**Inverse Optimization for Mathematical Programming Problem: An Introduction**

Sanjay Jain1, Nitin Arya2

1Government College Naand, Ajmer, 2Engineering College Jhalawar, Jhalawar

1drjainsanjay@gmail.com, 2nitin.arya1234@gmail.com

**Abstract**

An inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible. In this chapter, we have considered a mathematical programming problem and discussed its various inverse problems and their formulation, reported in the literature.

**Introduction**

A variety of real life problems can be formulated as a mathematical programming problem and solved by using suitable techniques. Whenever we model these problems mathematically, it is assumed that all the parameters associated with the problem are known exactly and we wish to find the solution which is optimal for the present values of parameter. However, in practice, there are many situations when we are not very much sure about these parameters or we only have some estimates of these parameters, but we may have a solution from the observation, experiment or experiences. The known solution may or may not be optimal for the present values of parameters, so we need to adjust these parameters to make the given solution optimal. This problem can be considered as an inverse problem, but whenever we talk about optimization, we always look for the best solution i.e. the adjustment of the parameters should be minimum or the cost associated behind it should be minimum. Thus, an inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible. The original problem is called the forward problem.

Thus in a forward problem, we identify the values ​​of the observable parameters (decision variables) given the values ​​of the model parameters (cost coefficients, right-hand side vector and constraint matrix). The inverse optimization problem involves estimating the values ​​of model parameters (cost coefficients, right-hand side vector, and constraint matrix) given the values ​​of observable parameters (decision variables).

General speaking, an inverse optimization problem is to find the values of parameters (cost, capacity, travel time, etc.) which makes the given solution optimum and which are differ from the given parameters as little as possible.

Within the mathematical programming community, interest in inverse optimization problems was generated by Burton and Toint's paper [14] in 1992, they consider a directed graph with a set of nonnegative costs on its arcs and modify these costs as little as possible to make the given path between the origin and destination, the shortest path. An important contribution on inverse optimization in the field of mathematical programming was Ahuja and Orlin [1]. They consider a general linear programming problem and prove that the inverse problem of linear program under l1 and l∞ norm is also linear program. Although Zhang and Liu [16, 5] has first been investigated the inverse linear programming problem and Huang and Liu [15] also obtained the same result, but Ahuja and Orlin used more general approach that can be apply to solve many inverse problems.

**Inverse Optimization: Classification and Formulation**

We can divide Inverse optimization problem into two categories: inverse solution optimization problem and inverse objective value optimization problem.

1. **Inverse solution optimization problem**

In the inverse solution optimization problem, we have a desired solution and we want to optimize it by adjusting the parameters associated with the decision variables in the objective function or in the set of constraints, so that the adjusted values ​​of the parameters vary with the given parameters as less as possible

These types of problems are known as inverse problems and most of the problems on inverse optimization, available in the literature are generally belongs to this group of problems. We have also considered the same type of problems in our work[10,13].

Let us consider the following optimization problem

Min

s.t. (1)

where is the feasible solution's set, the given cost vector. If is the given solution, is the modified value of *c* and is some selected norm then the inverse version of problem (1) is

Min

s.t.

 (2)

1. **Inverse objective value optimization problem**

In an inverse objective value optimization problem, we have the desired value of objective function instead off the desired solution of given mathematical programming problem and our aim is to adjust the parameter values which make the optimal objective value equal to its desired value and differ from the given values of parameters as little as possible. In order to avoid any confusion, we use the term “Reverse problem” for these problems and this type of problems have reported by Cai et al. [2] and Zhang et al. [8]. We have also considered the similar problem in our work.

If is the desired objective value of problem (1) then the inverse problem can be formulated as:

Min

s.t.

 (3)

 **Related problems**

There are many variants of the inverse version of optimization problems, which have been reported by various researchers. We are briefly discussing some of them.

1. **Adjustment Problem**

This type of problem is reported by Libura [4]. It is basically a generalization of inverse problem, where a subset *F* of the set of feasible solution *S* is given and we wish make the minimum adjustment in the objective coefficients so that the optimal solution of this problem belongs to set *F*.

If we consider the optimization problem (1) as the forward problem then the adjustment problem can be formulated as:

Min

s.t.

 , (4)

1. **Partial Inverse Problem**

In this type of problems, a partial solution is given and we wish to make a minimum adjustment in the cost vector so that the modified problem has a full solution which contains the partial solution and the full solution is also optimal for the problem. Yang [6], Yang and Zhang [7] have considered the partial inverse problems of assignment problem and minimum cut problem.

Let us consider a partial solution defined as, where , then the inverse problem to (1) is

Min

s.t.

, (5)

1. **Reverse Problem with Prescribed Objective Function Range**

It is the generalization of inverse objective value optimization problem, in which, instead of a single objective value, a set of objective value or a range of objective value is given. Heuberger [3] have considered this type of problem in his survey.

If (1) is the original problem, be the given range of the objective function, then the reverse problem can be formulated as:

Min

s.t.

 (6)

1. **Reverse Problem with Budget constraints**

In this problem, we specify a solution, but instead of making it optimal, we wish to obtain the best improvement that does not exceed the given budget *B*.

Let is the given solution, is the given budget, then the reverse problem to (1) with budget constraints is

Min

s.t. (7)

**5**. **Improvement Problem with Budget constraints**

In this type of problem, neither a solution nor the objective function value are given, but a certain limitation on budget is given. If we consider the original problem (1) then the inverse problem with budget constraints is formulated as:

Min

s.t.

 , (8)

This type of problem is reported in Heuberger [3].

1. **Inverse problem with given objective value**

In this type of problem, we have a solution that we want to optimize and we also have the desired objective value. If we consider (1) as the original problem then it's inverse version can be formulated as

Min

s.t.

 (9)

Where is the given solution and the desired objective value. Jain and Arya [10,13] proposed some inverse optimization models for this types of problem

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