A heuristic algorithm for 2 machines, n- jobs flow shop problem to find minimum total rental cost with setup time, probabilistic processing time, job block and transportation time.

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Abstract

In this paper we developed a heuristic algorithm for 2 machines, n- jobs flow shop production scheduling problem to find optimum/near optimum (minimum) total rental cost by taking the concepts of setup time, job block, transportation and probabilistic processing times.

Keywords

Probabilistic Processing Time, Job Block, Setup Time, Transportation Time, Rental Cost.

Introduction

The initial proposal of sequencing was posited by Johnson [15] in 1954 as a means of addressing the dilemma of flow shop scheduling for two machines and n jobs. Jackson [14] subsequently expanded upon Johnson's findings concerning job lot scheduling. The sequencing of two machines and n jobs with arbitrary time lags (namely, start lag and stop lag) was explored by Mitten [19] and Johnson's [16], acting independently of one another. Furthermore, Mitten [20] undertook an examination of scheduling predicaments, ultimately providing an analytical solution for the challenge of two machines and n jobs with arbitrary time lags. The utilization of the branch and bound methodology in the context of flow shop scheduling dilemmas has been thoroughly examined by Ignall and Schrage [13]. Furthermore, Lomnicki [17] has expatiated upon the notion of a branch and bound algorithm for the precise determination of scheduling solutions for three machines. Bagga [4] examined the intricacies two stage production scheduling flow shop problem. Additionally, Bagga [5] delved into the study of rental situations in sequencing. Yoshida and Hitomi [29] formulated the optimal algorithm for the scheduling of two stage production, taking into account separated setup time. Sule [24] expounded upon the topic of sequencing in the context

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of two machines and n jobs, incorporating considerations of setup time, processing time, and removal time, all with the aim of minimizing the flow shop scheduling problem. Bansal [6], in turn, advanced the field by imposing job restrictions on the two machine n jobs flow shop scheduling problem, yielding a consequential job. Panwalker [22] then extended the discussion by exploring the concept of travel time within the framework of two machines and n jobs flow shop scheduling problem. Finally, in 1992, Singh and Chandramouli [25] tackled the challenging problem of two machines and n jobs flow shop scheduling, introducing random processing and transportation times and group restrictions on jobs. Allahverdi, Gupta and Aldowaisan [3] presented an extensive analysis of scheduling research issues, incorporating various innovative concepts. Chandramouli [7] examined the intricate three machines and n jobs flow shop scheduling problem, which encompasses factors such as transportation time, breakdown time, and job weight. Narian and Bagga [21] explored scheduling predicaments in rental scenarios. Chandramouli, Gupta and Bhargava [8] explore the integration of probabilistic processing time, job block, and breakdown time with rental cost in the context of the two machines and n jobs flow shop scheduling problem. Additionally, Sharma [26] investigates the reduction of rental cost in scheduling of two machines and n jobs flow shop, considering the probabilities associated with transportation time and job block criteria. Vanchipura and Sridharan [28] deliberated upon the notion of diverse levels of setup time's impact on the efficacy of algorithms utilized for the scheduling of a flow shop with sequence dependent setup time. Gupta and Goyal [11] examined the minimization of the aggregate waiting time of tasks in a specially structured two stage flow shop scheduling, wherein processing is separated from setup time. Tyagi, Tripathi and Chandramouli [27] established sequencing and scheduling methodologies in the year 2017.

Fuchigami and Rangel [10] conducted an analysis on a survey encompassing case studies pertaining to production scheduling problems. Prata, Rodrigues, and Framinan [23] proposed a differential evolution algorithm for the problem of customer order scheduling, accounting for sequence dependent setup time.

Researchers that are interested in studying robot manipulators and fractional operators might consult the studies that are expressly discussed in the references [30–37].

There are the following situations for renting and then they are called the policies as:

P₁: All the machines are taken on rent at the same time and are returned at the same time.

P₂: All the machines are taken on rent at the same time but are returned as and when they are no longer required.

P₃: All the machines are taken on rent as and when they are required and returned as and when the requirement is over.

In this paper, a heuristic algorithm has been developed for 2- machines, n- jobs flow shop scheduling problem taking the P_3 policy and using C.S. Swapping Method. Here the concept of

probabilistic processing time with setup time, transportation time and job block has been combined in this problem.

