**CHAPTER – 4**

**A constant deterioration with shortage and trade credit inflation-affected EOQ inventory model with nonlinear stock dependent holding costs, nonlinear stock dependent demand, and constant degradation**

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**Abstract:** This work discusses a nonlinear stock-dependent demand and continuous deterioration economic order quantity (EOQ) inventory model. The merchant receives a trade credit term from the supplier, and the retailer is the one that creates this inventory model. Essentially, two inventory models are developed here: (i) a model of shortage and (ii) a model of shortfalls without shortage. Both inventory models aim to maximise the retailer's total profit per unit of time by determining the optimal ordering amount and ending inventory level.

**Keywords: EOQ model, Trade credit, Stocks**

1. **Introduction:**

There are a lot of application of mathematic in various domains such as numerical analysis [1-5], approximation theory [6-8], control theory [9-10], inventory handling [11-12] etc. In this chapter, an economic order quantity (EOQ) inventory model with continual degradation and nonlinear stock dependent demand are discussed. The supplier grants the merchant a trade credit term, and this inventory model is created from the retailer's perspective. Here, we deviate from the conventional presumption of a zero-ending inventory level and instead assume a non-zero ending inventory level (which may be positive, negative, or zero). A negative ending inventory level indicates that shortages are allowed and that the backlog is partially maintained at a constant rate of "δ." This chapter essentially develops two inventory models: (i) a model of shortage and (ii) a model of shortfall without shortage. Finding the ideal ordering quantity and ending inventory level that maximizes the retailer's overall profit per unit of time is the main goal of both inventory models.

Several prior articles were studied for this work, some of which are included below: the paper on an optimum replenishment policy for non-instantaneous degrading products with stock-dependent demand and partial backlogging, written in [13-14]. In order to expand it to include models with allowable payment delays as provided [15], In [16] developed an inventory model for non-instantaneous degrading products with allowable payment delays. Following several advancements in this subject and a substantial volume of publications, [17] created a lot-size model for deteriorating and faulty goods with trade credit. They believed that the demand varied with time. The ideal pricing and lot-sizing strategy for a supply chain system that takes into account deteriorating commodities under limited storage capacity was established in [18]. The inventory model with trade credit policy, non-zero ending inventory level, shortages, and partial backlog with constant backlog rate were all taken into consideration in this study.

This chapter expands on the study "An EOQ inventory model with nonlinear stock dependent demand, nonlinear stock dependent holding cost, and trade credit" [19]. We propose inflation and continuous degradation as two new variables in this study. This chapter presents the development of an inventory model titled "An EOQ inventory model with nonlinear stock-dependent holding cost, nonlinear stock-dependent demand, constant deterioration with shortage, and trade credit under the effect of inflation." A very practical inventory replenishment policy is provided through the analysis of inventory models. Once this model has been ultimately solved, we go through a few numerical instances that offer answers to the theoretical ideas. It is also done to analyze the optimal solution's sensitivity to key factors.

The notations and assumptions were first discussed in section 2 of this chapter. The mathematical modeling was presented in section 3. The cost components with shortage and without shortage were presented in sections 4 and 5. The solution process is presented in section 6. Numerical examples were provided in sections 7 and 8. Sensitivity analysis was presented in section 8. Finally, the conclusion to the model developed in this chapter is provided in section 9.

1. **Assumptions and Notations:**

**Notations:**

*Co*  replenishment cost per order.

*c*  purchasing cost per unit.

*s*  selling price per unit.

*h*  unit time holding cost per unit per unit time.

*cs* unit time shortage cost per unit per unit time.

*cl*  lost sale cost per unit.

γ the elasticity of holding cost; 0 < γ < 1.

β demand elasticity; 0 < β < 1.

a Scale parameter of the demand rate.

*q(t)* units inventory level at any time *t* where *0* ≤ *t* ≤ *T.*

*M* unit time the retailer’s trade credit period provided by the supplier.

r Rate of inflation; 0 < r < 1.

θ Constant rate of deterioration; 0 < θ < 1.

*Ie*  unit time Interest earned by the retailer.

