPC(X).

**3.2 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.2,0.1,0.6,0.6, 0.7 >} is an PNPGPCS in X.

**3.3 Theorem**

Every PNPCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPCS in X and let A ⊆ U and U is an PNPOS in (X, τ). Since PNPPCl(A) ⊆ PNPCl(A) and A is an PNPCS in X, PNPPCl(A) ⊆ PNPCl(A) = A ⊆ U. Therefore A is an PNPGPCS in X.

**3.4 Example**

 Let X = {a, b} and let $τ$ = 0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.2, 0.1,0.6,0.6,0.7 >} is an PNPGPCS in X but not an PNPCS in X.

**3.5 Theorem**

Every PNP$α$CS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNP$α$CS in X and let A ⊆ U and U is an PNPOS in (X, τ). By hypothesis, PNPCl(PNPInt(PNPCl(A))) ⊆ A. Since A ⊆ PNPCl(A), PNPCl(PNPInt(A)) ⊆ PNPCl(PNPInt(PNPCl(A))) ⊆ A. Hence PNPPCl(A) ⊆ A ⊆ U. Therefore A is an PNPGPCS in X.

**3.6 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4,0.3,0.6,0.5, 0.6 >, < a, 0.2,0.1,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.3, 0.2,0.6,0.0.6,0.7 >, < a, 0.1,0,0.6,0.7, 0.8 >} is an PNPGPCS in X but not an PNP$α$CS in X. Since PNPCl(PNPInt(PNPCl(A))) = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7,0.6,0.6,0.1, 0.2 >} $⊈ $A.

**3.7 Theorem**

Every PNPGCS is an PNPGPCS but not conversely.

**Proof:**

 Let A be an PNPGCS in X and let A ⊆ U and U is an PNPOS in (X, τ). Since PNPPCl(A) ⊆ PNPCl(A) and by hypothesis, PNPPCl(A) ⊆ U. Therefore A is an PNPGPCS in X.

**3.8 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4, 0.3,0.6,0.5,0.6>, < a, 0.5,0.4,0.6,0.4, 0.5>}. Then the PNPS A = {< a, 0.3,0.2,0.6,0.6, 0.7 >, < a, 0.4, 0.3,0.6,0.5,0.6 >} is an PNPGPCS in X but not an PNP$G$CS in X since A ⊆ T but PNPCl(A) = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.5,0.4,0.6,0.4, 0.5 >} $⊈ $T.

**3.9 Theorem**

Every PNPRCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPRCS in X. By Definition , A = PNPCl(PNPInt(A)). This implies PNPCl(A) = PNPCl(PNPInt(A)). Therefore PNPCl(A) = A. That is A is an PNPCS in X. By Theorem 3.3, A is an PNPGPCS in X.

**3.10 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.7,0.6,0.6,0.1,0.2 >}. Then the PNPS A = {< a, 0.3,0.2,0.6,0.6, 0.7 >, < a, 0.2, 0.1,0.6,0.7,0.8 >} is an PNPGPCS but not an PNPRCS in X since PNPCl(PNPInt(A)) = 0 ≠ A.

**3.11 Theorem**

Every PNPPCS is an PNPGPCS but not conversely.

**Proof:**

Let A be an PNPPCS in X and let A ⊆ U and U is an PNPOS in (X, τ). By Definition, PNPCl(PNPInt(A)) ⊆ A. This implies that PNPPCl(A) = A $∪$ PNPCl(PNPInt(A)) ⊆ A. Therefore PNPPCl(A) ⊆ U. Hence A is an PNPGPCS in X.

**3.12 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.8,0.7,0.6,0.1, 0.2 >, < a, 0.3,0.2,0.6,0.6, 0.7 >} is an PNPGPCS but not an PNPPCS in X since PNPCl(PNPInt(A)) = 1 $⊈ $A.

**3.13 Theorem**

Every PNP$α$GCS is an PNPGPCS but not conversely.

**Proof:**

 Let A be an PNP$α$GCS in X and let A ⊆ U and U is an PNPOS in (X, τ). By Definition, A $∪$ PNPCl(PNPInt(PNPCl(A))) ⊆ U. This implies PNPCl(PNPInt(PNPCl(A))) ⊆ U and PNPCl(PNPInt(A)) ⊆ U. Therefore PNPPCl(A) = A $∪$ PNPCl(PNPInt(A)) ⊆ U. Hence A is an PNPGPCS in X.

