Bulk Transportation Problem with Multi-Objectives: A Modern Approach in Fuzzy Environment

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Abstract

Many researchers considered fuzzy parameters in the transportation problem but we dealt with intuitionistic fuzzy parameters in trapezoidal form. It is very difficult for transportation problem with multiple objectives to find an optimal solution for all objectives simultaneously. So some modern approaches are used to find the compromise optimal solution for all objectives simultaneously. In this paper, we discussed a different type multi-objective transportation problem, which is "Bulk Transportation Problem" in which demand of each destination is fulfilled from only one source but source may supply to any number of destinations. Firstly, we used a ranking function to convert fuzzy data into crisp data and proposed a different analytic method for bulk multi-objective transportation problem by which we get an efficient solution for the problem. We gave a numerical example for showing the efficiency and capability of our proposed approach for bulk transportation problem with multiple objectives and environment of the problem is uncertain .

1. Introduction:

Transportation problem (TP) is the problem in which we find the number of quantity of product which are distributed from given sources to given destinations at minimum cost which is firstly discussed by Hitchcock [1]. TP is a special type linear programming problem but because of its special structure simplex method is not well required for solving it. Now the special type (bulk) transportation problem in which demand of each destination is fulfilled by one only one source but source may supply the product to any number of destination is firstly developed by maio and roveda [2]. In real world there are many situations where more than one objective are to be considered and these objectives are optimized at the same time which is multi-objective transportation problem. This case of transportation problem with many objectives is solved by Prakash and Ram [3] using branch and bound method. Moreover the parameters of transportation problem is not exact due to real situations i.e. cost or source and demand may be uncertain. To deal such situation Zadeh [4] gave the fuzzy set theory and Bellman and Zadeh [5] firstly used this theory in decision making problems. Oheigeartaigh [6] gave an algorithm to solve the fuzzy transportation problem. Zimmermann [7] gave the optimal solution of fuzzy transportation problem by fuzzy linear programming approach with several objectives. Sukhveer *et al.* [8] gave an algorithm for solving the multi-objective bulk transportation problem and gave efficient solution for the problem and Sukhveer Singh and Singh [9] solved bi-objective TP by convert it into single objective TP and gave fuzzy optimal solution for fuzzy cost and fuzzy time. Vidhya and Ganesan [10] also gave the efficient solution for fuzzy transportation problem with multiple objectives. Atanassov [11] introduced the concept of intuitionistic fuzzy sets which is the extension of fuzzy sets and these sets are most useful to deal with vagueness. These sets are different from fuzzy sets because an intuitionistic set deals with membership as well as non-membership with hesitation part. Annie Christi [12] solved transportation problem with multiple objectives in intuitionistic fuzzy environment.

In this paper we discuss the efficient solution for Bulk transportation problem under intuitionistic fuzzy environment with multiple objectives and we find cost time trade off pair for intuitionistic fuzzy cost and intuitionistic fuzzy time and also compare our solution with the solution of Vogel's approximation method. Paper is divided into 6 sections. In this paper section 2 presents basic definitions and arithmetic operations on intuitionistic trapezoidal fuzzy number, in section 3 presents mathematical formulation of this problem. In section 3 presents proposed algorithm and in section 4 presents the numerical example solved by proposed algorithm. In section 5 discuss comparison between the solutions obtained by proposed algorithm with the obtained solution by Vogel's approximation method. In last section we discuss the conclusion.

2. Basic concepts:

- **Fuzzy set:** A fuzzy set is defined by a membership function from an element of universe of discourse X to the unit interval [0, 1]. A fuzzy set $\widetilde{A}$ in a universe of discourse X is defined as the following set of pair - $\widetilde{A}$={(x,$f_{\widetilde{A}}$(x))}. Here $f_{\widetilde{A}}$: X→ [0,1] is a mapping called the membership function of the fuzzy set $\widetilde{A}$ and $f_{\widetilde{A}}$(x) is called the membership grade of x∈X on the fuzzy set $\widetilde{A}$. These membership grades are belonged in the interval [0,1].

- **Fuzzy number:** A fuzzy set $\widetilde{A}$ defined on the set of real number R is said to be fuzzy number if fuzzy set $\widetilde{A}$ has the following characteristics-

i. $\widetilde{A}$ is normal.

ii. $\widetilde{A}$ is convex.

iii. The support of $\widetilde{A}$ is closed and bounded.

