ERROR ESTIMATION OF FUNCTION VIA $(C,\beta,\gamma)(E,1)$ MEANS OF ITS FOURIER-LAGUERRE SERIES

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ABSTRACT

In the present work, we proposed the error estimation of function belonging to $L[0,\infty)$-class by $(C,\beta,\gamma)(E,1)$ means using its Fourier-Laguerre series at point t=0. Our findings generalize earlier results by Krasniqi, who studied function approximation by $(C,1)(E,q)$ means, and Sonker, who assessed the degree of approximation by $(C,2)(E,q)$ means for $q=1$. We also introduced the error estimation theorem using product summation, along with some graphical interpretations via MATLAB software.

Keywords—Fourier-Laguerre approximation; (C,β,γ) mean; $(E,1)$ mean; Error estimation

I. INTRODUCTION AND MOTIVATION

It is believed that in the last few decades of the nineteenth century, a well-known Weierstrass theorem served as the foundation for approximation theory of functions. Quantifying the errors generated and figuring out how functions can be best approximated by simpler functions are the two main objectives of approximation theory. The field of signal approximation has been studied extensively and has been the interest of academics. Approximation theory is important tool in the field of robotics [1, 2], applied mathematics [3, 4, 5, 6] and operator theory [7, 8]. The theorems of fuzzy numbers and sequence spaces are also covered by summability theory [9]. Product summable operatos [10, 11, 12, 13] outperform single summable operators [14, 15, 16]. Numerous mathematicians studying approximation theory and summability were motivated by this finding.In a study on the error estimate of a function via the $(C,1)(E,q)$ approach, Krasniqi [17] presented a study that made use of the concept of product summable operatos. This paper was further improved in 2014 by Sonker [18]. She approximated the series at $t=0$ by using the $(C,2)(E,q)$ operator. In response to these findings, the error in the approximation of the same series function was found in 2015 by Mittal and Singh [19] using the $(T.E_q)$ summable approach. Later, in 2016, Khatri and Mishra [20] calculated the error estimation of the Fourier-Laguerre series by using the H^1E^1 product summable operator under appropriate conditions. In 2021, Sharma [21] conducted an investigation into the (T,C_β) approach of the Fourier-Laguerre series, building on earlier work in the field [22]. This inspired us to estimate a function's error at the frontier point of $t=0$ using the $(C,\beta,\gamma)(E,1)$ composite summation method of its Fourier-Laguerre series. Our findings are compared to the results given by Krasniqi[17] and Sonker [18] in order to show the efficiency of proposed summation method. We also introduced the error estimation theorem using product summability along-with some graphical interpretations. Also, using different values of β and γ, the existing summability methods can be derived.

A function $g(t)\in L(0,\infty)$ is expanded by Fourier-Laguerre method as

$$g(t)\equiv\sum_{m=0}^{\infty}c_m {L_m}^{\delta}(y), \tag{1}$$

where

$$c_m=\frac{1}{\Gamma(\delta+1)\binom{m+\delta}{m}}\int_0^{\infty}e^{-y}y^{\delta}g(y){L_m}^{\delta}(y)dy, \tag{2}$$

and ${L_m}^{\delta}(y)$ denotes the mth Laguerre polynomial of order $\delta\geq-1$, defined by generating function

$$\sum_{m=0}^{\infty}{L_m}^{\delta}(t)w^m=(1-w)^{-\delta-1}e^{\left(\frac{-tw}{1-w}\right)},$$

and the integral (2) exists. Also,

$$\sigma(y)=\frac{1}{\Gamma(\beta+1)}e^{-y}y^{\delta}[g(y)-g(0)]. \tag{3}$$

Let the sequence $\{s_m(g;t)\}$ be the mth partial sum of the Fourier-Laguerre series (1) given by

$$s_m(g;t)=\sum_{h=0}^{m}c_hL_h{}^{\delta}(t),$$

is also known as Fourier-Laguerre polynomial of degree (or order)$\geq m$. We denote $C_m^{\beta,\gamma}$ or (C,β,γ) the m^{th} Ces$\grave{a}$o mean of order (β,γ) with $\beta+\gamma>-1$ of the sequence $\{s_m(g;t)\}$ i.e.