*Ip*  unit time Interest paid by the retailer.

Q order quantity per cycle.

B the end inventory level at time ‘T’.

λ time at which inventory level is zero.

τ the length of replenishment cycle.

TPunit time the total profit per unit time.

δ partial backlogging parameter; a fraction of the demand within the stock our period that is backlogged, δ € [0,1].

**Assumptions:**

1. The inventory system has an unlimited planning horizon.

2. There is very little lead time and an instantaneous replenishment rate.

3. There is allowance for shortages, which are partially backlogged at a backlogging rate ‘δ’; δ> 0.

4. A single-tier trade credit policy is taken into account. Here, the manufacturer, supplier, or retailer extends credit to his or her client for a certain amount of time and on well stated terms and conditions.

5. The holding cost, which depends on stock level, is a nonlinear function that may be expressed as H(t) = h[I(t)]γ ; γ > 0.

6.The demand function is considered as a nonlinear stock dependent demand given by



7This model takes inflation into account along with the rate of inflation ‘r’; r > 0.

8. Deterioration rate must be regarded as constant with a parameter. ‘θ’; θ > 0.

1. **Mathematical modelling:**

In this inventory model we consider two cases with two sub cases each. The mathematical modeling for this inventory model is shown in the given graphs for all the four cases.

Here, in the first graph we consider shortage along with trade credit time less than the time at which inventory finished. In the second graph we consider shortage along with trade credit time greater than the time at which inventory finished. In the third graph shortage is not considered but trade credit policy is considered with trade credit time less than the time at which inventory finished. In the last graph shortage is not considered and trade credit time is greater than the time at which inventory finished.

In case 1, let at time ‘0’ inventory level is at ‘Q’ level. Due to deterioration and non- linear demand, after time ‘λ’ inventory level is finished and shortage occurs. In this modeling environment we considered trade credit policy so, there is two subcases for this and in first subcase we consider trade period ‘M’ is less than ‘λ’ and in another subcase we consider trade period ‘M’ is greater than ‘λ’.

In case 2, let at time ‘0’ inventory level is at ‘Q’ level. Due to deterioration and non- linear demand, after time ‘λ’ inventory level is finished and shortage is not considered. In this modeling environment we considered trade credit policy so, there is two subcases for this and in first subcase we consider trade period ‘M’ is less than ‘λ’ and in another subcase we consider trade period ‘M’ is greater than ‘λ’.

λ

0

B

Q

I(t)

δ

τ

λ

0

B

Q

I(t)

δ

τ

Case: 1 When Shortage is considered.

M

M

M ≤ λ

M ≥ λ

Fig :4.1

Fig :4. 2

I(t)

τ

0

λ

M

I(t)

τ

0

λ

M

M ≤ λ M ≥ λ

Fig :4.4

Fig :4.3

 Case: 2 When Shortage is not considered.

1. **Mathematical formulation:** The differential equation for first and second case is given as follows:

**When shortage is considered:** The differential equation for case ‘1’ is given as follows:

**Cost Components:**

The costs for case ‘1’ are given as follows:

 **Holding Cost:**

Holding cost is the amount of money required to keep the inventory. Since the formulation for determining holding cost in this model is time dependent, we evaluated holding cost to be as follows:

**Fixed Ordering cost:**

Fixed ordering cost is the amount of money required to order inventory materials. This expense is set, therefore please allow it to be ‘C0’.

**Purchasing cost:**

The purchasing cost is the total amount of money required to purchase inventory items. This sum is the product of the inventory level in the initial state and "c," or the purchase cost per unit time. This model's purchase price is listed as follows:

**Sales Revenue cost:**

Total revenue cost, sometimes referred to as sales revenue cost, is the amount of money received once inventory is sold. The following is the formula for the cost:

**Deteriorating cost:**

Deterioration cost is the amount of money lost as a result of inventory damage or deterioration. The following is the term indicating declining cost.

**Shortage cost:**

Shortage cost is the amount of money lost as a result of an inventory shortfall. The following is the shortfall cost for this model.

S.C.

