Numerical computation of heat and mass transfer in micropolar Casson fluid flow over non-linearly porous stretching sheet with heat generation

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**Abstract**

In the present study we investigation the numerical solution of heat and mass transfer in micropolar Casson fluid flow over non-linearly stretching sheet with heat generation effect in porous medium. The controlling partial differential equations are transformed into nonlinear ordinary differential equations by applying the proper similarity transformations. The aforementioned transformed nonlinear ordinary differential equations are numerically solved with the application of MATLAB software. Concentration, fluid velocity, micro-rotation, and temperature are provided graphically along with an analysis and discussion of the effects of various relevant factors such as the Casson fluid parameter, porosity parameter, micro-inertia density parameter, heat generation parameter, buoyancy parameter, diffusion parameter, spin gradient viscosity parameter, Schmidt number, etc. on the velocity, micro-rotation, concentration, and temperature profiles. The generated data's graphical representations are explained in depth.

**Keywords** Micropolar fluid; Heat and mass; Heat generation; Casson fluid; Porous medium; non-linearly stretching sheet;

**Introduction**

Magneto hydrodynamics or MHD investigates the behavior of fluids within electrically conducting environments. Alfven laid the groundwork for this field, which later saw researchers delving into its mathematical underpinnings and applying these models to various mechanical scenarios [1]. MHD's relevance extends to metallurgical and metalworking processes, where understanding the flow of electrically conducting fluids is crucial. It is especially valuable in situations where heat transfer is a critical factor. Moreover, micropolar fluid theory, introduced by Erigen, has captivated researchers for decades. Nazar explored micropolar fluid flow at stagnation points on stretching surfaces, while Chiam investigated the flow of micropolar fluids induced by stretchy surfaces. These fluids, characterized by non-Newtonian behavior, consist of colloidal elements such as large dumbbell molecules suspended in smaller fluid bodies. They are valuable for characterizing the flow behavior of suspension solutions, specialized lubricants, plyometric fluids, paints, animal blood, and liquid crystals, demonstrating their versatility across diverse fields.

Eringen [2] pioneered the thought of microstructure fluids, which integrate micro scale elements connecting the overall velocity vector field with particle rotation. These fluids comprise solid particles dispersed within a fluid medium and elucidate the behavior of colloidal fluids, liquid crystals, and biological systems. Effective cooling systems are crucial for product manufacturing due to the importance of cooling speed. Thermal radiation, a well-understood phenomenon, simplifies handling extreme heat in various industrial activities like nuclear reactors and spacecraft. Radiant heat energy from a heated surface is proportional to the fourth power of its absolute temperature, influencing the thermal change in energy equation. The fluid characterization of non-Newtonian fluids, which is not described by Navier-Stokes equations, has prompted the exploration of different non-Newtonian model types. Some fluids are discussed by Hayat and Mustafa [3], Hayat and Obaidat [4], Nadeem and Fang [5], Turkyilmazoglu and Pop [6], and Qasim [7]). Non-Newtonian liquids, such as Jeffrey fluid, exhibit unique properties like linear viscoelastic behavior, finding applications in industries like polymer production. Jeffrey fluid, including its forms like diluted polymer solution, is often utilized in fluid models due to its simplicity and representation of retardation and relaxation times. Numerous studies, including those by Sharma et al. [8], Nallapu and Radhakrishnamacharya [8], and Vandna et al. [9], have explored the effects of magnetic fields, flow in porous media, and heat transfer in MHD micropolar Jeffrey fluid flow. Other investigations by researchers such as Ellahi et al. [11], Khan et al. [12], and Vaidya et al. [13] have delved into different aspects of the Jeffrey model, including Radiative flow and thermal radiation effects [14]. Studies like those by Vandna et al. [15] and Dhermendar et al. [16] have examined MHD fluid flow over various geometries and mediums, analyzing parameters like curvature and porosity on temperature gradients. Mahabaleshwar et al. [17] have numerically investigated magnetohydrodynamics fluid flow and heat transfer over elastic or non-elastic sheets in the presence of heat sources/sinks and thermal effects.