$$C_m^{\beta,\gamma}=\frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}s_h,$$

where

$$A_m^{\beta+\gamma}=O(m^{\beta+\gamma}),\beta+\gamma>-1 \ and \ A_0^{\beta+\gamma}=1.$$

The Fourier-Laguerre series (1) is said to be (C,β,γ) summable to the definite number s if

$$C_m^{\beta,\gamma}=\frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}s_h\rightarrow s \ as \ m\rightarrow\infty.$$

Also, If

$$E_m^1=\frac{1}{2^m}\sum_{h=0}^{m}\binom{m}{h}s_h\rightarrow s \quad as \quad m\rightarrow\infty,$$

then $\{s_m(g;y)\}$ converges to a definite value 's' by E_m^1 means (by Hardy [23]), and we write it as,

$$s_m\rightarrow s(E_m^1).$$

We now introduce the Ces$\grave{a}$ro-Euler product summability mean of order $(\beta,\gamma,1)$ as follows.

A. Ces$\grave{a}$ro-Euler product summability means

- The (C,β,γ) transform of the $(E,1)$ transform defines $(C,\beta,\gamma)(E,1)$ transform of order $(\beta,\gamma,1)$ and we shall denote it by $(CE)_m^{1,\beta,\gamma}$. Moreover, if

$$\begin{aligned} t_m^{CE} &= (CE)_m^{q,\beta,\gamma} \\ &= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}E_h^q \\ &= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma} \\ &\quad .\frac{1}{(2)^h}\sum_{v=0}^{h}\binom{h}{v}s_v\rightarrow s \ as \ m\rightarrow\infty. \end{aligned} \tag{4}$$

- The regularity of (C,β,γ) and $(E,1)$ methods implies the regularity of $(CE)_m^{1,\beta,\gamma}$ method.

II. USEFUL LEMMAS

Lemma 1: Given by Szegö (1975, p.177, Theorem 7.6.4) [24], let δ is any real number ε are fixed +ve constant. Then

$$L_m^{\delta}(t)=O(m^{\delta}) \qquad if \quad 0<t<1/m, \tag{5}$$

$$=O(t^{-(2\delta+1)/4}m^{(2\delta-1)/4} \quad if \ 1/m<t<\varepsilon, \tag{6}$$

$$as \ m\rightarrow\infty.$$

Lemma 2: Given by Szegö (1975, p.177, Theorem 7.6.4) [24], let α and δbe an arbitrary real no., and $0<\chi<4$ and $\varepsilon>0$, then

$$\max e^{-t/2}t^{\alpha}\mid L_m^{\delta}(t)\mid=O(m^{Q})$$

where

$$Q=\max(\alpha-1/2,\delta/2-1/4), \ \varepsilon\leq t\leq(4-\chi)m, \tag{7}$$

$$=\max(\alpha-1/3,\delta/2-1/4), \ t>m. \tag{8}$$

Lemma 3: Let $\delta > -1$. If $q = 1$, then

$$\frac{1}{2^m}\sum_{h=0}^{m}\binom{m}{h}h^{(2\delta+1)/4} = O\left(m^{(2\delta+1)/4}\right), \tag{9}$$

and if $\beta+\gamma > -1$, then

$$I = \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}(1+h)^{\delta} = O\left((1+m)^{\delta}\right). \tag{10}$$