**Opportunity cost:**

The amount of money which needs to pay by the inventor for penalty due to not completing the demand on time is known as opportunity cost or penalty cost. The expression for this cost is given as follows:

** **

**Trade credit policy:**

Opportunity cost or penalty cost is the sum of money that the innovator must pay as compensation for missing the deadline for fulfilling the demand. The following is the phrase for this cost:

**For case: 1(M ≤ λ)**

**Interest pay:**

Interest payments are calculated based on the amount that the store pays the loan agency. This is the sum that the retailer deducts from any external lending agency in order to pay the supplier since the trading period is shorter than the time at which all of the inventory is sold.

**Interest earn:**

Interest earn is the sum of money that a merchant receives as interest on merchandise that is sold. The amount of interest earned is as follows:

**For case: 2 (M ≥ λ)**

**Interest pay :**

Interest payments are calculated based on the amount that the store pays the loan agency.

i.e. This is the sum that the retailer deducts from any external lending agency in order to pay the supplier since the trading period is shorter than the time at which all of the inventory is sold.

 There is no interest payable here, therefore I.P. = 0

**Interest earn :**

Interest earn is the sum of money that a merchant receives as interest on merchandise that is sold. The amount of interest earned is as follows:

**Total cost for case: 1.1**

**T.P1 =** Sales revenue cost + interest earn – ordering cost – purchasing cost – holding cost – deteriorating cost – shortage cost – opportunity cost – interest pay.

**Total cost for case: 1.2**

**T.P2 =** Sales revenue cost + interest earn – ordering cost – purchasing cost – holding cost – deteriorating cost – shortage cost –- Preservation technology investment cost - opportunity cost.

**Total average cost:** Total average cost for both cases are given as follows:

 

**(Case:2) When shortage is not considered:**

The differential equation for case 2 is given as follows: ****

**Cost Components:**

**Holding cost:**

Holding cost is the amount of money required to keep the inventory. We regarded holding cost as time-dependent in this model as the following statement may be used to calculate holding cost:

 **Fixed Ordering cost:**

The amount of money which needs to order the inventorymaterial is known as fixed ordering cost. This cost is fixed and let this cost is ‘C0’.

**Purchasing cost:**

The amount of money which needs for purchase the inventory goods is known as purchasing cost. This amount is equal to ‘c’(purchasing cost per unit time) times the invent tory level at first state. The purchasing cost for this model is given as follows:

**Sales Revenue cost:**

The amount of money which obtained after selling inventory is known as total revenue cost or sales revenue cost. 

**Deteriorating cost:**

The amount of money which is lost due to deterioration or damage of inventory is known as deterioration cost. The expression for deteriorating cost is given as follows:****

**Trade credit policy:**

In this model we consider trade credit policy for both cases when trade period is less than to the period when inventory finished and when trade period is greater than to the period when inventory finished. The costs for both case are given as follows:

**For case: 1(M ≤ λ)**

**Interest pay:**

The amount of money which is paid by retailer to the loan agency comes under consideration to interest pay. This is the amount which is taken by retailer, to pay supplier, from any external loan agency due to trade period is less than the time at which complete inventory sold.

****

**Interest earn:**

The amount of money which is earned as interest by retailer, on the sold inventory is known as interest earn. The interest earn is given as follows:

****

**For case: 2 (M ≥ λ)**

**Interest pay :**

The amount of money which is paid by retailer to the loan agency comes under consideration to interest pay.

i.e. This is the amount which is taken by retailer, to pay supplier, from any external loan agency due to trade period is less than the time at which complete inventory sold.

Here, there is no interest pay so I.P. = 0

**Interest earn :**

The amount of money which is earned as interest by retailer, on the sold inventory is known as interest earn. The interest earn is given as follows:

****

 **Total cost for case: 2.1**

**T.P'1 =** Sales revenue cost + interest earn – ordering cost – purchasing cost – holding cost – deteriorating cost– interest pay.