Assumptions

(a)   The jobs are processed through two machines A and B in the order AB.

(b)   All jobs are available simultaneously at time zero.

(c)   All the machines are taken on rent as and when they are required and returned as and when the requirement is over.

(d)   Both machines A and B are hired at some fixed rental costs H_A and H_B respectively.

(e)   Each machine operates independently.

Mathematical Formulation of the Problem:

Let us consider the following 2- machines, n- jobs flow shop problem. There is n- jobs to be processed on two machines A and B with processing time A_i and B_i with probabilities p_{i1} and p_{i2}, transportation time t_i and S_{i1} and S_{i2} are the setup times of machines A and B respectively.

The above problem in the tabular form is as:

Job	Machine A			Transportation time	Machine B		
i	S_{i1}	A_i	p_{i1}	t_i	S_{i2}	B_i	p_{i2}
1	S_{11}	A_1	p_{11}	t_1	S_{12}	B_1	p_{12}
2	S_{21}	A_2	p_{21}	t_2	S_{22}	B_2	p_{22}
3	S_{31}	A_3	p_{31}	t_3	S_{32}	B_3	p_{32}
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
-	-	-	-	-	-	-	-
n	S_{n1}	A_n	p_{n1}	t_n	S_{n2}	B_n	p_{n2}

Heuristic Algorithm

Step 1: First we introduce two fictitious machines G and H with processing time of job i on these two machines as

$$G_i = A_i \, q_{i1} \quad \text{and } H_i = B_i q_{i2} \text{ respectively.}$$

Where, $q_{i1} = 1 - p_{i1}$ and $q_{i2} = 1 - p_{i2}$.

Step 2: Structural Conditions

a) $\text{Min} \, (S_{i1} + G_i) \geq \text{Max} \, (t_i + G_i)$

b) $\text{Min} \, (S_{i2} + H_i) \geq \text{Max} \, (t_i + S_{i2})$

If one or both structural conditions satisfied then go to the step 3 and otherwise no solution.

Step 3: Again, take two fictitious machines G' and H' with processing times of job i on these two machines as, $G_i' = G_i + S_{i1}$ and $H_i' = H_i + S_{i2}$ respectively.

Step 4: Now two fictitious machines M and N with processing time of job i on these two machines as, $M_i = G_i + t_i$ and $N_i = H_i + t_i$ respectively.

Step 5: Now taking the job block $\alpha = (\alpha_1, \alpha_2)$ then the processing time of machines M and N will be taken as

Where, $M_\alpha = M\alpha_1 + M\alpha_2 - \text{min} \, (M\alpha_1, N\alpha_1)$

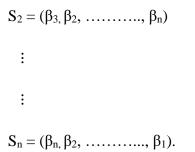and $N_\alpha = N\alpha_1 + N\alpha_2 - \text{min} \, (M\alpha_1, N\alpha_1)$.

Step 6: Now, applying Johnson's algorithm if the above sequencing problem we can find a sequence as $S_o = (\beta_1, \beta_2, \ldots\ldots, \beta_n)$.

Where, β_i is the ith positioned of job *i*.

Step 7: Now, using C. S. Swapping method on S_0,

We have (n-1) new sequences such as,

$S_1 = (\beta_2, \beta_3, \ldots\ldots, \beta_n)$

$$S_2 = (\beta_3, \beta_2, \ldots\ldots, \beta_n)$$

$$\vdots$$

$$\vdots$$

$$S_n = (\beta_n, \beta_2, \ldots\ldots, \beta_1).$$

Step 8: Now, we find total elapsed time T and utilization times U_A and U_B of machine A and machine B respectively for the above sequences.

Where, $U_A = T(\alpha_n)^A_{out}$, $U_B = T - T(\alpha_n)^B_{in}$ and $T = T(\alpha_n)^B_{out}$.

Step 9: Arranging the above results in tabular form we can find an optimal/near optimal sequence for which T, U_A and U_B is minimum. Now we find the minimum rental cost as

$$H_{AB} = H_A(U_A)_{min} + H_B (U_B)_{min}$$

where, H_A and H_B are the rental cost of machine A and B respectively.

