**An EOQ Model for Deteriorating item with Carbon Emission and Optimal Economic Circular Index Policy**

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**Abstract**

This paper presents an Economic Order Quantity (EOQ) model for deteriorating items that integrates carbon emission considerations and the development of an Optimal Economic Circular Index (OECI) policy. The model addresses the complexities associated with perishable goods, whose value diminishes over time, by optimizing inventory management strategies that balance both economic efficiency and environmental sustainability. The proposed EOQ model incorporates carbon emission costs, highlighting the trade-offs between traditional cost minimization and the reduction of environmental impact. Additionally, the model introduces the OECI policy, which promotes circular economy principles by encouraging the reuse, recycling, and efficient disposal of deteriorating items. Through analytical and numerical analysis, the study demonstrates how integrating carbon emissions and circular economy principles can lead to more sustainable inventory practices. The results underscore the potential for businesses to achieve cost savings while simultaneously reducing their carbon footprint, thereby aligning economic objectives with environmental stewardship.

**Key words-** EOQ Model, Carbon emission, circular index policy, Partial backlogging

**Introduction and Literature review**

To address global warming, biodiversity loss, carbon emissions, and waste, the circular economy has emerged as a viable alternative to industries' current linear economic framework. Every year, a massive amount of waste is dumped into the environment as a result of the current linear economic system, which begins with the production of goods from raw materials and ends with their disposal into the environment. Many raw materials are limited and require a significant amount of energy to extract. The circular economy is a systematic framework in which production is reused, recycled, remanufactured, and returned to the market at an economic cost. As a result, a circular economy addresses both environmental and financial issues. In (2016) Lenwandowski developed a comprehensive model to introduce the characteristics of a circular economy based on the business environment. In (2018) De Angelis developed a framework for circular supply chain to eliminate the drawbacks of the present linear supply chain. In (2020) Rabta for the first time, developed an economic order quantity model to investigate the results of products’ circularity level. In (2022) Thomas and Mishra developed a sustainable circular economy model to reduce Carbon emission and waste with the help of 3D printing. Further, Khan et al. in (2022) studied a production system with carbon emission to optimize the circular economy index policy. Mohan and Prakash (2023) developed research in this study looks into the generalised time-fractional Cattaneo model. The numerical solution for this model is obtained using the homotopy perturbation transform technique. The stability is examined using the Lyapunov function, and the error analysis is explained. Chaudhary and Kumar (2023) developed a research paper and in this paper, they proposed a fractional order fast terminal sliding mode control approach to handle the trajectory tracking problem that emerges when robot manipulators are subjected to uncertainties and external disturbances. A unique fractional order fast terminal sliding surface is proposed to achieve rapid finite time convergence, and an exact definition for the settling time is also provided. Mohan and Prakash (2023) developed research this article they investigated the fractional model of brain tumor. The numerical solution for this model is derived using a modified technique known as the Natural transform homotopy perturbation technique. Kumar and Chaudhary (2024) developed research in which they investigated the position tracking control problem for nonholonomic mobile robots with system uncertainties and external disturbances. In the design method, a fractional-order sliding surface is proposed that provides asymptotic stability of the system states as they approach equilibrium. Mohan and Prakash (2024) developed research in this work they presented a highly efficient technique for analyzing the fractional BBM-Burger equation and fractional Diffusion-Wave equation. These equations are used to simulate a variety of real-world phenomena, including acoustic gravity waves, diffusion theory, anomalous diffusive systems, and wave propagation. Mohan and Prakash (2024) developed research and in this article, they examined the fractional Cattaneo heat equation to investigate heat conduction in porous media. This equation is also used to investigate extended irreversible thermodynamics, materials, plasma, cosmological models, computational biology, and diffusion theory in crystalline solids. Mohan and Prakash (2024) developed research and in this paper, the time-fractional Tricomi equation is studied using two efficient computing techniques. This equation explains the nearly sonic speed gas dynamics phenomenon. The time-fractional Tricomi equation is solved using the Homotopy perturbation transform methodology, which combines the Laplace transform and a semi-analytical technique, as well as the Homotopy analysis method. From this literature survey we find that there are no one research about that circular economy takes with carbon emission and shortage and also consider deterioration so, here we developed an inventory model with consider these concepts together.