Proof: The first result is on similar lines as given by Lenski and Szal [25].Regarding the latter result, A. Zygmund [26][7, Vol. I (1.15) and Theprem 1.17] have stated that

$$A_m^{\beta+\gamma} = \binom{m+\beta+\gamma}{m} \equiv O\left((m+1)^{\delta}\right),$$

is positive for $\beta+\gamma > -1$. Moreover, $A_m^{\beta+\gamma}$ is decreasing for $-1 < \beta+\gamma < 0$ and increasing for $\beta+\gamma > 0$. Hence for $\delta < 0$,

$$\begin{aligned}
I &= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{[m/2-1]}A_{m-h}^{\beta-1}A_h^{\gamma}(1+h)^{\delta} \\
&\quad + \frac{1}{A_m^{\beta+\gamma}}\sum_{h=[m/2]}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}(1+h)^{\delta} \\
&= O\left(\frac{(m+1)^{\beta-1}(m+1)^{\gamma}}{(m+1)^{\beta+\gamma}}\right)\sum_{h=0}^{[m/2-1]}(1+h)^{\delta} \\
&\quad + O\left((1+m)^{\delta}\right)\frac{1}{A_m^{\beta+\gamma}}\sum_{h=[m/2]}^{m}A_{m-h}^{\beta-1}A_h^{\gamma} \\
&= O\left((1+m)^{-1}\right)\sum_{h=0}^{m}(1+h)^{\delta}\int_h^{h+1}dz \\
&\quad + O\left((1+m)^{\delta}\right)\frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma} \\
&= O\left((1+m)^{-1}\right)\sum_{h=0}^{m}\int_h^{h+1}z^{\delta}dz + O\left((1+m)^{\delta}\right) \\
&= O\left((1+m)^{-1}\right)\int_0^{m+1}z^{\delta}dz + O\left((1+m)^{\delta}\right) \\
&= O\left((1+m)^{-1}\right)\frac{(m+1)^{\delta+1}}{\delta+1} + O\left((1+m)^{\delta}\right) \\
&= O\left((1+m)^{\delta}\right)
\end{aligned}$$

If $\delta > 0$, the outcome is obvious. Our proof is thus finished.

Additional Results:

Also, we will use

$$A_m^{\beta+\gamma}(t) = \frac{L_m^{(\beta+\gamma+1)}}{\Gamma(\beta+\gamma+1)},$$

and also using this we can prove $\beta+\gamma > -1$, then

$$\frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^{m}A_{m-h}^{\beta-1}A_h^{\gamma}(h^{(2\delta+1)/4}) = O\left(m^{(2\delta+1)/4}\right). \tag{11}$$

III. Error Estimation Theorem

Let g be a lebesgue integrable function then the error estimation of g at $t = 0$ by the Cesàro-Euler means of order $(\beta,\gamma,1)$ with $\beta+\gamma \geq -1$, $q = 1$ of the Fourier-Laguerre series of g is given by

$$\mid (CE)_m^{1,\beta,\gamma}(g;0) - g(0) \mid = o(\tau(m)) \tag{12}$$

with conditions

$$\sigma(x)=\int_0^x \mid \sigma(y)\mid dy = o(x^{\delta+1}\tau(1/x)),\ x\to 0, \tag{13}$$

$$\int_\varepsilon^m e^{y/2}y^{-(2\delta+3)/4}\mid \sigma(y)\mid dy = o(m^{(-2\delta+1)/4}\tau(m)), \tag{14}$$

and

$$\int_m^\infty e^{y/2}y^{-1/3}\mid \sigma(y)\mid dy = o(\tau(m)),\ m\to\infty, \tag{15}$$

where $\tau(x)$ is positive and monotonically increasing signal of x such that $\tau(m)\to\infty$ as $m\to\infty$.