For clarification of the above Heuristic Algorithm, we have a numerical problem as

Consider a two machine and six jobs problem with setup time, processing time and transportation time is given as:

Job i	Machine A			Transportation time t_i	Machine B		
	S_{i1}	A_i	p_{i1}		S_{i2}	B_i	p_{i2}
1	2	5	0.32	3	3	16	0.05
2	1	10	0.18	4	2	6	0.15
3	2	14	0.12	1	1	8	0.25
4	1	12	0.08	4	1	10	0.35
5	3	13	0.06	5	2	7	0.12
6	2	9	0.24	2	3	11	0.08

Rental cost of machine A and B are Rs.12/hr and Rs.14.50/hr respectively and $\alpha = (6,3)$ is a job block.

Solution:

Job i	Machine A				Transportation time	Machine B			
	S_{i1}	A_i	p_{i1}	q_{i1}		S_{i2}	B_i	p_{i2}	q_{i2}

					t_i				
1	2	5	0.32	0.68	3	3	16	0.05	0.95
2	1	10	0.18	0.82	4	2	6	0.15	0.85
3	2	14	0.12	0.88	1	1	8	0.25	0.75
4	1	12	0.08	0.92	4	1	10	0.35	0.65
5	3	13	0.06	0.94	5	2	7	0.12	0.88
6	2	9	0.24	0.76	2	3	11	0.08	0.92

Job i	Machine G		Transportation time t_i	Machine H	
	S_{i1}	$G_i = A_i q_{i1}$		S_{i2}	$H_i = B_i q_{i2}$
1	2	3.4	3	3	15.2
2	1	8.2	4	2	5.1
3	2	12.32	1	1	6
4	1	11.04	4	1	6.5
5	3	12.22	5	2	6.16
6	2	6.84	2	3	10.12

The structural condition satisfies. Hence,

Job i	Machine G' $G_i' = G_i + S_{i1}$	Transportation time t_i	Machine H' $H_i' = H_i + S_{i2}$
1	5.4	3	18.2
2	9.2	4	7.1
3	14.32	1	7
4	12.04	4	7.5
5	15.22	5	8.16
6	8.84	2	13.12

Job i	Machine M $M_i = G_i' + t_i$	Machine N $N_i = H_i' + t_i$
1	8.4	21.2
2	13.2	11.1
3	15.32	8
4	16.04	11.5
5	20.22	13.16
6	10.84	15.12

Taking $\alpha = (6,3)$ as a job block. Hence,

$$M_\alpha = 11.04 \text{ and } N_\alpha = 8.$$

Now the, new reduced problem in tabular form is as,

Job i	Machine M M_i	Machine N N_i
α	11.04	8
1	8.4	21.2
2	13.2	11.1
4	16.04	11.5
5	20.22	13.16

Now, applying Johnson's Algorithm to find a sequence,

$$S_0 = (1, 5, 4, 2, α)$$

By using C. S. Swapping Method, we have

$$S_1 = (5, 1, 4, 2, α)$$

$$S_2 = (4, 5, 1, 2, α)$$

$$S_3 = (2, 4, 5, 1, α)$$

$$S_4 = (α, 2, 4, 5, 1).$$

For, $S_0 = (1, 5, 4, 2, α) = (1, 5, 4, 2, (6, 3))$.

Job i	Machine G$'$ In - Out	Transportation time t_i	Machine H$'$ In - Out
1	0 – 5.4	3	8.4 – 26.6
5	5.4 – 20.62	5	26.6 – 34.76
4	20.62 – 32.66	4	36.66 – 44.16
2	32.66 – 41.86	4	45.86 – 52.96
6	41.86 – 50.7	2	52.96 – 66.08
3	50.7 – 65.02	1	66.08 – 73.08

The total elapsed time T, utilization times U_A and U_B of machines A and B respectively.

$$T = 73.08, U_A = 65.02 \text{ and } U_B = 73.08 – 8.4$$

$$= 64.68.$$

Similarly, we can calculate T, U_A and U_B for the sequences S_1, S_2, S_3 & S_4.

