**1. Introduction and Literature review**

To address global warming, biodiversity loss, carbon emissions, and waste, the circular economy has emerged as a viable alternative to industries' current linear economic framework. Every year, a massive amount of waste is dumped into the environment as a result of the current linear economic system, which begins with the production of goods from raw materials and ends with their disposal into the environment. Many raw materials are limited and require a significant amount of energy to extract. The circular economy is a systematic framework in which production is reused, recycled, remanufactured, and returned to the market at an economic cost. As a result, a circular economy addresses both environmental and financial issues. In (2016) Lenwandowski developed a comprehensive model to introduce the characteristics of a circular economy based on the business environment. In (2018) De Angelis developed a framework for circular supply chain to eliminate the drawbacks of the present linear supply chain. In (2020) Rabta for the first time, developed an economic order quantity model to investigate the results of products’ circularity level. In (2022) Thomas and Mishra developed a sustainable circular economy model to reduce Carbon emission and waste with the help of 3D printing. Further, Khan et al. in (2022) studied a production system with carbon emission to optimize the circular economy index policy.

Table:1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Authors** | **Circular economy** | **Demand depended on** | **Per unit gross profit depended on** | **Carbon emission** | **Shortages** | **Deterioration** |
| Chang *et al.* (2003) |  | Time  | Constant  |   |  |  |
| Ouyang *et al.* (2013) |  | Constant  | Constant  |  |  |  |
| Sharma *et al.* (2016) |  | Constant  | Constant  |   | Allowed with partial backlog |  |
| Rabta (2020) | *
 | Circularity index | Circularity index |  |  |  |
| Thomas and Mishra (2022) |  | Circularity index | Circularity index |  |  |   |
| Khan *et al.* (2023) |  | Circularity index | Circularity index |  |  |  |
| This paper |  | Circularity index | Circularity index |  | Allowed with partial backlog |  |

 **2. Assumptions**

Assumptions are made as follows.

1. The demand rate is dependent on circularity level

Where ,, and are constant parameters.

1. The unit gross profit is dependent on circularity level

Where ,, and are constant parameters.

1. This model contains a single item and single retailer.
2. Replenishment rate is instantaneous.
3. The inventory system has an infinite planning horizon.
4. Shortages are allowed with partial backlogging and the partial backlogging parameter is *ρ,* 0 < *ρ <* 1.
5. Inflation is taken into consideration

**3. Notations**

* A Ordering cost per order
* selling price per unit
* selling price per unit when circularity index
* demand rate
* demand rate when circularity index
* r rate of inflation
* Ce  reduce carbon emissions
* Q retailer’s order quantity
* θ deterioration rate
* B back order level
* I1(t) inventory level in interval [0,T1]
* I2(t) inventory level in interval [T1, T]
* *ρ* partial backlogging parameter
* Ch holding cost
* Cd deterioration cost
* Sc shortage cost
* Ci lost sale cost

**4. Mathematical model formulation**

The behaviour of the inventory level is shown in figure 1.



Fig:1

The differential equation representing the inventory level is:

With boundary conditions , , .

After solving these equations (1) and (2) with the help of boundary conditions, we get

Ordered quantity and backorder for this model is

and .

The total profit for the retailer contains the following terms,

1. Ordering cost (OC)
2. Holding cost (Ch) =
3. Deterioration cost (Cd)=
4. Green investment = Ig
5. Carbon emission cost (CE) =
6. Shortage cost =
7. Lost sale cost =
8. Sales revenue =

Total profit =

**2.5. Numerical illustration**

|  |  |  |  |
| --- | --- | --- | --- |
| Parameters  | Value  | Parameters  | Values  |
| p0 | 1.5 | b | 0.1 |
| β | 0.21 | r | 0.05 % |
| Do | 1 | A | 0.01 |
| γ | 0.01 | ρ | 0.11 |
| a | 1.1 | ch | 100$ |
| θ | 0.001 | cd | 10$ |
| ce | 310 Rs | sc | 0.001$ |

Table:2

Table:3

Final optimal solution is

|  |  |  |
| --- | --- | --- |
|  | Time taken to finish the inventory level *(t)* | 0.00381725 year |
| *T* | Replenishment cycle length | 0.0927898 year |
| *ω* | Circularity index 0 *≤ ω ≤* 1 | 0.000707859  |
|  | Total profit  | 0.0160897 Lakh |

**6.Concavity**

Fig: 2 concavity Between (Circularity index) and T1(Time taken to finish the inventory level)



Fig:3 concavity Between (circularity index) and T (cycle length)



Fig:4 concavity Between T (cycle length) and T1(Time taken to finish the inventory level)

**7. Sensitivity Analysis**

Table:4

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Parameter | % value | Total cost | T | T1 | ω |
| p0 | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | -0.00560 | 0.003502 | 0.09542 | 0.00056 |
| -20% | -0.02673 | 0.00318 | 0.09769 | 0.000130 |
| b | +20% | 0.01853 | 0.00388 | 0.09317 | 0.01630 |
| +10% | 0.01738 | 0.00387 | 0.09269 | 0.02830 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
| β | +20% | 0.01560 | 0.00381 | 0.09285 | 0.000503 |
| +10% | 0.01584 | 0.003812 | 0.09282 | 0.0005011 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01634 | 0.003821 | 0.09277 | 0.000565 |
| -20% | 0.0165 | 0.00382 | 0.09275 | 0.000557 |
| r | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| Do | +20% | 0.01853 | 0.00388 | 0.09317 | 0.01630 |
| +10% | 0.01738 | 0.00387 | 0.09269 | 0.02830 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
| a | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
| γ | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| ρ | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
| A | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01853 | 0.00388 | 0.09317 | 0.01630 |
| -20% | 0.01738 | 0.00387 | 0.09269 | 0.02830 |
| ch | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| θ | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |
| cd | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| ce | +20% | 0.06138 | 0.00443 | 0.08645 | 0.00088 |
| +10% | 0.03839 | 0.00412 | 0.08982 | 0.00050 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| -20% | 0.01728 | 0.00383 | 0.09269 | 0.31492 |
| sc | +20% | 0.016126 | 0.003817 | 0.09283 | 0.000960 |
| +10% | 0.01610 | 0.00381 | 0.09281 | 0.000502 |
| 0 | 0.01648 | 0.00385 | 0.09266 | 0.00175 |
| -10% | 0.01490 | 0.00380 | 0.09293 | 0.000532 |
| -20% | 0.013715 | 0.00378 | 0.09307 | 0.000468 |

**8. Conclusion**

This paper is developed for single retailer. In this paper we developed inventory model with the effect of inflation and Circularity Index. In which demand is depends on circularity index. The unit gross profit is dependent on circularity level. Wee also consider shortage in this model which is partially backlogged. Effect of carbon emission is also taken into account and to reduce the carbon emission investment in green technology is also taken into consideration. Numerical example is given to validate the model mathematically. Sensitivity analysis is carried out for showing the behaviour of different parameters on optimal solutions.

On increases in p0 the total cost, cycle length increases while circularity index decreases. On increases in the para meter b total cost, cycle length and circularity index increased. On increases in the parameter β total cost decreases cycle length fluctuating and circularity index decreases. On increases in the parameter r total cost, cycle length and circularity index decreases. On increases in the parameter D0 total cost, cycle length and circularity index increases. On increases in the parameter a total cost, cycle length increases while circularity index fluctuating. On increases in the parameter ρ the total cost, cycle length and circularity index increases.

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