- **Trapezoidal fuzzy number:** A fuzzy number $\widetilde{A}= (a_1, a_2, a_3, a_4)$ is said to be trapezoidal fuzzy number if its membership function is represented by-

$$f_{\widetilde{A}}(X) = \begin{Bmatrix} 0; & x \leq a_1 \ and \ x \geq a_4 \\ \frac{x-a_1}{a_2-a_1}; & a_1 \leq x \leq a_2 \\ 1; & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}; & a_3 \leq x \leq a_4 \end{Bmatrix}$$

- **Trapezoidal intuitionistic fuzzy number:** A trapezoidal intuitionistic fuzzy number is represented by $\widetilde{A}' = (a_1, a_2, a_3, a_4; a_1', a_2', a_3', a_4')$ whose membership and non-membership function is represented by as follows:

$$\mu_{\widetilde{A}}(X)= \begin{Bmatrix} 0; & x \leq a_1 \ and \ x \geq a_4 \\ \frac{x-a_1}{a_2-a_1}; & a_1 \leq x \leq a_2 \\ 1; & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}; & a_3 \leq x \leq a_4 \end{Bmatrix} \quad \text{(membership function)}$$

$$\vartheta_{\widetilde{A}}(X) = \begin{Bmatrix} 1; & x \leq a_1' \ and \ x \geq a_4' \\ \frac{a_2'-x}{a_2'-a_1'}; & a_1' \leq x \leq a_2' \\ 0; & a_2' \leq x \leq a_3' \\ \frac{x-a_3'}{a_4'-a_3'}; & a_3' \leq x \leq a_4' \end{Bmatrix} \quad \text{(non-membership function)}$$

Where $a_1' \leq a_1 \leq a_2' \leq a_2 \leq a_3 \leq a_3' \leq a_4 \leq a_4'$.

- **Ranking function on trapezoidal intuitionistic fuzzy number:**

let $\widetilde{A}^{'}= (a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'}, a_4^{'})$ be trapezoidal intuitionistic fuzzy number then:

$$r(\widetilde{A}^{'})= \frac{a_1+a_2+a_3+a_4+a_1^{'}+a_2^{'}+a_3^{'}+a_4^{'}}{8}$$

If $R(\widetilde{A}^{'})\leq R(\widetilde{B}^{'})$ then $\widetilde{A}^{'} \leq \widetilde{B}^{'}$ and if $R(\widetilde{A}^{'})\geq R(\widetilde{B}^{'})$ then $\widetilde{A}^{'} \geq \widetilde{B}^{'}$.

3. Arithmetic operations on trapezoidal intuitionistic fuzzy number:

Let $\widetilde{A}^{'}= (a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'},a_4^{'})$ and $\widetilde{B}^{'}= (b_1,b_2,b_3,b_4; b_1^{'},b_2^{'},b_3^{'},b_4^{'})$ be two trapezoidal intuitionistic fuzzy number then:

- **Addition**: $\widetilde{A}^{'}+\widetilde{B}^{'}= (a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'}, a_4^{'})+ (b_1,b_2,b_3,b_4; b_1^{'},b_2^{'},b_3^{'}, b_4^{'})$

$$=(a_1+ b_1,\ a_2+ b_2,\ a_3+ b_3,\ a_4+ b_4;\ a_1^{'}+ b_1^{'},\ a_2^{'}+ b_2^{'},\ a_3^{'}+ b_3^{'},\ a_4^{'}+ b_4^{'}).$$

- **Subtraction**: $\widetilde{A}^{'}- B^{'}= (a_1,a_2,a_3,a_4; a_1^{'},a_2^{'},a_3^{'}, a_4^{'}) - (b_1,b_2, b_3,b_4; b_1^{'},b_2^{'},b_3^{'}, b_4^{'})$

$$=(a_1- b_4,\ a_2- b_3,\ a_3- b_2,\ a_4- b_1;\ a_1^{'}- b_4^{'},\ a_2^{'}- b_3^{'},\ a_3^{'}- b_2^{'},\ a_4^{'}- b_1^{'}).$$

- **Multiplication:** $\widetilde{A}^{'} \times B^{'}= (c_1,c_2,c_3,c_4; c_1^{'},c_2^{'},c_3^{'}, c_4^{'})$

where $c_1=min(a_1b_1,a_1b_4,a_4b_1, a_4b_4)$, $c_2=min(a_2b_2,a_2b_3,a_3b_2, a_3b_3)$,

$c_3=max(a_2b_2,a_2b_3,a_3b_2, a_3b_3)$, $c_4=max(a_1b_1,a_1b_4,a_4b_1, a_4b_4)$,

$c_1^{'}=min(a_1^{'}b_1^{'},\ a_1^{'}b_4^{'},\ a_4^{'}b_1^{'}, a_4^{'}b_4^{'})$, $c_2^{'}=min(a_2^{'}b_2^{'},\ a_2^{'}b_3^{'},\ a_3^{'}b_2^{'}, a_3^{'}b_3^{'})$,

$c_3^{'} =max(a_2^{'}b_2^{'},\ a_2^{'}b_3^{'},\ a_3^{'}b_2^{'}, a_3^{'}b_3^{'})$, $c_4^{'} =max(a_1^{'}b_1^{'},\ a_1^{'}b_4^{'},\ a_4^{'}b_1^{'}, a_4^{'}b_4^{'})$

4. Mathematical formulation of multi-objective transportation problem with intuitionistic fuzzy cost and fuzzy time:

Let there be m source and n destination and each source has a_i (where i= 1,2,...M) available quantity of product respectively and each destination has b_j (where j= 1,2,…n) requirement of product respectively. Let C and T be the total intuitionistic fuzzy cost and total intuitionistic fuzzy time respectively.