**Table:1**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Authors** | **Circular economy** | **Demand depended on** | **Per unit gross profit depended on** | **Carbon emission** | **Shortages** | **Deterioration** |
| Chang *et al.* (2003) |  | Time | Constant |  |  |  |
| Ouyang *et al.* (2013) |  | Constant | Constant |  |  |  |
| Sharma *et al.* (2016) |  | Constant | Constant |  | Allowed with partial backlog |  |
| Rabta (2020) |  | Circularity index | Circularity index |  |  |  |
| Thomas and Mishra (2022) |  | Circularity index | Circularity index |  |  |  |
| Khan *et al.* (2023) |  | Circularity index | Circularity index |  |  |  |
| This paper |  | Circularity index | Circularity index |  | Allowed with partial backlog |  |

**Assumptions**

Assumptions are made as follows.

1. The demand rate is dependent on circularity level

Where ,, and are constant parameters.

1. The unit gross profit is dependent on circularity level

Where ,, and are constant parameters.

1. This model contains a single item and single retailer.
2. Replenishment rate is instantaneous.
3. The inventory system has an infinite planning horizon.
4. Shortages are allowed with partial backlogging and the partial backlogging parameter is *ρ,* 0 < *ρ <* 1.
5. Inflation is taken into consideration

**Notations**

* A Ordering cost per order
* selling price per unit
* selling price per unit when circularity index
* demand rate
* demand rate when circularity index
* r rate of inflation
* Ce  reduce carbon emissions
* Q retailer’s order quantity
* θ deterioration rate
* B back order level
* I1(t) inventory level in interval [0,T1]
* I2(t) inventory level in interval [T1, T]
* *ρ* partial backlogging parameter
* Ch holding cost
* Cd deterioration cost
* Sc shortage cost
* Ci lost sale cost

**Mathematical model formulation**

The behaviour of the inventory level is shown in figure 1.

Inventory

q

B

Time

**Fig:1- Inventory level with respect to time**

The differential equation representing the inventory level is:

(1)

(2)

With boundary conditions , ,

After solving these equations (1) and (2) with the help of boundary conditions, we get

(3)

(4)

Ordered quantity and backorder for this model is

(5)

And

(6)

The total profit for the retailer contains the following terms,

Ordering cost (OC) (7)

Holding cost (Ch) =

(8)

Deterioration cost (Cd)=

(9)

Green investment = (10)

Carbon emission cost (CE) =

(11)

Shortage cost =

(12)

Lost sale cost =

(13)

Sales revenue =

(14)

Total profit = (15)

**Numerical illustration**

The results from numerical analysis not only validate the theoretical model but also provide actionable insights for businesses. By analysing different scenarios, companies can better understand the trade-offs between economic and environmental objectives and make informed decisions that align with both cost efficiency and sustainability goals. The use of numerical analysis thus bridges the gap between theoretical development and practical application, ensuring that the EOQ model is both relevant and reliable in real-world settings.

**Table: 2**

|  |  |  |  |
| --- | --- | --- | --- |
| Parameters | Value | Parameters | Values |
| p0 | 1.5 | b | 0.1 |
| β | 0.21 | r | 0.05 % |
| Do | 1 | A | 0.01 $ |
| γ | 0.01 | ρ | 0.11 |
| a | 1.1 | ch | 100$ |
| θ | 0.001% | cd | 10$ |
| ce | 310 $ | sc | 0.001$ |

Final optimal solution is

Table:2

|  |  |  |
| --- | --- | --- |
|  | Time taken to finish the inventory level *(t)* | 0.00381725 year |
| *T* | Replenishment cycle length | 0.0927898 year |
| *ω* | Circularity index 0 *≤ ω ≤* 1 | 0.000707859 |
|  | Total profit | 0.0160897 $ |

**Concavity**



*ω*

TP

Fig:2 concavity Between *ω* (cycle length) and T1(Time taken to finish the inventory level)



*ω*

T

TP

Fig: 3 concavity Between (Circularity index) and T(Time taken to finish the inventory level)



T

TP

Fig:4 concavity Between (circularity index) and T (cycle length)