**Total cost for case: 2.2**

**T.P'2 =** Sales revenue cost + interest earn – ordering cost – purchasing cost – holding cost – deteriorating cost.



**Total average cost:** Total average cost for both cases are given as follows:

****

1. **Solution Procedure:**

Our aim is to find out the best possible value of Q, B, τ, λ such that **AP1, AP2, AP’1 and AP’2** is maximal. For this we use bordered hessian matrix method for solving non-linear programming problem. There are four variables on which objective function depends.so, there will be 4\*4 order hessian matric. Since, this problem is of maximization, so for optimality bordered hessian matrix should be negative definite. The given bordered hessian matrix is as follows:



The given matrix is negative definite. So, optimality will obtained on its critical points which will obtained by putting partial derivative of A.P with respect to parameters Q, B, , λ.

That is,



This is analytical method to solve this problem, but i did all these calculations by the help of mathematica and develop a numerical on it, which are given as follows.

1. **Numerical Example:**

 For analysing the facts which we introduced in this chapter, we develop some numerical examples in which four are given as follows.

**4.6.1 (Case 1a):** The above given result are illustrated through the numerical examples.To illustrate the model we consider the following input data for subcase ‘1’ of first case.

Let s = 70, d = 0.3, h = 0.5, a = 1, r = 0.5, C = 50, c = 50, M = 0.5, ρ = 50, γ = 1.2, δ = 0.8, θ = 0.5, β = 0.1, *Ip* = 0.12, *Ie* = 0.09, Cl = 10, Cs = 20.

**Ans**: Applying solution process for case ‘1a’. we get following results;

A.P1 = 708.6355, Q = 99.99, λ = 2.6421, τ = 3, B = -40.

**4.6.2 (Case 1b):** For subcase ‘2’ of first case we consider the following data for illustrate the model in mathematical form.

Let s = 70, d = 0.3, h = 0.5, a = 1, r = 0.5, C = 50, c = 50, M = 0.5, ρ = 50, γ = 1.2, δ = 0.8, θ = 0.5, β = 0.1, *Ip* = 0.12, *Ie* = 0.09, Cl = 10, Cs = 20.

**Ans**: Applying solution process for case ‘1a’. we get following results;

A.P2 = 695.23, Q = 78.89, λ = 2.3421, τ = 3.2, B = -35.

**4.6.3 (Case 2a):** For subcase ‘1’ of second case we consider the following data for illustrate the model in mathematical form.

Let s = 70, d = 0.3, h = 0.5, a = 1, r = 0.5, C = 50, c = 50, M = 0.5, ρ = 50, γ = 1.2, δ = 0.8, θ = 0.5, β = 0.1, *Ip* = 0.12, *Ie* = 0.09, Cl = 10, Cs = 20.

**Ans**: Applying solution process for case ‘2a’. we get following results;

A.P’1 = 911.825, Q = 100, λ = 0.5, τ = 3, B = -40.

**4.6.4 (Case 2b):** For subcase ‘2’ of second case we consider the following data for illustrate the model in mathematical form.

Let s = 70, d = 0.3, h = 0.5, a = 1, r = 0.5, C = 50, c = 50, M = 0.5, ρ = 50, γ = 1.2, δ = 0.8, θ = 0.5, β = 0.1, *Ip* = 0.12, *Ie* = 0.09, Cl = 10, Cs = 20.

**Ans**: Applying solution process for case ‘2a’. we get following results;

A.P’2 = 893.690, Q = 97, λ = 0.52, τ = 3.75, B = -38.

1. **Behavior of optimum cost function:**

**For case: 1**



A.P1.

Q

τ

**For case: 2**



A.P1'.

Q

τ

1. **Sensitivity Analysis:**

To see, how optimal solution is affected by the values of parameters, we originate the sensitivity analysis for some of the parameters. The particular values of some parameter decreased to -5%, -10%, -15%, -20% and then increased to 5%, 10%, 15%, 20%.