The influence of carbon nanotubes (CNTs) on radiation effects has been investigated, revealing that the magnetic field profile of the upper branch solutions exhibits greater analytical stability compared to the lower branch solutions. Researchers, such as Panday et al. [18], have explored the relationship between Convection combined with thermal radiation in the flow of a Newtonian fluid is a nonlinear stretching sheet, demonstrating that an increase in the linear convection parameter leads to higher stream velocity near the surface and lower velocity different from it. Megahed and colleagues [19] explored the investigation of magnetohydrodynamics flow close to an unstable elastic sheet under conditions of changing fluid properties, thermal emission, and heat flow. In a separate study, Krishna [20] investigated the MHD unsteady free convective rotating flow of Jeffrey fluid with a gradually increasing wall temperature, examining ion slip and Hall effects. Some more important relevant studies can be seen through [21-29] and the references therein. Khader et al. [30] presented numerical solutions for MHD unsteady micropolar fluid flow induced by either an elastic or non-elastic surface, while considering the effects of thermal radiation and heat source. Numerous research investigations utilize Magnetohydrodynamics (MHD) flow coupled with heat and mass transfer due to the predominant influence of the magnetic field [31-34]. Magnetohydrodynamics (MHD), the study of electrically conducting fluid motion under magnetic field influence, holds significant importance across a range of applications including MHD generators, nuclear reactors, and biometric pumps. Adegbie et al. [35] and Malik et al. [36] examined the varied characteristics of MHD non-Newtonian fluid flow over a stretching surface, highlighting the impact of changing thermal conductivity on fluid temperature. The analysis and numerical exploration of electrically conductive fluids are of great importance in both engineering and scientific fields, with applications ranging from MHD generators and nuclear reactors to technological innovations such as intelligent system of lubrication and equipment segregation. Within the domain of biological fluids, many scholars [37-39] have been conducted magnetohydrodynamics studies. Additionally, the investigation of convective heat transfer within porous media saturated with fluid has garnered significant attention due to its broad industrial applications, including geothermal energy recovery, food processing, and various chemical industry processes. Ahmed and colleagues [40] investigated the influence of radiation on convective heat transport in the MHD boundary layer of a porous medium in small pressure gradient. Similarly, Butt et al. [41] examined the consequence of heat production of the flow of magnetohydrodynamics Jeffrey fluids through porous media.

**Mathematical formulation**

We consider a two dimensional boundary layer flow of incompressible micropolar Casson fluid towards a non-linear porous stretching surface at y=o. According to the plane x-axis taken along the surface and the y-axis perpendicular to it, fluid flow is confined in the positive y region towards a surface which is shown in fig. 1. Furthermore In is accepted that power-low surface velocity distribution $u\_{w}=ax^{n}$ for the positive value of a. physically $n>1$ indicates non-linear stretching case. Temperature at the plate is regulated by heated fluid of temperature $T\_{f}$ below the surface of the wall and heat transfer coefficient is$ h\_{f}$. The concentration of species at the surface kept constant$ C\_{w}$. The concentration and temperature beyond the boundary layer are$ C\_{\infty }$ and$ T\_{\infty }$ respectively.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is given by:

$τ\_{ij}=\left\{\begin{array}{c}2\left(μ\_{B}+^{P\_{y}}/\_{\sqrt{2π}}\right)e\_{ij}, π>π\_{c},\\2\left(μ\_{B}+^{P\_{y}}/\_{\sqrt{2π\_{c}}}\right)e\_{ij}, π>π\_{c},\end{array}\right.$

Where $μ\_{B}$ plastic dynamic viscosity, $π=e\_{ij}e\_{ij}$ and $e\_{ij}$ is the $(i,j)^{th}$component of deformation rate,$ π$ denotes deformation rate, $π\_{c}$is a critical value of non Newtoniam model, $P\_{y}$is the yield stress of fluid. Under these assumptions, the governing equations are

$\frac{∂u}{∂x}+\frac{∂v}{∂y}=0$ (1)

