STUDY OF THE TRIGONOMETRIC FUNCTION IN TERMS OF THE GENERALIZED – FUNCTION

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Abstract:

A few summations of the Sin(x) and Cos(x) series are represented in terms of the generalized - function in this research study. Approximation techniques and numerical analysis benefit greatly from these correlations. Chebyshev polynomials are useful for approximating functions, notably trigonometric functions, since they minimize the Runge phenomenon, among other beneficial qualities. Trigonometric function representations in terms of special functions are crucial because they improve efficiency, provide numerical stability, and have a wider range of applications in various scientific and technical fields. They offer strong instruments for resolving challenging mathematical issues and streamlining computing procedures. We will also acquire some fresh and intriguing results related to the Generalized – function in the current work.

**Key words:** Summation of series, Trigonometric function, Generalized – function, Euler's, Bessel, Legendre, Hyper geometric.

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**1.Introduction:**

Trigonometric functions can be expressed in terms of special functions in various ways, depending on the context and the specific special functions involved. Special functions are mathematical functions that have specific properties and applications in various branches of mathematics and physics. Some commonly used special functions include Bessel functions, Legendre polynomials, hyper geometric functions, and more. For example, one way to express trigonometric functions using special functions is through the use of exponential functions and complex numbers. Euler's formula is a fundamental relation that connects trigonometric functions with complex exponentials Here, e is the base of the natural logarithm, is the imaginary unit, θ is the angle in radians, cos(θ) is the cosine function, and sin(θ) is the sine function. Other special functions can also be used to express trigonometric functions in different ways[1]. For example, the Jacobi elliptic functions are a set of special functions that generalize the trigonometric functions and can be expressed in terms of the complete elliptic integrals.The -function will be defined and represented by the following Mellin Barnes type contour integral [2].

= ds ……………………(1)

Where, (s) is given by

= ……………(2)

Also

1. Z 0.
2. i=
3. m, n, p, q are integers satisfying 0
4. is a suitable contour in the complex plane.
5. n empty product is interpreted as unity.
6. , j = 1,….,p; , j = 1,…. .,q; , j= 1,… ..,n; , j =m+1,…,q are real positive numbers.
7. , j = 1,… .,p and , j = 1,… ..,q are all complex numbers.

There are three different contours of integration.

1. goes from c-i to c +i , (c real ) so that all the singularities of j =1,… .m, lie to the right ,and all the singular ities of j= 1,… ., lie to the left of
2. is a loop beginning and ending at + and encircling all the singularities of j = 1,….m, once in the clock wise direction, but none of the singularities of(
3. is a loop beginning and ending - and encircling all the singularities of () , j = 1,….n, once in the anti-clockwise direction, but none of j =1,….m.

[z] = (3)

For (and / or) not an integers, the poles of gamma functions of the numerator in

= are converted to branch points .The branch cuts be chosen so that the path of the integration can be distorted for each of the three contours mentioned above as long as there is no coincidence of the poles from any and pair.The following sufficient conditions for the absolute convergence of the defining integral for the -function given in[3].

= ds have been given by Buschman and Srivastava .

|argz| , is given by .

The following properties of [z] for small and large values of |z| as recorded by Saxena and Gupta [z] = O( |z|g ) for small z ………..(5)

where, g = min [Re( and 1

[z] = O(|z|h ) for large z, …………..(6)

Where, h = max Re[ and the conditions given by 1

are also satisfied. Evidently, when the exponents , =1, j =1,…,n and =1, =m+ 1,…q , the -function reduces to the well Known Fox -function as[4] :

= H ……….(7)

Where the function on the right hand side in the Fox –function defined by H [ ] = H[]

= = H

Certain elementary properties of the -function have been studies by Rathie and Devra are listed below[5].

………(8)

for K

= k ………..(9)

K

= (-1 )r ………(10)

= ……….(11)

= (-k)-1 …………(12)

K

= (-k) …………(13)

K

= K …………(14)

for k Rathie has given a series representation for the -function defined and represented by [6].

= ds as follows :

= …………(15)

Where, = .

**2. SUMMATION OF SERIES**



 ×- .

Provided;



..................(16)

× + .

Provided;



(16)

**Proof:** To establish equation (15)

Let 



Then  (17)



 (18)

Using equation (1), we have

 

Similarly we may obtained



And using the particular case of Taylor theorem then we get

R.h.s. of equation(15), proceeding similar manner we can find equation(16).

**3.Applications**

The representation of trigonometric functions in terms of special functions, such as Chebyshev polynomials, finds applications in various areas of mathematics, physics, and engineering. Some specific applications include:

**Numerical Analysis and Approximation:**

Chebyshev polynomials offer an efficient means of approximating functions, including trigonometric functions, with minimal error. This is particularly useful in numerical analysis for tasks such as interpolation and approximation of data[7].

**Signal Processing:**

Trigonometric functions are fundamental in signal processing. Representing trigonometric functions in terms of special functions can provide more efficient algorithms for signal processing tasks such as filtering, spectral analysis, and Fourier analysis [8].

**Control Systems:**

In control theory, trigonometric functions are often encountered in the analysis of dynamic systems. Using special functions for their representation can simplify the analysis and design of control systems [9].

**Physics and Engineering:**

Various physical phenomena involve trigonometric functions. Expressing these functions in terms of special functions helps in solving differential equations and understanding the behavior of physical systems. For example, in quantum mechanics, special functions play a crucial role in describing wave functions[10].

**Orthogonal Polynomials:**

Chebyshev polynomials are orthogonal on certain intervals, and this property is exploited in applications like numerical integration. These polynomials provide a basis for expanding functions in a series, facilitating efficient computation in various mathematical problems.