PROOF OF THEOREM:

Based on the equality

$$L_m^\delta(0)=\binom{m+\delta}{\delta},$$

we obtain

$$\begin{aligned}
s_m(0) &= s_m(g;0)\\
&= \sum_{h=0}^m c_h {L_h}^\delta(0)\\
&= \frac{1}{\Gamma(\delta+1)\binom{m+\delta}{m}}{L_m}^\delta(0)\int_0^\infty e^{-y}y^\delta g(y)\sum_{h=0}^m {L_h}^\delta(y)dy\\
&= \frac{1}{\Gamma(\delta+1)}\int_0^\infty e^{-y}y^\delta g(y){L_m}^{\delta+1}(y)dy.
\end{aligned}$$

Now

$$\begin{aligned}
(CE)_m^{1,\beta,\gamma}(g;0) &= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^m A_{m-h}^{\beta-1}A_h^\gamma\frac{1}{2^h}\sum_{v=0}^h\binom{h}{v}s_v(0)\\
&= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^m A_{m-h}^{\beta-1}A_h^\gamma\frac{1}{2^h}\sum_{v=0}^h\binom{h}{v}\\
&\quad \cdot\frac{1}{\Gamma(\delta+1)}\int_0^\infty e^{-y}y^\delta g(y){L_v}^{\delta+1}(y)dy.
\end{aligned}$$

Therefore using (3), we have

$$\begin{aligned}
\mid (CE)_m^{1,\beta,\gamma}(g;0)-g(0)\mid &= \frac{1}{A_m^{\beta+\gamma}}\sum_{h=0}^m A_{m-h}^{\beta-1}A_h^\gamma\frac{1}{2^h}\\
&\quad \cdot\sum_{v=0}^h\binom{h}{v}\int_0^\infty \mid\sigma(y)\mid {L_v}^{\delta+1}(y)dy\\
&= \left(\int_0^{1/m}+\int_{1/m}^\varepsilon+\int_\varepsilon^m+\int_m^\infty\right)\frac{1}{A_m^{\beta+\gamma}}\\
&\quad \cdot\sum_{h=0}^m A_{m-h}^{\beta-1}A_h^\gamma\frac{1}{2^h}\sum_{v=0}^h\binom{h}{v}\\
&\quad .\mid\sigma(y)\mid {L_v}^{\delta+1}(y)dy\\
&= J_1+J_2+J_3+J_4. \qquad (16)
\end{aligned}$$

Using orthogonal property (13), Lemma [1][condition 5] and Lemma [3] we get

$$\begin{aligned}
J_1 &= \int_0^{1/m} \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \\
&\quad . \mid \sigma(y) \mid L_v{}^{\delta+1}(y)dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} O(m^{\delta+1}) \\
&\quad . \int_0^{1/m} \mid \sigma(y) \mid dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} O(m^{\delta+1}) o\Big(\tau(m)/m^{\delta+1} \Big) \\
&= O(m^{\delta+1}) o\Big(\tau(m)/m^{\delta+1} \Big) \\
&= o(\tau(m)).
\end{aligned} \tag{17}$$

Further using the orthogonal property (14), Lemma [1][condition 6], Lemma (3) and using the argument as in Nigam and Sharma [16] and Krasniqi [17] then integrating by parts, we get

$$\begin{aligned}
J_2 &= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \int_{1/m}^{\varepsilon} \mid \sigma(y) \mid L_v{}^{\delta+1}(y)dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \\
&\quad .O(v^{(2\delta+1)/4}) \int_{1/m}^{\varepsilon} \mid \sigma(y) \mid y^{-(2\delta+3)/4} dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} O(h^{(2\delta+1)/4}) \int_{1/m}^{\varepsilon} \mid \sigma(y) \mid y^{-(2\delta+3)/4} dy \\
&= O(m^{(2\delta+1)/4}) \int_{1/m}^{\varepsilon} \mid \sigma(y) \mid y^{-(2\delta+3)/4} dy \\
&= o(\tau(m)).
\end{aligned} \tag{18}$$