**3.14 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6,0.5,0.6,0.3, 0.4 >}. Then the PNPS A = {< a, 0.4,0.3,0.6,0.5, 0.6 >, < a, 0.5,0.4,0.6,0.4, 0.5 >} is an PNPGPCS but not an PNP$α$GCS in X since $PNPαcl$(A) = 1 $⊈ $T.

**3.15 Proposition**

PNPSCS and PNPGPCS are independent to each other.

**3.16 Example**

 Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.2,0.1,0.6,0.5,0.6 >}. Then the PNPS A = T is an PNPSCS but not an PNPGPCS in X since A ⊆ T but PNPPCl(A) = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6, 0.5,0.6,0.1,0.2 >} $⊈ $T.

**3.17 Example**

 Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.8,0.7,0.6,0.1, 0.2 >}. Then the PNPS A = {< a, 0.8,0.7,0.6,0.1, 0.2 >, < a, 0.7,0.6,0.6,0.1,0.2 >} is an PNPGPCS but not an PNPSCS in X since PNPInt(PNPCl(A)) $⊈$ A.

**3.18 Proposition**

 PNPGSCS and PNPGPCS are independent to each other.

**3.19 Example**

 Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.2,0.1,0.6,0.5,0.6 >}. Then the PNPS A = T is a PNPSCS but not a PNPGPCS in X since A ⊆ T but PNPPCl(A) = {< a, 0.5, 0.4,0.6,0.4,0.5 >, < a, 0.6, 0.5,0.6,0.1,0.2 >} $⊈ $T.

**3.20 Example**

 Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.7, 0.6,0.6,0.2,0.3 >, < a, 0.9, 0.8,0.6,0,0.1 >}. Then the PNPS A = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7,0.6,0.6,0.2, 0.3 >} is an PNPGPCS but not an PNPGSCS in X since A ⊆ T but PNPscl(A) = 1 $⊈ $T.

**3.21 Remark**

The union of any two PNPGPCSs is not an PNPGPCS in general as seen in the following example.

**3.22 Example**

Let X = {a, b} be an PNPTS and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.8,0.7,0.6,0.1, 0.2 >}. Then the PNPSs A = {< a, 0.1, 0,0.6,0.8,0.9 >, < a, 0.8,0.7,0.6,0.1,0.2 >}, B = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.7, 0.6,0.6.0.2,0.3 >} are PNPGPCSs but A $∪$ B is not an PNPGPCS in X.

**4. Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets**

In this section we introduce Pentapartitioned Neutrosophic Pythagorean generalized pre-open sets and studied some of its properties.

**4.1 Definition**

 An PNPS A is said to be an Pentapartitioned Neutrosophic Pythagorean generalized pre-open set (PNPGPOS in short) in (X,$ τ)$ if the complement $A^{c}$ is an PNPGPCS in X.

The family of all PNPGPOSs of an PNPTS (X,$τ)$ is denoted by PNPGPO(X).

**4.2 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.7, 0.3,0.6,0.1,0.2 >, < a, 0.6, 0.5,0.6,0.2,0.3 >}. Then the PNPS A = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.7, 0.6,0.6,0.1,0.2 >} is an PNPGPOS in X.

**4.3 Theorem**

 For any PNPTS (X,$τ)$, we have the following:

* Every PNPOS is an PNPGPOS
* Every PNPSOS is an PNPGPOS
* Every PNP$α$OS is an PNPGPOS
* Every PNPPOS is an PNPGPOS.

But the converses is not true in general

**Proof**: Straight forward.

The converse of the above statements need not be true which can be seen from the following examples.

**4.4 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.2, 0.1,0.6,0.7,0.8 >, < a, 0.3,0.2,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.8, 0.7,0.6,0.1,0.2 >, < a, 0.7, 0.6,0.6,0.1,0.2 >} is an PNPGPOS in (X,$ τ)$ but not an PNPOS in X.

**4.5 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.4, 0.3,0.6,0.5,0.6 >}. Then the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.7, 0.6,0.6,0.2,0.3 >} is an PNPGPOS but not an PNPSOS in X.

**4.6 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.4, 0.3,0.6,0.2,0.6 >, < a, 0.2,0.1,0.6,0.6, 0.7 >}. Then the PNPS A = {< a, 0.7,0.6,0.6,0.2, 0.3 >, < a, 0.8,0.7,0.6,0, 0.1 >} is an PNPGPOS but not an PNP$α$OS in X.

**4.7 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.6, 0.5,0.6,0.3,0.4 >, < a, 0.5,0.4,0.6,0.4, 0.5 >}. Then the PNPS A = {< a, 0.7, 0.6,0.6,0.2,0.3 >, < a, 0.6, 0.3,0.6,0.2,0.4 >} is an PNPGPOS but not an PNP$P$OS in X.