**8.1 Sensitivity Analysis for case ‘1’ is given as follows:**

**8.1.1 Sensitivity analysis for parameter ‘r’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  R |  τ  |  Q  |   |  B |    |
| 0.400 | 3.002 | 100.00 | 0.501 | -3.48×10-7 | 662.630 |
| 0.425 | 3.002 | 100.00 | 0.500 | 0.0000 | 661.998 |
| 0.450 | 3.001 | 99.990 | 0.500 | 0.0000 | 661.396 |
| 0.475 | 3.000 | 99.990 | 0.500 | -4.9×10-8 | 660.821 |
| 0.500 | 3.000 | 99.980 | 2.642 | -40.00 | 708.630 |
| 0.525 | 3.000 | 99.980 | 2.568 | -40.00 | 714.080 |
| 0.550 | 3.001 | 99.990 | 2.503 | -40.00 | 720.580 |
| 0.575 | 3.001 | 100.00 | 2.445 | -40.00 | 727.870 |
| 0.600 | 3.002 | 100.00 | 2.395 | -40.00 | 735.760 |

**8.1.2 Sensitivity analysis for parameter ‘θ’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  Θ |  τ  |  Q  |   |  B |   |
| 0.400 | 3.001 | 100.00 | 0.500 | -3.47×10-7 | 662.630 |
| 0.425 | 3.001 | 100.00 | 0.500 | 0.0000 | 661.998 |
| 0.450 | 3.001 | 100.00 | 0.499 | 0.0000 | 661.396 |
| 0.475 | 3.000 | 99.999 | 0.499 | -4.9×10-8 | 660.821 |
| 0.500 | 3.000 | 99.999 | 2.642 | -40.00 | 708.630 |
| 0.525 | 3.000 | 99.999 | 2.594 | -40.00 | 714.760 |
| 0.550 | 3.000 | 100.00 | 2.552 | -39.99 | 720.203 |
| 0.575 | 3.001 | 100.00 | 2.515 | -39.98 | 726.664 |
| 0.600 | 3.001 | 100.00 | 2.482 | -39.98 | 733.464 |

**8.1.3 Sensitivity analysis for parameter ‘γ’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  **Γ** |  τ  |  Q  |   |  B |   |
| 0.96 | 3.002 | 100.01 | 2.637 | -40.002 | 706.06 |
| 1.02 | 3.001 | 100.00 | 2.638 | -40.001 | 706.70 |
| 1.08 | 3.001 | 99.999 | 2.639 | -40.001 | 707.35 |
| 1.14 | 3.001 | 99.998 | 2.641 | -40.001 | 707.99 |
| 1.20 | 3.000 | 99.998 | 2.642 | -40.000 | 708.64 |
| 1.26 | 3.000 | 99.999 | 2.643 | -40.000 | 709.28 |
| 1.32 | 3.000 | 100.00 | 2.644 | -40.001 | 709.92 |
| 1.38 | 3.001 | 100.01 | 2.646 | -40.002 | 710.57 |
| 1.44 | 3.001 | 100.01 | 2.647 | -40.002 | 711.22 |

**8.1.4 Sensitivity analysis for parameter ‘β’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  Β |  τ  |  Q  |   |  B |   |
| 0.080 | 3.002 | 100.00 | 2.646 | -40.002 | 707.62 |
| 0.085 | 3.001 | 100.00 | 2.645 | -40.001 | 707.87 |
| 0.090 | 3.001 | 99.999 | 2.644 | -40.001 | 708.13 |
| 0.095 | 3.001 | 99.998 | 2.643 | -40.001 | 708.38 |
| 0.100 | 3.000 | 99.998 | 2.642 | -40.000 | 708.68 |
| 0.105 | 3.000 | 99.999 | 2.641 | -40.000 | 708.88 |
| 0.110 | 3.000 | 100.00 | 2.640 | -40.001 | 709.18 |
| 0.115 | 3.001 | 100.01 | 2.339 | -40.002 | 709.38 |
| 0.120 | 3.001 | 100.01 | 2.638 | -40.002 | 709.62 |