For all sequences the calculation can be put in tabular form as

Sequence	T	U_A	U_B	$H_{AB} = H_A U_A + H_B U_B$ (Apply P_3 policy)
$S_0 = (1542(63))$	73.08	65.02	64.68	1,718.1
$S_1 = (5142(63))$	81.3	65.02	61.08	1,665.9

$S_2 = (4512(63))$	85.84	65.02	69.8	1,792.34
$S_3 = (2451(63))$	87.94	65.02	74.74	1,863.97
$S_4 = ((63)2451)$	90.98	65.02	80.14	1,942.27

After analysing the above table, we have the sequence S_1 has the minimum rental cost.

(1)   The minimum flow time is 73.08 for the sequence S_0.

(2)   The minimum total rental cost is 1665.9 for the sequence S_1 for which U_B is minimum.

$H_{AB} = $ Rs. 1665.9.

Conclusion

(1)   This problem can be extended taking three or more machines.

(2)   This problem can be extended introducing the different concepts such as due date, tardiness and weights of jobs.

(3)   This problem can be extended by using the assumption P_1 or P_2.

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Appendix

S.No.	Sequence	Total Rental Cost
1	$S_1' = (\alpha 1245)$	1,756.67
2	$S_2' = (\alpha 1425)$	1,756.67
3	$S_3' = (\alpha 1254)$	1,732.60
4	$S_4' = (\alpha 1524)$	1,732.60
5	$S_5' = (\alpha 1452)$	1726.80
6	$S_6' = (\alpha 1542)$	1726.80
7	$S_7' = (\alpha 2145)$	1756.67
8	$S_8' = (\alpha 2415)$	1815.83
9	$S_9' = (\alpha 2154)$	1744.20
10	$S_{10}' = (\alpha 2541)$	1876.44
11	$S_{11}' = (\alpha 2451)$	1942.27
12	$S_{12}' = (\alpha 2541)$	1918.20
13	$S_{13}' = (\alpha 4125)$	1785.38

14	$S_{14}' = (\alpha4215)$	1810.03
15	$S_{15}' = (\alpha4152)$	1785.38
16	$S_{16}' = (\alpha4512)$	1911.82
17	$S_{17}' = (\alpha4215)$	1942.27
18	$S_{18}' = (\alpha4521)$	1912.40
19	$S_{19}' = (\alpha5124)$	1845.99
20	$S_{20}' = (\alpha5214)$	1846.57
21	$S_{21}' = (\alpha5142)$	1845.99
22	$S_{22}' = (\alpha5412)$	1887.75
23	$S_{23}' = (\alpha5241)$	1918.20
24	$S_{24}' = (\alpha5421)$	1912.40
25	$S_{25}' = (1\alpha245)$	1792.05
26	$S_{26}' = (1\alpha425)$	1792.05
27	$S_{27}' = (1\alpha254)$	1767.98
28	$S_{28}' = (1\alpha524)$	1767.98
29	$S_{29}' = (1\alpha452)$	1762.18
30	$S_{30}' = (1\alpha542)$	1762.18
31	$S_{31}' = (2\alpha145)$	1722.45
32	$S_{32}' = (2\alpha415)$	1781.61
33	$S_{33}' = (2\alpha154)$	1698.38
34	$S_{34}' = (2\alpha514)$	1842.22
35	$S_{35}' = (2\alpha451)$	1908.05
36	$S_{36}' = (2\alpha541)$	1883.98
37	$S_{37}' = (4\alpha125)$	1681.27
38	$S_{38}' = (4\alpha215)$	1734.63
39	$S_{39}' = (4\alpha152)$	1665.90
40	$S_{40}' = (4\alpha512)$	1836.42
41	$S_{41}' = (4\alpha251)$	1866.87
42	$S_{42}' = (4\alpha521)$	1837.00
43	$S_{43}' = (5\alpha124)$	1665.90
44	$S_{44}' = (5\alpha142)$	1665.90
45	$S_{45}' = (5\alpha412)$	1751.74
46	$S_{46}' = (5\alpha241)$	1782.19
47	$S_{47}' = (5\alpha421)$	1776.39
48	$S_{48}' = (5\alpha214)$	1710.56
49	$S_{49}' = (12\alpha45)$	1792.05
50	$S_{50}' = (14\alpha25)$	1792.05
51	$S_{51}' = (12\alpha54)$	1767.98
52	$S_{52}' = (14\alpha52)$	1762.18
53	$S_{53}' = (21\alpha45)$	1722.45
54	$S_{54}' = (21\alpha54)$	1698.38