$u\frac{∂u}{∂x}+v\frac{∂u}{∂y}=ν\left(1+\frac{1}{β}\right)\frac{∂^{2}u}{∂y^{2}}+\frac{κ}{ρ}\left(\frac{∂N}{∂y}\right)+gβ\_{T}\left(T-T\_{\infty }\right)+gβ\_{C}\left(C-C\_{\infty }\right)+\frac{μ}{ρk^{\*}}u$ (2)

$u\frac{∂N}{∂x}+v\frac{∂N}{∂y}=\frac{γ}{ρ}\left(\frac{∂^{2}N}{∂y^{2}}\right)-\frac{κ}{ρ}\left(2N+\frac{∂u}{∂y}\right)$ (3)

$u\frac{∂T}{∂x}+v\frac{∂T}{∂y}=∝\left(\frac{∂^{2}T}{∂y^{2}}\right)+\frac{Q}{ρC\_{P}}\left(T-T\_{\infty }\right)$ (4)

$u\frac{∂C}{∂x}+v\frac{∂C}{∂y}=D\_{B}\left(\frac{∂^{2}C}{∂y^{2}}\right)+\frac{D\_{T}}{D\_{w}}\left(\frac{∂^{2}T}{∂y^{2}}\right)-k\_{2}\left(C-C\_{\infty }\right)$ (5)

Where u and v are the velocity components in the x-and y-direction,$ β$ is the Casson fluid parameter,$ ρ$ is the density of the fluid, $ν$ are the kinematic viscosity,$β\_{C}$,$ β\_{T}$ and $k\_{2}$ are the coefficient of mass expansion, the coefficient of thermal expansion and the reaction rate of the solute, respectively, g is acceleration due to gravity and D is the diffusion coefficient.

**Boundary conditions**

$\left.\begin{array}{c}u=u\_{w}=ax^{n}, v=0, N=-m\_{0}\frac{∂u}{∂y} , -k\_{2}\frac{∂T}{∂y}=h\_{f}\left(T\_{f}-T\right), C=C\_{w} , as y\rightarrow 0\\u\rightarrow 0, N\rightarrow 0, T\rightarrow T\_{\infty }, C\rightarrow C\_{\infty }, as y\rightarrow \infty \end{array}\right\}$ (6)

Now consider the non-dimensional variables given below.

$$η=\sqrt{\frac{a\left(n+1\right)}{2υ}}x^{\frac{n-1}{2}}y , N=ax^{n}\sqrt{\frac{a\left(n+1\right)}{2υ}}x^{\frac{n-1}{2}}G\left(η\right) , ϕ\left(η\right)=\frac{C-C\_{\infty }}{C\_{w}-C\_{\infty }} ,θ\left(η\right)=\frac{T-T\_{\infty }}{T\_{w}-T\_{\infty }} , $$

$$u=ax^{n}F^{'}\left(η\right) , v=-υ\sqrt{\frac{a\left(n+1\right)}{2υ}}x^{\frac{n-1}{2}} \left[F\left(η\right)+\frac{n-1}{n+1}ηF^{'}\left(η\right)\right]$$

Substitute similarity variable into equation (1), (2), (3), (4) and (5), the transformed equation are

$\left(1+\frac{1}{β}\right)F^{'''}\left(η\right)+F\left(η\right)F^{''}\left(η\right)-\frac{2}{n+1}\left[nF^{'}\left(η\right)^{2}-λ^{\*}θ\left(η\right)-λϕ\left(η\right)+KpF^{'}\left(η\right)\right]+KG^{'}\left(η\right)=0$ (7)

$λ\_{0}G^{''}\left(η\right)+F\left(η\right)G^{'}\left(η\right)-\left(\frac{3n-1}{n+1}\right)G\left(η\right)F^{'}\left(η\right)-\frac{2}{\left(n+1\right)}.KB\left[2G\left(η\right)+F^{''}\left(η\right)\right]=0$ (8)

$θ^{''}\left(η\right)+Prθ^{'}\left(η\right)F\left(η\right)+\left(\frac{2}{n+1}\right)Pr ε θ\left(η\right)=0$ (9)