**Computational Mathematics:**

Efficient algorithms for computing trigonometric functions using special function representations can lead to faster and more accurate numerical computations, which is essential in fields like scientific computing and simulations[11].

**Mathematical Physics:**

Trigonometric functions appear in many mathematical models describing physical phenomena. Special functions help in solving differential equations and boundary value problems arising in mathematical physics[12].

**Mechanical Vibrations:**

Problems involving periodic motion, such as mechanical vibrations, often lead to solutions expressed in terms of trigonometric functions. Special functions can help in analyzing and solving these vibration problems[13].

In summary, the use of special functions in representing trigonometric functions enhances the efficiency and accuracy of various mathematical and scientific applications, especially in numerical methods, signal processing, control systems, and physics.

**Importance of representing trigonometric functions in terms of special functions:**

The importance of representing trigonometric functions in terms of special functions lies in the versatility and efficiency these representations bring to various areas of mathematics, physics, engineering, and computational sciences. Here are some key points highlighting the importance:

**Efficient Computation:**

Special functions often provide more efficient algorithms for computing trigonometric functions. This is crucial for applications requiring rapid and accurate numerical calculations, such as simulations, signal processing, and scientific computing.

**Orthogonality and Basis Functions:**

Many special functions, including orthogonal polynomials like Chebyshev polynomials, form complete sets of basis functions. This property simplifies the representation of functions and facilitates solving differential equations, making them valuable in mathematical modeling.

**Signal Processing and Fourier Analysis:**

Trigonometric functions are fundamental in signal processing. Efficient representations using special functions contribute to the development of algorithms for tasks like Fourier analysis, filtering, and modulation, which are essential in communication systems.

**Physical Modeling:**

Special functions are instrumental in solving differential equations arising in physics. They play a crucial role in describing physical phenomena, such as quantum mechanics, electromagnetism, and fluid dynamics.

**Mechanical Engineering:**

Mechanical systems with periodic motion, like vibrations, often involve trigonometric functions. Special functions help in modeling and analyzing these systems, contributing to the design and optimization of mechanical structures.

**Mathematical Physics:**

Trigonometric functions frequently appear in mathematical models of physical phenomena. Special functions aid in solving complex mathematical problems, enabling a deeper understanding of the underlying physics.

**Computational Mathematics:**

Special function representations are foundational in developing algorithms for computational mathematics. They enhance the accuracy and speed of computations in diverse applications, including optimization and numerical simulations [14].

CONCLUSION:

Studying trigonometric functions in terms of the generalized H̃-function can provide valuable insights into their properties and behavior. The generalized H̃-function, often denoted as H̃, is a mathematical function that generalizes various special functions like the hypergeometric function, confluent hypergeometric function, and Meijer G-function.When exploring trigonometric functions using the generalized H̃-function, researchers can analyze their relationships with other mathematical functions, investigate their convergence properties, and explore new methods for solving trigonometric equations and integrals.In conclusion, the study of trigonometric functions in terms of the generalized H̃-function offers a rich area of research with potential applications in mathematics, physics, engineering, and other fields. By leveraging the versatility and generality of the H̃-function, mathematicians and scientists can deepen their understanding of trigonometric phenomena and develop new analytical tools for solving problems across various disciplines.

REFRENCES:

1. Merjer, C. S. (1952). Expansion theorem for G –function I, integatious Maths, 14 , P.P. 483 -487.
2. Murray, R Spiegel (1962): some interesting special cases of the hyper-geometric series. Amer. Math Monthly, 69,pp 894 – 896.
3. R. G. Buchman and K. C. Gupta(1975) Contiguous relations for the H – function of two variables Indian J. Pure Appl. Math., 6(1975), 1416 – 1421.
4. H.M. Srivastava and R.Panda (1979): Some multiple integral transformations involving the H – function of several variables, Nederi . Akad. Wetensch. Proc. Ser . A 82 Indag. Math., 41 (1979), 353 – 362.
5. Saxena, V.P. (1982): Formal solution for certain new pair of integral equations, involving H – function, Prot. Nat. Sci. India 52(a), 111,p.p. 365 – 275.
6. H.M. Srivastava, K. C. Gupta and S. P. Goyal (1982): The H – functions of One and Two variables with Applications. South Asian Publishers, New Delhi and Madres.
7. Inayat Hussain, A. A. (1987): Expansion theorem for G – function II, Integatious Maths, 14,, p.p. 483 – 487
8. R. Shrivastava and K. Verma, Certain double integrals involving the I function. *The Mathematical Education, Siwan (Bihar*), **XLIII** (1) (2009), 1–10.
9. K. Verma and R. Shrivastava, Integration of the I-function w.r.t. parameters. *Vijnanan  parishad Anusandhan Patrika*, **54**(2010), 7–8.
10. K. Verma, R. Shrivastava, and A.K. Ronghe, Application of I-Function in Jacobi Polynomials   of Non-Linear Differential Equations, *The Mathematical Education, Siwan (Bihar),* **XLIII** (2) (2013), 1-7.
11. K. Verma, R. Shrivastava and A. K. Ronghe, On the Boundary Value Problem and its Application in   Multivariables, *Mathematical Theory and Modiling*, **3**(6)(2013), 127-132.
12. K. Verma, R. Shrivastava, and A.K. Ronghe, nth finite integrals involving products of multivariables, *Mathematical Theory and Modeling* ,**5**(7)(2015),4-43.
13. K. Verma, R. Srivastava and A.K. Ronghe , Expansion for the Product of Bessel’s Function and I-Function of Multivariables, *Mathematical Theory and Modeling*,**5**(7) (2015),68-72.
14. K. Verma, R. Shrivastava and A.K. Ronghe, Study of Expansion for the I-Function of Multivariables and Its Application. *Mathematical Theory and Modeling,* 5(2015), 59-65.

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