55	$S_{55}' = (45\alpha12)$	1792.34
56	$S_{56}' = (54\alpha12)$	1707.66
57	$S_{57}' = (45\alpha21)$	1837.00
58	$S_{58}' = (54\alpha21)$	1776.39
59	$S_{59}' = (15\alpha24)$	1767.98
60	$S_{60}' = (51\alpha24)$	1665.90
61	$S_{61}' = (15\alpha42)$	1762.18
62	$S_{62}' = (51\alpha42)$	1665.90
63	$S_{63}' = (24\alpha15)$	1737.53
64	$S_{64}' = (42\alpha15)$	1695.48
65	$S_{65}' = (24\alpha51)$	1908.05
66	$S_{66}' = (42\alpha51)$	1866.87
67	$S_{67}' = (52\alpha14)$	1671.41
68	$S_{68}' = (52\alpha41)$	1782.19
69	$S_{69}' = (41\alpha25)$	1681.27
70	$S_{70}' = (41\alpha52)$	1665.90
71	$S_{71}' = (25\alpha14)$	1798.14
72	$S_{72}' = (25\alpha41)$	1883.98
73	$S_{73}' = (124\alpha5)$	1792.05
74	$S_{74}' = (214\alpha5)$	1722.45
75	$S_{75}' = (421\alpha5)$	1690.55
76	$S_{76}' = (241\alpha5)$	1737.53
77	$S_{77}' = (142\alpha5)$	1792.05
78	$S_{78}' = (412\alpha5)$	1681.27
79	$S_{79}' = (125\alpha4)$	1767.98
80	$S_{80}' = (215\alpha4)$	1698.38
81	$S_{81}' = (152\alpha4)$	1767.98
82	$S_{82}' = (512\alpha4)$	1665.90
83	$S_{83}' = (251\alpha4)$	1798.14
84	$S_{84}' = (521\alpha4)$	1666.48
85	$S_{85}' = (145\alpha2)$	1762.18
86	$S_{86}' = (415\alpha2)$	1667.06
87	$S_{87}' = (154\alpha2)$	1762.18
88	$S_{88}' = (514\alpha2)$	1665.90
89	$S_{89}' = (451\alpha2)$	1792.34
90	$S_{90}' = (541\alpha2)$	1707.66
91	$S_{91}' = (245\alpha1)$	1863.97
92	$S_{92}' = (425\alpha1)$	1822.79
93	$S_{93}' = (254\alpha1)$	1839.90
94	$S_{94}' = (524\alpha1)$	1738.11
95	$S_{95}' = (452\alpha1)$	1797.85

96	$S_{96}' = (542\alpha 1)$	1737.24
97	$S_{97}' = (1245\alpha)$	1747.97
98	$S_{98}' = (2145\alpha)$	1678.37
99	$S_{99}' = (1425\alpha)$	1747.97
100	$S_{100}' = (4125\alpha)$	1665.90
101	$S_{101}' = (2415\alpha)$	1737.53
102	$S_{102}' = (4215\alpha)$	1690.55
103	$S_{103}' = (1254\alpha)$	1723.90
104	$S_{104}' = (2154\alpha)$	1665.09
105	$S_{105}' = (1524\alpha)$	1723.90
106	$S_{106}' = (5124\alpha)$	1665.90
107	$S_{107}' = (2514\alpha)$	1798.14
108	$S_{108}' = (5214\alpha)$	1666.48
109	$S_{109}' = (1452\alpha)$	1718.10
110	$S_{110}' = (4152\alpha)$	1665.90
111	$S_{111}' = (1542\alpha)$	1718.10
112	$S_{112}' = (5142\alpha)$	1665.90
113	$S_{113}' = (4512\alpha)$	1792.34
114	$S_{114}' = (5412\alpha)$	1707.66
115	$S_{115}' = (2451\alpha)$	1863.97
116	$S_{116}' = (4251\alpha)$	1829.17
117	$S_{117}' = (2541\alpha)$	1839.90
118	$S_{118}' = (5241\alpha)$	1738.11
119	$S_{119}' = (4521\alpha)$	1792.92
120	$S_{120}' = (5421\alpha)$	1732.31