**Sensitivity Analysis**

Sensitivity analysis is a crucial step in evaluating the robustness and reliability of an EOQ model, particularly when dealing with deteriorating items, carbon emissions, and an Optimal Economic Circular Index (OECI) policy. It involves systematically varying key parameters within the model to observe how changes in these variables impact the optimal solutions, such as order quantity, total cost, and carbon emissions. This analysis not only provides insights into the stability of the optimal solutions under different scenarios but also helps in identifying critical parameters that may require more precise estimation or monitoring.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter | % value | Total cost | T | T1 | ω |
|  | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | -0.00560 | 0.003502 | 0.09542 | 0.00056 |
| -20% | -0.02673 | 0.00318 | 0.09769 | 0.000130 |
|  | +20% | 0.01853 | 0.00388 | 0.09317 | 0.01630 |
| +10% | 0.01738 | 0.00387 | 0.09269 | 0.02830 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
|  | +20% | 0.01560 | 0.00381 | 0.09285 | 0.000503 |
| +10% | 0.01584 | 0.003812 | 0.09282 | 0.0005011 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01634 | 0.003821 | 0.09277 | 0.000565 |
| -20% | 0.0165 | 0.00382 | 0.09275 | 0.000557 |
|  | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
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| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
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| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
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| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |

1. On increasing in p0 the total profit, cycle length increases while circularity index fluctuating and is decreases.

2. On increasing in the parameter , total profit, cycle length and are all increases while circularity index decreases.

3. On increasing in the parameter , total profit decreases cycle length fluctuating and circularity index decreases.

4. On increasing in the parameter , total profit is decreasing, cycle length fluctuating while is decreasing and circularity index also decreases.

5. On increasing in the parameter , total profit, cycle length and circularity index increases.

6. On increasing in the parameter , total profit, cycle length increases while circularity index fluctuating.

7. On increasing in the parameter , the total profit, cycle length and circularity index increases.

8. On increasing in parameter , the total profit and fluctuating, cycle length and circularity index decreasing.

9. On increasing in parameter , total profit fluctuating cycle length, circularity index and decreases.

10. On increasing in parameter , total profit is increases, cycle length is also increases, is decreases and circularity index fluctuating.

11. On increasing in parameter , total profit is increases slightly decreases same situation for cycle length and is increases while circularity index is decreases.

12. On increasing in parameter , total profit is increases same for cycle length, is also decreases while circularity index is increases.

13. On increasing in parameter , total profit is increases, cycle length is fluctuating, is decreases circularity index is increases.

14. On increasing in parameter , total profit is slightly increases same for cycle length, is decreases and circularity index is also decreases.

**Conclusion**

This study develops an EOQ model for deteriorating items that integrates carbon emission considerations and introduces an Optimal Economic Circular Index (OECI) policy. The model effectively balances economic efficiency with environmental sustainability, demonstrating that it is possible to reduce costs and carbon footprints simultaneously through optimized inventory management. The inclusion of carbon emission costs within the EOQ framework highlights the importance of considering environmental impact in traditional inventory models. The introduction of the OECI policy further emphasizes the value of circular economy principles, encouraging businesses to adopt practices such as recycling, reuse, and efficient disposal of deteriorating items. Sensitivity analysis confirms the model's robustness and provides valuable insights into the influence of key parameters, such as deterioration rate and emission costs, on optimal inventory strategies. This analysis also underscores the importance of accurately estimating these parameters to achieve optimal results. several avenues for future research can be pursued. One potential direction is extending the model to multi-item and multi-echelon inventory systems, exploring how different items with varying deterioration rates and carbon footprints can be managed collectively and how inventory policies can be coordinated across different levels of a supply chain. Incorporating stochastic elements such as random demand fluctuations and variable deterioration rates would also enhance the model's applicability to real-world scenarios, necessitating the development of robust optimization techniques to handle uncertainty.

Overall, this study offers a comprehensive approach to managing deteriorating inventories in a way that supports both economic and environmental objectives. The proposed model can serve as a valuable tool for businesses seeking to enhance their sustainability efforts while maintaining economic viability, providing a framework for integrating circular economy practices into inventory management.

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