The mathematical formulation of multi-objective transportation problem with total intuitionistic fuzzy cost and total intuitionistic fuzzy time is as follows:

Minimize

C = $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^{'} x_{ij}^{'}$ and T = max$\{t_{ij}^{'}$: $x_{ij}^{'}$=1; i=1,2,…m, j=1,2,…n}

With the constraints

$\sum_{j=1}^{n} b_j x_{ij}^{'} \leq a_i$ (i=1,2,….M),

$\sum_{i=1}^{m} x_{ij}^{'}$=1 (j=1,2,…n) And $x_{ij}^{'}$ = 1 or 0 (i=1,2,…m, j=1,2,…n).

Where a_i denotes the available quantity of product at i^{th} source and b_j denotes the demand of j^{th} destination.

$c_{ij}^{'}$ Denotes the intuitionistic fuzzy cost of b_j units for transport from i^{th} source to j^{th} destination and $t_{ij}^{'}$ denotes time taken for transport b_j units from i^{th} source to j^{th} destination.

$x_{ij}^{'}$ Is the decision variable with the assuming value 1 or 0 depending upon that i^{th} source fulfilled the requirement of j^{th} destination or not respectively.

Moreover $c_{ij}^{'}$ and $t_{ij}^{'}$ expressed as:

$c_{ij}^{'}$= (c_1,c_2,c_3,c_4; $c_1^{'}$,$c_2^{'}$,$c_3^{'}$, $c_4^{'}$) and $t_{ij}^{'}$= (t_1,t_2,t_3,t_4; $t_1^{'}$,$t_2^{'}$,$t_3^{'}$, $t_4^{'}$).

5. Proposed Algorithm:

Algorithm as follows:

Step 1: Construct the multi-objective transportation problem in intuitionistic form.

Step 2: Now remove that cell for which available quantity at i^{th} source is less than the required quantity for j^{th} destination in initial cost table.

Step 3: Now convert the cost into crisp form by ranking function. Then choose the minimum element from each row and subtract this cost from each cost in corresponding row. Do the same process for each column.

Step 4: Now select the zero of the cell that contains the least zero for the corresponding row and column in the cost table and assign 1 to this cell.

In case of equality, select the cell with the highest sum of entries in the relevant row and column. Again in case of equality, the cell which has highest requirement is selected.

Step 5: Now delete the destination which demand is fulfilled and source for which supply is less than demand for each destination. Repeat 2 to 4 step unless all the demands are fulfilled. Then first efficient solution of the problem is obtained.

Step 6: Now for find the next efficient solution leave the cell whose rank of time is greater than the rank of time for obtained solution for first cost time trade-off pair. Repeat same process and find all possible efficient solution for intuitionistic fuzzy cost time trade-off pair.

In the next multi-objective transportation problem with intuitionistic fuzzy cost and fuzzy time is considered such that in each cell upper entries are cost and lower entries are time as trapezoidal intuitionistic fuzzy number. Supply and demand are in crisp form.

6. Numerical example:

1. Step 1:

Table: 1

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	(0,1,2,5; 0,.5,1.5,5) (1,3,4,8; 0,2,5,9)	(1,2,3,6; 0,1,4,7) (1,3,4,8; 0,2,5,9)	(1,2,3,6; 0,1,4,7) (3,7,10,20; 2,6,11,21)	(2,5,7,14; 1,4,8,15) (3,5,8,16; 2,4,9,17)	(0,.5,1.5,2; 0,.5,1.5,2) (2,5,7,14; 1,4,8,15)	7
O_2	(1,3,4,8; 0,2,5,9) (1,3,4,8; 0,2,5,9)	(0,.5,1.5,2; 0,.5,1.5,2) (2,5,7,14; 1,4,8,15)	(0,.5,1.5,2; 0,.5,1.5,2) (5,7,12,24; 4,6,13,25)	(0,1,2,5; 0,.5,1.5,5) (5,9,14,28; 4,8,15,29)	(3,5,8,16; 2,4,9,17) (3,5,8,16; 2,4,9,17)	9
O_3	(0,.5,1.5,2; 0,.5,1.5,2) (3,5,8,16; 2,4,9,17)	(2,5,7,14; 1,4,8,15) (0,1,2,5; 0,.5,1.5,5)	(5,6,11,22; 4,5,12,23) (1,3,4,8; 0,2,5,9)	(0,.5,1.5,2; 0,.5,1.5,2) (1,3,4,8; 0,2,5,9)	(1,4,5,10; 0,3,6,11) (1,3,4,8; 0,2,5,9)	10
Demand	3	5	4	6	2	