**8.1.5 Sensitivity analysis for parameter ‘s’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  S |  τ  |  Q  |   |  B |   |
| 56.00 | 3.002 | 100.00 | 0.5000 | -1.15×10-8 | 193.545 |
| 59.50 | 3.001 | 100.00 | 0.4999 | -1.16×10-8 | 310.230 |
| 63.00 | 3.001 | 99.999 | 0.4999 | -1.21×10-8 | 426.910 |
| 66.50 | 3.001 | 99.998 | 2.6422 | -40.00 | 545.289 |
| 70.00 | 3.000 | 99.998 | 2.6421 | -40.00 | 708.640 |
| 73.50 | 3.000 | 99.999 | 2.6421 | -39.99 | 871.980 |
| 77.00 | 3.000 | 100.00 | 0.5000 | -39.99 | 957.289 |
| 80.50 | 3.001 | 100.01 | 2.6421 | -39.98 | 1198.68 |
| 84.00 | 3.001 | 100.01 | 2.6421 | -39.98 | 1362.03 |

**8.1.6 Sensitivity analysis for parameter ‘m’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  M |  τ  |  Q  |   |  B |   |
| 0.400 | 3.0001 | 100.01 | 2.64219 | -40.00 | 672.816 |
| 0.425 | 3.0001 | 100.01 | 2.64219 | -39.99 | 682.213 |
| 0.450 | 3.0001 | 100.00 | 2.64218 | -39.99 | 691.312 |
| 0.475 | 3.0001 | 100.00 | 2.64218 | -39.98 | 700.118 |
| 0.500 | 3.0000 | 100.00 | 2.64217 | -39.99 | 708.636 |
| 0.525 | 3.0000 | 100.00 | 2.64218 | -39.99 | 716.870 |
| 0.550 | 3.0001 | 100.00 | 2.64219 | -40.00 | 724.826 |
| 0.575 | 3.0001 | 100.00 | 2.64219 | -40.00 | 732.508 |
| 0.600 | 3.0002 | 100.01 | 2.64219 | -40.01 | 739.921 |

**8.1.7 Sensitivity analysis for parameter ‘δ’.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  Δ |  τ  |  Q  |   |  B |   |
| 0.784 | 3.0002 | 100.01 | 2.64271 | -40.00 | 708.595 |
| 0.788 | 3.0001 | 100.01 | 2.64258 | -39.99 | 708.605 |
| 0.792 | 3.0001 | 100.01 | 2.64245 | -39.98 | 708.615 |
| 0.796 | 3.0001 | 100.01 | 2.64232 | -39.98 | 708.625 |
| 0.800 | 3.0000 | 100.01 | 2.64219 | -39.99 | 708.636 |
| 0.804 | 3.0000 | 100.00 | 2.64206 | -39.99 | 708.646 |
| 0.808 | 3.0001 | 100.00 | 2.64193 | -40.00 | 708.656 |
| 0.812 | 3.0001 | 100.00 | 2.64180 | -40.01 | 708.666 |
| 0.816 | 3.0002 | 100.00 | 2.64167 | -40.01 | 708.676 |

**8.1.8 Sensitivity analysis for parameter ‘ip’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  ip |  τ  |  Q  |   |  B |   |
| 0.096 | 3.0002 | 100.01 | 2.70972 | -40.00 | 744.918 |
| 0.102 | 3.0001 | 100.01 | 2.69082 | -39.99 | 735.747 |
| 0.108 | 3.0001 | 100.01 | 2.67338 | -39.98 | 726.648 |
| 0.114 | 3.0001 | 100.01 | 2.65722 | -39.98 | 717.613 |
| 0.120 | 3.0000 | 100.01 | 2.64219 | -39.99 | 708.636 |
| 0.126 | 3.0000 | 100.00 | 2.62816 | -39.99 | 699.709 |
| 0.132 | 3.0001 | 100.00 | 2.61501 | -40.00 | 690.828 |
| 0.138 | 3.0001 | 100.00 | 2.60267 | -40.01 | 681.987 |
| 0.144 | 3.0002 | 100.00 | 2.59105 | -40.01 | 673.184 |