Using (14), Lemma [2][condition 7] and Lemma [3] we get

$$\begin{aligned}
J_3 &= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \\
&\quad . \int_{\varepsilon}^{m} \mid \sigma(y) \mid L_v{}^{\delta+1}(y)dy \\
&\leq \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \int_{\varepsilon}^{m} e^{y/2} \\
&\quad .y^{-(2\delta+3)/4} \mid \sigma(y) \mid e^{-y/2} y^{(2\delta+3)/4} L_v{}^{\delta+1}(y)dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} O(v^{(2\delta+1)/4}) \\
&\quad . \int_{\varepsilon}^{m} e^{y/2} y^{-(2\delta+3)/4} \mid \sigma(y) \mid dy \\
&= O(m^{(2\delta+1)/4}) o\Big(m^{-(2\delta+1)/4} \tau(m) \Big) \\
&= o(\tau(m)).
\end{aligned} \tag{19}$$

Finally, using (15), Lemma [2][condition 7] and Lemma [3], we get

$$\begin{aligned}
J_4 &= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{2^h} \sum_{v=0}^{h} \binom{h}{v} \int_m^{\infty} e^{y/2} \\
&\quad .y^{-(3\delta+5)/6} \mid \sigma(y) \mid e^{-y/2} y^{(3\delta+5)/6} {L_v}^{\delta+1}(y)dy \\
&= \frac{1}{A_m^{\beta+\gamma}} \sum_{h=0}^{m} A_{m-h}^{\beta-1} A_h^{\gamma} \frac{1}{(2^h} \sum_{v=0}^{h} \binom{h}{v} O(m^{(\delta+1)/2}) \\
&\quad \cdot \int_m^{\infty} \frac{e^{y/2} y - 1/3 \mid \sigma(y) \mid}{y^{(\delta+1)/2}} dy \\
&= O(m^{(\delta+1)/2}) o(m^{-(\delta+1)/2} \tau(m)) \\
&= o(\tau(m)). \qquad (20)
\end{aligned}$$

Combining (16), (17), (18), (19) and (20), we get

$$\mid (CE)_m^{1,\beta,\gamma}(g;0) - g(0) \mid = o(\tau(m)).$$

IV. COROLLARY

The novelty of the work is that by using different values of β and γ, the existing summability methods can be derived given as follows:

- If we take $\beta = 1, \gamma = 0$ *and* $q = 1$, Our findings reduces to the results given by Krasniqi [17] for $q = 1$.
- If we take $\beta = 2, \gamma = 0$ *and* $q = 1$, Our findings reduces to the results given by Sonker [18] for $q = 1$.
- If we take $\beta = 0, \gamma = 0$ *and* $q = 1$, Our findings reduces to the results given by Nigam and Sharma [16] and many other.

V. GRAPHICAL ANALYSIS

Here, we consider the function

$$g(t) = t^6,$$

with its Fourier-Laguerre series

$$g(t) = \sum_{m=0}^{\infty} (-1)^m \binom{6}{m} \Gamma(7) L_m^{\beta}(t).$$

Here, $\{c_m\}$ is the cofficient sequence in the Fourier-Laguerre expansion. $(CE)_m^{1,\beta,\gamma}$ is the proposed mean about the point t=0. We are plotting g and $(CE)_m^{1,\beta,\gamma}$ verses Number of terms.

The Fourier-Laguerre series for above function about the point t=0 is plotted in Figure 1 and we can analyse that oscillations can be seen only for very small values about point t=0.