2. After applying step 2 and 3 we get the initial cost:

Table 2:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	2	6	0	7
O_2	3	0	0	1	7	9
O_3	0	6	10	0	4	10
Demand	3	5	4	6	2	

- By applying step 4 and 5, we get x_{15}=1. Then by repeat this process we get the first efficient solution of intuitionistic fuzzy transportation problem with multiple objectives. Solution is as follows $\{x_{15}, x_{34}, x_{31}, x_{12}, x_{23}\}$. By this the cost for transportation is C_1= $(\ 0,2.5,7.5,10; 0,2.5,7.5,10\ )$ and corresponding time T_1 = (5,7,12,24;4,6,13,25), then obtained first cost-time trade-off pair is (C_1, T_1).
- Now by applying step 6 and then step 3 the initial cost table for second efficient solution of the problem:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	0	6	0	7
O_2	3	0	-	-	7	9
O_3	0	6	8	0	4	10
Demand	3	5	4	6	2	

- Now by applying step 6, we get the next efficient solution for the cost time problem. Second efficient solution is as follows $\{x_{22}, x_{34}, x_{31}, x_{13}, x_{15}\}$. Then the cost for transportation is C_2 = $(\ 0,2.5,7.5,10; 0,2.5,7.5,10\ )$ and corresponding time T_2 =(3,7,10,20;2,6,11,21), then second cost-time trade-off pair is (C_2, T_2) . By using this algorithm we get all possible efficient solution and cost-time trade-off pair.

- Now again by applying step 6 and step 3 the initial cost table for next efficient solution of the problem:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	-	6	0	7
O_2	3	0	-	-	7	9
O_3	0	6	0	0	4	10
Demand	3	5	4	6	2	

- Then next efficient solution is as follows $\{x_{15}, x_{22}, x_{34}, x_{33}, x_{11}\}$ and the cost for transportation is C_3 ={5,8.5,17.5,33;4,7,18,34} and corresponding time T_3 = {2,5,7,14;1,4,8,15} then obtained second cost-time trade-off pair is (C_3, T_3).
- By using this algorithm we get all possible efficient solution and cost-time trade-off pair which is in next table:

Optimal solution	Total intuitionistic fuzzy cost	Total intuitionistic fuzzy time
$X_1 = \{x_{15}, x_{34}, x_{31}, x_{22}, x_{23}\}$ $X_2 = \{x_{22}, x_{34}, x_{31}, x_{13}, x_{15}\}$ $X_3 = \{x_{15}, x_{22}, x_{34}, x_{33}, x_{11}\}$	C_1 = {0,2.5,7.5,10; 0,2.5,7.5,10} C_2= {1,4,9,14;0,3,10,15} C_3= { 5,8.5,17.5,33;4,7,18,34}	T_1 = {5,7,12,24; 4,6,13,25} T_2= {3,7,10,20;2,6,11,21} T_3= {2,5,7,14;1,4,8,15}

- Now the efficient solution and cost time trade-off pair by using Vogel's Approximation Method:

Optimal solution	Total intuitionistic fuzzy cost	Total intuitionistic fuzzy time
$X_1 = \{x_{34}, x_{31}, x_{22}x_{23}, x_{15}\}$ $X_2 = \{x_{13}, x_{22}, x_{34}, x_{31}, x_{15}\}$ $X_3 = \{x_{33}, x_{34}, x_{15}, x_{22}, x_{11}\}$	C_1 = {0,2.5,7.5,10; 0,2.5,7.5,10} C_2= {1,4,9,14;0,3,10,15} C_3= { 5,8.5,17.5,33;4,7,18,34}	T_1 = {5,7,12,24; 4,6,13,25} T_2= {3,7,10,20;2,6,11,21} T_3= {2,5,7,14;1,4,8,15}

7. Comparison: We get that the cost time trade-off pair obtained by proposed algorithm is same as the cost time trade-off pair obtained by Vogel's Approximation Method. That's means proposed approach is efficient to find the compromise optimal solution for solving such type multi-objective transportation problem under intuitionistic fuzzy environment.

8. Conclusion: In this paper, a modern approach is developed to find an efficient solution of multi-objective transportation problem by representing cost and time as trapezoidal intuitionistic fuzzy number and cost, time are converting into crisp form by using the ranking function of trapezoidal intuitionistic fuzzy number. This method gives the optimal solution of special type transportation problem as Vogel's Approximation Method (VAM). The obtained result by proposed algorithm is same as the VAM. By this method this special type problem is solved in less computational work and it is very easy to apply.

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