**8.1.9 Sensitivity analysis for parameter ‘ie’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  ie |  τ  |  Q  |   |  **B** |   |
| 0.0720 | 3.0002 | 100.01 | 2.64219 | -40.00 | 708.574 |
| 0.0765 | 3.0001 | 100.01 | 2.64219 | -39.99 | 708.589 |
| 0.0810 | 3.0001 | 100.01 | 2.64219 | -39.98 | 708.605 |
| 0.0855 | 3.0001 | 100.01 | 2.64218 | -39.98 | 708.620 |
| 0.0900 | 3.0000 | 100.01 | 2.64218 | -39.99 | 708.636 |
| 0.0945 | 3.0000 | 100.00 | 2.64217 | -39.99 | 708.651 |
| 0.0990 | 3.0001 | 100.00 | 2.64218 | -40.00 | 708.666 |
| 0.1035 | 3.0001 | 100.00 | 2.64219 | -40.01 | 708.682 |
| 0.1080 | 3.0002 | 100.00 | 2.64219 | -40.01 | 708.697 |

**8.2 Sensitivity analysis for case ‘2’ is given as follows:**

**8.2.1 Sensitivity analysis for parameter ‘r’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  R |  τ  |  Q  |   |  B |  AP1' |
| 0.400 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.716 |
| 0.425 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.743 |
| 0.450 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.771 |
| 0.475 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.798 |
| 0.500 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.525 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.852 |
| 0.550 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.878 |
| 0.575 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.904 |
| 0.600 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.930 |

**8.2.2 Sensitivity analysis for parameter ‘θ’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  Θ |  τ  |  Q  |   |  B |  AP1' |
| 0.400 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.909 |
| 0.425 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.887 |
| 0.450 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.865 |
| 0.475 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.845 |
| 0.500 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.525 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.852 |
| 0.550 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.878 |
| 0.575 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.904 |
| 0.600 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.930 |

**8.2.3 Sensitivity analysis for parameter ‘γ’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  **Γ** |  τ  |  Q  |   |  B |  AP1' |
| 0.96 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.645 |
| 1.02 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.690 |
| 1.08 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.735 |
| 1.14 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.780 |
| 1.20 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 1.26 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.870 |
| 1.32 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.915 |
| 1.38 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.959 |
| 1.42 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.004 |

**8.2.4 Sensitivity analysis for parameter ‘β’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  Β |  τ  |  Q  |   |  B |  AP1' |
| 0.080 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.297 |
| 0.085 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.433 |
| 0.090 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.566 |
| 0.095 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.697 |
| 0.100 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.105 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.950 |
| 0.110 | 3.0001 | 100.00 | 0.500 | -40.00 | 912.073 |
| 0.115 | 3.0001 | 100.00 | 0.500 | -40.00 | 912.194 |
| 0.120 | 3.0002 | 100.00 | 0.500 | -40.00 | 912.312 |

**8.2.5 Sensitivity analysis for parameter ‘s’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  S |  τ  |  Q  |   |  B |  AP1' |
| 56.00 | 3.0002 | 100.01 | 0.500 | -40.00 | 258.430 |
| 59.50 | 3.0001 | 100.01 | 0.499 | -40.00 | 421.779 |
| 63.00 | 3.0001 | 100.01 | 0.499 | -40.00 | 585.127 |
| 66.50 | 3.0001 | 100.01 | 0.498 | -39.99 | 748.476 |
| 70.00 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 73.50 | 3.0000 | 100.00 | 0.499 | -39.99 | 1075.17 |
| 77.00 | 3.0001 | 100.00 | 0.500 | -40.00 | 1238.52 |
| 80.50 | 3.0001 | 100.00 | 0.500 | -40.00 | 1401.87 |
| 84.00 | 3.0002 | 100.00 | 0.500 | -40.00 | 1565.22 |

**8.2.6 Sensitivity analysis for parameter ‘m’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  M |  τ  |  Q  |   |  B |  AP1' |
| 0.400 | 3.0002 | 100.01 | 0.500 | -40.00 | 899.818 |
| 0.425 | 3.0001 | 100.01 | 0.499 | -40.00 | 902.966 |
| 0.450 | 3.0001 | 100.01 | 0.499 | -40.00 | 906.016 |
| 0.475 | 3.0001 | 100.01 | 0.498 | -39.99 | 908.968 |
| 0.500 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.525 | 3.0000 | 100.00 | 0.499 | -39.99 | 914.588 |
| 0.550 | 3.0001 | 100.00 | 0.500 | -40.00 | 917.259 |
| 0.575 | 3.0001 | 100.00 | 0.500 | -40.00 | 919.839 |
| 0.600 | 3.0002 | 100.00 | 0.500 | -40.00 | 922.330 |