$ϕ^{''}\left(η\right)+Sc.F\left(η\right)ϕ^{'}\left(η\right)+ScSrθ^{''}\left(η\right)-\frac{2}{n+1}Sc.Kr.ϕ\left(η\right)=0 $ (10)

Boundary condition after using similarity variable

 $\left.\begin{array}{c}F\left(η\right)=0,F^{'}\left(η\right)=1,G\left(η\right)=-m\_{0}F^{''}\left(η\right),ϕ\left(η\right)=-γ\left(1-θ\left(η\right)\right) at y=0\\F^{'}\left(η\right)\rightarrow 0,G\left(η\right)\rightarrow 0,ϕ\left(η\right)\rightarrow 0,θ\left(η\right)\rightarrow 0 at y\rightarrow \infty \end{array}\right\}$ (11)

$$λ^{\*}=\frac{G\_{r}}{R\_{e}^{2}} , λ=\frac{G\_{c}}{R\_{e}^{2}} , Kp=\frac{ν}{ak^{\*}x^{n-1} } , K=\frac{κ}{ρυ} , λ\_{0}=\frac{γ}{ρjυ} , B=\frac{v}{jax^{n-1}} , Pr=\frac{υ}{∝} , ε= \frac{Q}{ax^{n-1}ρC\_{P}} , Kr=\frac{k\_{2}}{ax^{n-1}} , Sc=\frac{υ}{D\_{B}} , Sr=\frac{D\_{T}}{υD\_{w}}\left(\frac{T\_{w}-T\_{\infty }}{C\_{w}-C\_{\infty }}\right) , γ=\frac{-h\_{f}}{k\_{2}}\sqrt{\frac{2ν}{a(1+n)}}x^{-\left(\frac{n-1}{2}\right)}$$

Where the primes denotes differentiation with respect to$ η, λ^{\*}=\frac{G\_{r}}{R\_{e}^{2}} buoyancy parameter , λ=\frac{G\_{c}}{R\_{e}^{2}} diffusion parameter , Kp=\frac{ν}{ak^{\*}x^{n-1}} porosity parameter, K=\frac{κ}{ρυ}$ micropolar parameter $λ\_{0}=\frac{γ}{ρjυ} $spin gradient viscosity parameter,$ B=\frac{v}{jax^{n-1}} micro inertia density parameter$ $Pr=\frac{υ}{∝} $ Prandtl number, $ε= \frac{Q}{ax^{n-1}ρC\_{P}}$ heat generation/absorption parameter, $ Kr=\frac{k\_{2}}{ax^{n-1}}$ chemical reaction parameter, $Sc=\frac{υ}{D\_{B}} $ Schmidt number, $Sr=\frac{D\_{T}}{υD\_{w}}\left(\frac{T\_{w}-T\_{\infty }}{C\_{w}-C\_{\infty }}\right)$ Soret number and $γ=\frac{-h\_{f}}{k\_{2}}\sqrt{\frac{2ν}{a\left(1+n\right)}}x^{-\left(\frac{n-1}{2}\right)}$ Biot number.

The physical quantities of interest are skin friction coefficients$C\_{f}$ , local Nusselt number $Nu\_{x}$ and Sherwood number $Sh\_{x}$ which are defined as

$C\_{f}=\frac{τ\_{w}}{ρu\_{w}^{2}} , Nu\_{x}=\frac{xq\_{w}}{k\left(T\_{w}-T\_{\infty }\right)},$ $ Sh\_{x}=\frac{xq\_{m}}{D\_{B}(C\_{w}-C\_{\infty })}$ (12)

Where the wall friction$τ\_{w}$, heat and mass transfer at wall $q\_{w}$ and $q\_{m}$ are expressed as

$τ\_{w}=\left[κN+\left(μ+^{P\_{y}}/\_{\sqrt{2π}}+κ\right)\frac{∂u}{∂y}+\right]\_{y=0}$, $q\_{w}=-k\left(\frac{∂u}{∂y}\right)\_{y=0}$ and $ q\_{m}=-D\_{B}\left(\frac{∂C}{∂y}\right)\_{y=0}$ (13)

Skin friction coefficients

$ C\_{f}\sqrt{ Re\_{x}}=\sqrt{\frac{n+1}{2}}\left(1+\frac{1}{β}+K(1-n)\right)F^{''}\left(0\right)$ (14)

Local Nusselt number

$ Nu\_{x}Re\_{x}^{-^{1}/\_{2}}=-\sqrt{\frac{n+1}{2}}θ^{'}(0)$ (15)

And Sherwood number

$ Sh\_{x}Re\_{x}^{-^{1}/\_{2}}=-ϕ^{'}(0)$ (16)

Where $ Re\_{x}=\frac{U\_{w}x}{v}$ is the local Reynolds number?