**4.8 Theorem**

 Let (X,$ τ)$ be an PNPTS. If A $\in $ PNPGPO(X) then V ⊆ PNPInt(PNPCl(A)) whenever V $⊆A$ and V is PNPCS in X.

**Proof**:

 Let A $\in $ PNPGPO(X). Then $A^{c}$ is an PNPGPCS in X. Therefore PNPPCl($A^{c}$) ⊆ U whenever $A^{c}$ ⊆ U and U is an PNPOS in X. That is PNPCl(PNPInt($A^{c}$)) ⊆ U. This implies $U^{c}$ ⊆ PNPInt(PNPCl(A)) whenever $U^{c}$ ⊆ A and $U^{c}$ is PNPCS in X. Replacing $U^{c}$ by V, we get V ⊆ PNPInt(PNPCl(A)) whenever V ⊆ A and V is PNPCS in X.

**4.9 Theorem**

 Let (X,$ τ)$ be an PNPTS. Then for every A $\in $ PNPGPO(X) and for every B $\in $ PNPS(X), PNPPInt(A) ⊆ B ⊆ A implies B $\in $ PNPGPO(X).

**Proof:**

 By hypothesis $A^{c}$ ⊆ $B^{c}$ ⊆ $(PNPPInt\left(A\right))^{c}$ . Let $B^{c}$ ⊆ U and U be an PNPOS. Since $A^{c}$ ⊆ $B^{c}$ , $A^{c}$ ⊆ U. But $A^{c}$ is an PNPGPCS, PNPPCl($A^{c}$) ⊆ U. Also $B^{c}$ ⊆ $(PNPPInt\left(A\right))^{c}$ = PNPPCl($A^{c}$). Therefore PNPPCl($B^{c}$) ⊆ PNPPCl($A^{c}$) ⊆ U. Hence $B^{c}$ is an PNPGPCS. Which implies B is an PNPGPOS of X.

**4.10 Remark**

The intersection of any two PNPGPOSs is not an PNPGPOS in general.

**4.11 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPTS be an PNPT on X, where T = {< a, 0.6,0.5,0.6,0.3, 0.4 >, < a, 0.8,0.7,0.6,0.5, 0.2 >}. Then the PNPSs A = {< a, 0.9, 0.8,0.6,0,0.1 >, < a, 0.2,0.1,0.6,0.7, 0.8 >} and B = {< a, 0.4, 0.3,0.6,0.5,0.6 >, < a, 0.3, 0.2,0.6,0.6,0.7 >} are PNPGPOSs but A $∩ $B is not an PNP$GP$OS in X.

**4.12 Theorem**

An PNPS A of an PNPTS (X,$ τ)$ is an PNPGPOS if and only if F ⊆ PNPPInt(A) whenever F is an PNPCS and F ⊆ A.

**Proof:**

 Necessity: Suppose A is an PNPGPOS in X. Let F be an PNPCS and F ⊆ A. Then

 $F^{c}$ is an PNPOS in X such that $A^{c}$ ⊆ $F^{c}$. Since $A^{c}$ is an PNPGPCS, we have PNPPCl($A^{c})$

⊆ $F^{c}$. Hence $(PNPPInt\left(A\right))^{c}$⊆ $F^{c}$. Therefore F ⊆ PNPPInt(A).

Sufficiency: Let A be an PNPS of X and let F ⊆ PNPPInt(A) whenever F is an PNPCS and F ⊆ A. Then $A^{c}$ ⊆ $F^{c}$ and $F^{c}$ is an PNPOS. By hypothesis, $(PNPPInt\left(A\right))^{c} $⊆ $F^{c}$. Which implies PNPPCl($A^{c})$ ⊆ $F^{c}$. Therefore $A^{c}$ is an PNPGPCS of X. Hence A is an PNPGPOS of X.

**4.13 Corollary**

An PNPS A of an PNPTS (X,$ τ)$ is an PNPGPOS if and only if F ⊆ PNPInt(PNPCl(A)) whenever F is an PNPCS and F ⊆ A.

**Proof:**

 Necessity: Suppose A is an PNPGPOS in X. Let F be an PNPCS and F ⊆ A. Then

$F^{c}$ is an PNPOS in X such that $A^{c}$ ⊆ $F^{c}$. Since $A^{c}$ is an PNPGPCS, we have PNPPCl($A^{c})$

⊆ $F^{c}$. Therefore PNPCl(PNPInt($A^{c}$) ⊆ $F^{c}$.Hence $(PNPInt\left(PNPCl\left(A\right))\right)^{c}$ ⊆ $F^{c}$. Therefore F

⊆ PNPInt(PNPCl(A)).