**8.2.7 Sensitivity analysis for parameter ‘δ’.**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  **δ**  |  τ  |  Q  |   |  B |  AP1' |
| 0.784 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.825 |
| 0.788 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.825 |
| 0.792 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.825 |
| 0.796 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.800 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.804 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.825 |
| 0.808 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.825 |
| 0.812 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.825 |
| 0.816 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.825 |

**8.2.8 Sensitivity analysis for parameter ‘ip’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  ip |  τ  |  Q  |   |  B |  AP1' |
| 0.098 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.825 |
| 0.106 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.825 |
| 0.108 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.825 |
| 0.114 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.120 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.126 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.825 |
| 0.132 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.825 |
| 0.138 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.825 |
| 0.144 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.825 |

**8.2.9 Sensitivity analysis for parameter ‘ie’**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  ie |  τ  |  Q  |   |  **B** |  AP1' |
| 0.0720 | 3.0002 | 100.01 | 0.500 | -40.00 | 911.763 |
| 0.0765 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.778 |
| 0.0810 | 3.0001 | 100.01 | 0.499 | -40.00 | 911.794 |
| 0.0855 | 3.0001 | 100.01 | 0.498 | -39.99 | 911.809 |
| 0.0900 | 3.0000 | 100.01 | 0.498 | -39.99 | 911.825 |
| 0.0945 | 3.0000 | 100.00 | 0.499 | -39.99 | 911.840 |
| 0.0990 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.856 |
| 0.1005 | 3.0001 | 100.00 | 0.500 | -40.00 | 911.871 |
| 0.1030 | 3.0002 | 100.00 | 0.500 | -40.00 | 911.887 |

1. **Conclusion:**

The globe is getting smaller these days due to technological advancements. The goods are also very accessible to organisations worldwide, in addition to the information flow. As a result, the businesses must use technical and strategic advances to penetrate farther into the supply chain they are a part of. The current global marketplaces, characterized by fluctuating markets and changing output, compel the many supply chain players to leverage technology to make tailored judgements. The management of supply chain difficulties pertaining to inventories, trade credit, and customer satisfaction is difficult due to corporate practices and governmental restrictions as the global economy grows and encompasses more different markets. In this context, the nonlinear stock holding cost, the nonlinear demand with continuous degradation, and trade credit under the influence of inflation are all attempted to be included in this chapter. This chapter looks at the best course of action for the store when receiving a credit term from their supplier under inflation, taking into account non-linear holding and stock-dependent demand of their goods. In essence, this research creates two inventory models that allow for shortages and partial backlogs, with nonlinear stock dependent holding costs, nonlinear stock dependent demand, and a single level credit policy with flexible ending inventory level. Finding the ideal ordering quantity and ending inventory level that maximizes the retailer's overall profit per unit of time is the main goal of both inventory models.

It is reasonable to assume that the rapidly evolving market conditions will result in a fundamental movement away from traditional supply chain models and towards customized ones. Because of this, the suggested inventory models take into account a number of aspects of the actual market situation, assisting merchants in working with their business partners. Furthermore, the suggested inventory models enable merchants to leverage the business and efficiently manage their stock levels. It is crucial to note that while some previously published models are specific instances of these, the two suggested inventory models serve as a basic framework.

Additionally, these inventory models are quite helpful for a variety of businesses, including 3PL organizations, retail sectors, and shopping. The following recommendations are made for potential future study directions: Among these are the two-level trade credit policy, the partial trade credit for customers who pose a credit risk, the credit-dependent demand function, the fuzzy-valued and interval-valued inventory costs. An integrated inventory model with several suppliers, distributors, and retailers in the supply chain may also be worthwhile to take into account. The suggested inventory models, which are the single player solution to the two players non-cooperative Nash or Stackelberg solution, can also be expanded in an interesting way. In the near future, academics and researchers might investigate these active study avenues.

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