**Solution Algorithm**

In order to address the boundary value problems (7)-(10) which are accompanied in (11), we employed the bvp4c function, a pre-existing solver, within the MATLAB software package. The basis of the algorithm is established on a methodology equation (7)-(10) in conjunction with the boundary condition (11) via a process of diminution, resulting in a system of first order nonlinear differential equation as presented below.

$F=y\_{1},F^{'}=y\_{2},F^{''}=y\_{3}, G=y\_{4}, G^{'}=y\_{5}, θ=y\_{6}, θ^{'}=y\_{7}, ϕ=y\_{8}, ϕ^{'}=y\_{9}$ (17)

By means of similarity transformation, equation (7), (8), (9) and (10) can be reduced to first- order ordinary differential equations.

$y\_{1}^{'}=F^{'}=y\_{2}$ (18)

$y\_{2}^{'}=F^{''}=y\_{3}$ (19)

$y\_{3}^{'}=\frac{1}{\left(1+\frac{1}{β}\right)}\left[\frac{2}{n+1}\left(ny\_{2}^{2}-λ^{\*}y\_{6}-λy\_{8}+Kpy\_{2}\right)-Ky\_{5}-y\_{1}y\_{3}\right]$ (20)

$y\_{4}^{'}=G^{'}=y\_{5}$ (21)

$y\_{5}^{'}=\frac{1}{λ\_{0}}\left⌊\left(\frac{3n-1}{n+1}\right)y\_{2}y\_{4}+\frac{2}{\left(n+1\right)}KB\left(2y\_{4}+y\_{3}\right)-y\_{1}y\_{5}\right⌋$ (22)

$y\_{6}^{'}=θ^{'}=y\_{7}$ (23)

$y\_{7}^{'}=-\left[Pry\_{1}y\_{7}+\frac{2}{\left(n+1\right)}Pr\in y\_{6}\right]$ (24)

$y\_{8}^{'}=ϕ^{'}=y\_{9}$ (25)

$y\_{9}^{'}=\frac{2}{\left(n+1\right)}Sc.Kr.y\_{8}+Sc.Sr\left[Pry\_{1}y\_{7}+\frac{2}{\left(n+1\right)}Pr\in y\_{6}\right]-Sc.y\_{1}y\_{9}$ (26)

**Result and discussion**

The equation (7), (8), (9) and (10), following their transformation were subjected to numerical, solutions through the bvp4c routine in MATLAB, along with due consideration of the boundary conditions (11). The result thus obtained were subsequently analyzed and presented in the form of graphs, which demonstrate the behavior of non-dimensional parameters such as $λ^{\*}, λ , Kp, K, λ\_{0}$,$ B,Pr$, $ε$, $ Kr$, $Sc$ and $Sr$, in relation to the simulated fluid velocity, along with temperature , concentration and micro rotation profiles. These graphs are vividly depicted in figure 1-19.



Fig.1 Casson fluid parameter with velocity profile

Figure1. Show that effect of Casson fluid parameter on fluid velocity. We can see that increase in Casson fluid parameter, the velocity profile decreases.



Fig.2 porosity parameter with velocity profile

Figure 2 is drafted to the variation in porosity parameter with respect to velocity profile. For raising the values of porosity parameter when found that velocity profile increased.



Fig.3 $K on f^{'}$

Figure 3 represent the effect of Micropolar parameter in velocity profile. It is seen from this graph that fluid velocity decrease for raising values of Micropolar parameter.