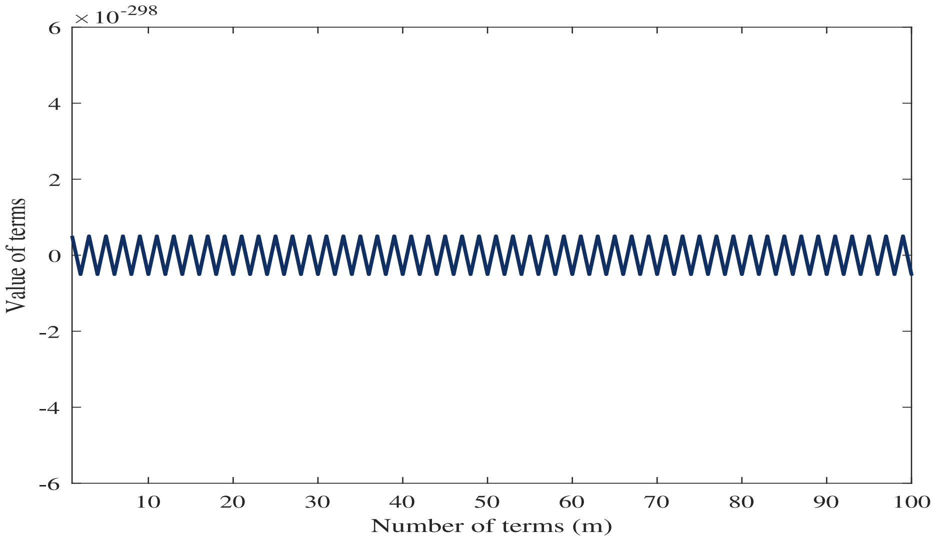

FIGURE 1. Fourier-Laguerre series about the point t=0

Here, we discuss our results in following cases:

Case 1: When $\beta+\gamma<0$ *and* $q=1$, we interpret that after applying $(CE)_m^{1,\beta,\gamma}$ mean the Fourier-Laguerre polynomial is approximating $g(t)$ from negative side and larger the value of $\beta+\gamma$ better will be the approximation.

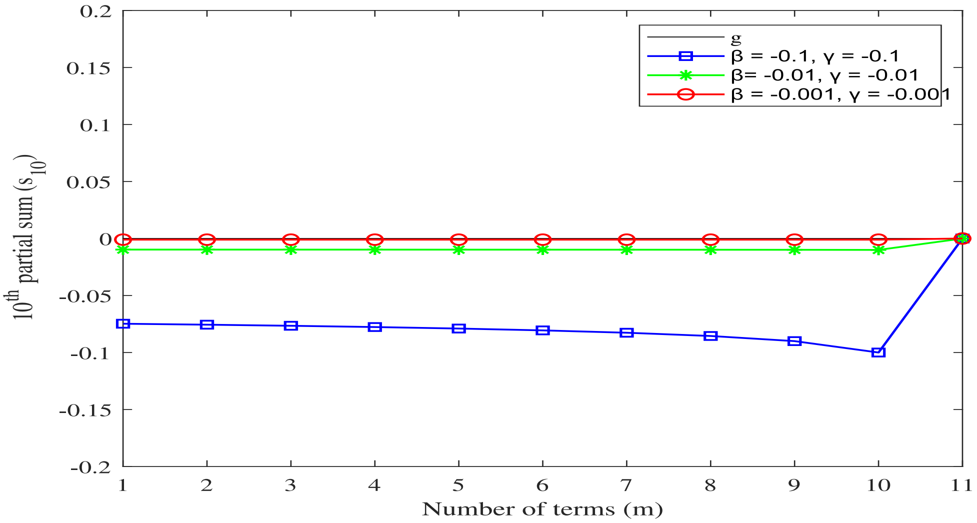

FIGURE 2. Approximation of function when $\beta+\gamma<0$

Case 2: When $\beta+\gamma>0$ *and* $q=1$, we interpret that after applying $(CE)_m^{1,\beta,\gamma}$ mean the Fourier-Laguerre polynomial is approximating $g(t)$ from positive side and smaller the value of $\beta+\gamma$ better will be the approximation.