Sufficiency: Let A be an PNPS of X and let F ⊆ PNPInt(PNPCl(A)) whenever F is an

PNPCS and F ⊆ A. Then $A^{c}$ ⊆ $F^{c}$ and $F^{c}$ is an PNPOS. By hypothesis,

$(PNPInt\left(PNPCl\left(A\right))\right)^{c} $⊆ $F^{c}$.Hence PNPCl(PNPInt($A^{c}$)) ⊆ $F^{c}$ , which implies pcl($A^{c})$ ⊆

 $F^{c}$. Hence A is an PNPGPOS of X.

**4.14 Theorem**

For an PNPS A, A is an PNPOS and an PNPGPCS in X if and only if A is an PNPROS in X.

**Proof:**

 Necessity: Let A be an PNPOS and an PNPGPCS in X. Then pcl(A) ⊆ A. This implies PNPCl(PNPInt(A)) ⊆ A. Since A is an PNPOS, it is an PNPPOS. Hence A ⊆ PNPInt(PNPCl(A)). Therefore A = PNPInt(PNPCl(A)). Hence A is an PNPROS in X.

Sufficiency: Let A be an PNPROS in X. Therefore A = PNPInt(PNPCl(A)). Let A ⊆ U and U is an PNPOS in X. This implies pcl(A) ⊆ A. Hence A is an PNPGPCS in X.

**5. Applications of Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets**

In this section we provide some applications of Pentapartitioned Neutrosophic Pythagorean generalized pre-closed sets.

**5.1 Definition**

An PNPTS (X,$ τ)$ is said to be an Pentapartitioned Neutrosophic Pythagorean pT1/2 (PNPpT1/2 in short) spaceif every PNPGPCS in X is an PNPCS in X.

**5.2 Definition**

An PNPTS (X,$ τ)$ is said to be an Pentapartitioned Neutrosophic Pythagorean gpT1/2 (PNPgpT1/2 in short) spaceif every PNPGPCS in X is an PNPPCS in X.

**5.3 Theorem**

Every PNPpT1/2 space is an PNPgpT1/2 space. But the converse is not true in general.

**Proof:**

 Let X be an PNPpT1/2 space and let A be an PNPGPCS in X. By hypothesis A is an PNPCS in X. Since every PNPCS is an PNPPCS, A is an PNPPCS in X. Hence X is an PNPgpT1/2 space.

**5.4 Example**

Let X = {a, b} and let $τ$ = {0, T, 1} be an PNPT on X, where T = {< a, 0.9, 0.8,0.6,0,0.1 >, < a, 0.9, 0.8,0.6,0,0.1 >}.Then (X,$ τ)$ is an PNPgpT1/2 space. But it is not an PNPpT1/2 space since the PNPS A = {< a, 0.2,0.1,0.6,0.7, 0.8 >, < a, 0.3, 0.2,0.6,0.6,0.7 >} is PNPGPCS but not an PNPCS in X.

**5.5 Theorem**

Let (X,$ τ)$ be an PNPTS and X is an PNPpT1/2 space then

1. Any union of PNPGPCSs is an PNPGPCS.
2. Any intersection of PNPGPOSs is an PNPGPOS.

**Proof:**

(i) Let $\{A\_{i}\}\_{i \in J}$ is a collection of PNPGPCSs in an PNPpT1/2 space (X,$ τ)$. Therefore every PNPGPCS is an PNPCS. But the union of PNPCS is an PNPCS. Hence the union of PNPGPCS is an PNPGPCS in X.

(ii) It can be proved by taking complement in (i).

**5.6 Theorem**

An PNPTS X is an PNPgpT1/2 space if and only if PNPGPO(X) = PNPPO(X).

**Proof:**

 Necessity: Let A be an PNPGPOS in X, then $A^{c}$ is an PNPGPCS in X. By hypothesis $A^{c}$ is an PNPPCS in X. Therefore A is an PNPPOS in X. Hence PNPGPO(X) = PNPPO(X).

Sufficiency: Let A be an PNPGPCS in X. Then $A^{c}$ is an PNPGPOS in X. By hypothesis $A^{c}$ is an PNPPOS in X. Therefore A is an PNPPCS in X. Hence X is an PNPgpT1/2 space.