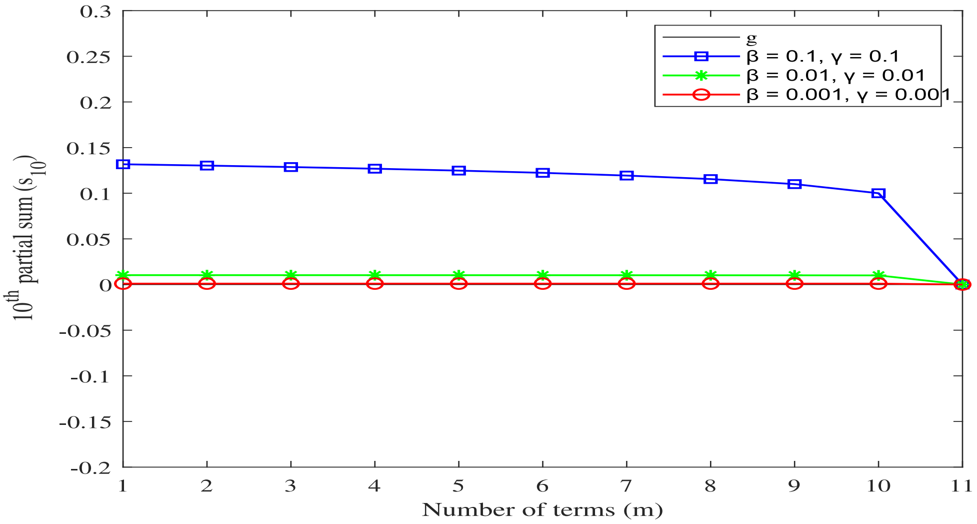

FIGURE 3. Approximation of function when $\beta+\gamma>0$

Comparison with existing methods: From the graph given below it can be analysed that the rate of convergence of proposed method is much faster than the existing methods given by Krasniqi[17] and Sonker [18] for $q=1$.

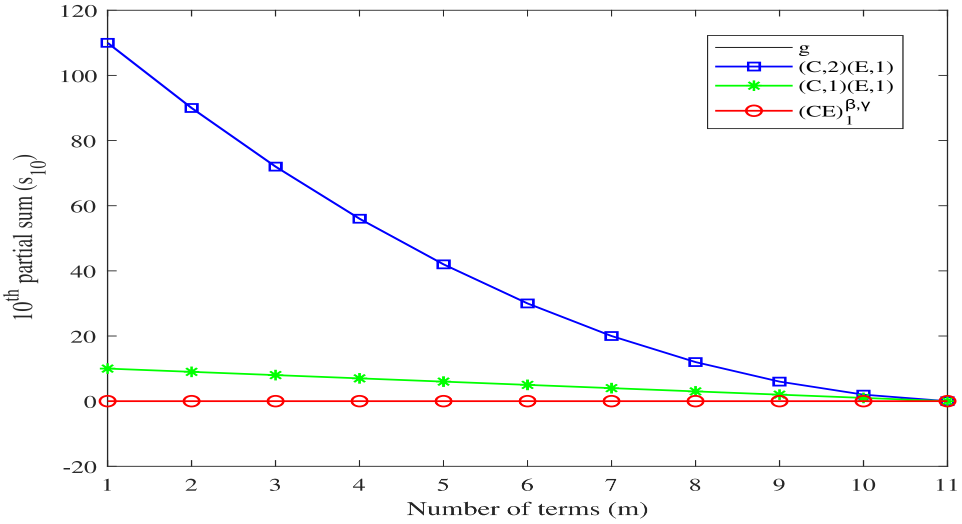

FIGURE 4. Comparison with existing method

From above graphical interpretation, we can say that $(CE)_m^{1,\beta,\gamma}$ product summability method is much efficient. Also, the change in the value of β and γ changes the behaviour of approximation.

CONCLUSION

The use of $(CE)_m^{1,\beta,\gamma}$ product summability of order $(\beta,\gamma,1)$ generalised the results discussed in corollary and add flexibility to convergence as with the change in values of β,γ changes the behaviour of approximation. The rate of convergence is improved with the help of proposed method. Also, using different values of β and γ, the existing summability methods can be derived. We can infer that our result is much efficient and useful.

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