Fig.4 $λ on f^{'}$

Figure 4 is designed to show how the diffusion parameter affects the velocity. It can be identified that velocity profile is rising for enhances the value diffusion parameter.



Fig.5 $λ^{\*} on f^{'}$

Figure 5 describes the behavior of buoyancy parameter on velocity profile. It is seen that, velocity increases near to the surface with increase in buoyancy parameter.



Fig.6 $n on f^{'}$

Figure 6 shows effect of power index on velocity. Due to the raising the value of power index parameter, velocity of the fluid is decrease.



Fig.7 $B on micro rotation parameter$

Figure 7 shows effect of micro inertia density parameter B on micro-rotation profile. For higher values of micro inertia density parameter, it is seen that increases the value of micro-rotation profile.



Fig.8 $K on micro rotation parameter$

From fig.8 it is clear that for higher values of micropolar parameter, the micro-rotation parameter also increase.



Fig.9 effect $λ\_{0} on G\left(η\right)$

Figure 9 effect of Spin gradient viscosity parameter on microrotation parameter. It can b realized from the figure that the microrotation parameter turns up for rise in spin gradient viscosity parameter.



Fig.10 effect $n on G\left(η\right)$

From figure 10 shows that effect of power index parameter on micro-rotation parameter. It is seen that micro-rotation improve with increase in power index near the surface, whereas reverse effects on throughout the flow field.



Fig.11 effect $ε on θ\left(η\right)$

Figure 11 specify the temperature profile have been plotted for various values of heat generation/absorption parameter by keeping other parameter fixed. Physically when we increase the values of heat generation/absorption parameter, temperature of the fluid is also increase.



Fig.12 effect $Pr on θ\left(η\right)$

Figure 12 shows the effect of Prandtl number on temperature profile. Physically, when the amount of Prandtl number parameter increases, the values of temperature profile will decrease.



Fig.13 effect $n on θ\left(η\right)$

Figure 13 determine the effect of power index on temperature profile. Show that as the power index increases, the temperature profile decreases gradually.



Fig.14 effect $ε on ϕ\left(η\right)$

Figure 14 exhibits the concentration profiles for different values of heat generation/absorption parameter. It I observed that concentration profiles oscillate with increase in heat generation/absorption parameter.



Fig.15 effect $Sr on ϕ\left(η\right)$

Figure 15 shows that concentration profile decreases near the boundary and after$ η=0.1961$ sometime we observe the opposite action it means concentration increases with increases in the Soret number.



Fig.16 effect $Pr on ϕ\left(η\right)$

Figure 16 impact of various value of Prandtl number on concentration profile. It can be realized from the picture that the concentration profiles oscillate for increasing the value of Prandtl number.



Fig.17 effect $Sc on ϕ\left(η\right)$

From figure 17 Soret number defines the effects of the concentration profile. We found fall in concentration parameter when Soret number value increasing.



Fig.18 effect $Kr on ϕ\left(η\right)$

Figure 18 is plotted in order to check the effect of chemical reaction parameter on concentration profile. When we increase the value of chemical reaction parameter there is concentration profile decreasing.



Fig.19 effect $n on ϕ\left(η\right)$

Figure 19 illustrates the effect of the power index on concentration profile. When we increase the values of power index parameter there is initial raise and then fall in the concentration profile.

**Table 1: Effect of various parameters on**$C\_{f}$

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $$β $$ | $$λ^{\*}$$ | $$λ$$ | $$K\_{p} $$ | **K** |  ***n*** | $$f^{''}(0)$$ | $$\sqrt{\left(\frac{n+1}{2}\right)} \left(1+\frac{1}{β}+K(1-n)\right)f^{''}\left(0\right)$$ |
| **0.5** | 1 | 0.5 | 1 |  1 | 1.5 | -0.4227 | **-1.1835** |
| **1.0** | 1 | 0.5 | 1 | 1 | 1.5 | -0.4413 | **-0.7413** |
| **2.0** | 1 | 0.5 | 1 | 1 | 1.5 | -0.4507 | **-0.5047** |
| 1 | **0.7** | 0.5 | 1 | 1 | 1.5 | -0.5518 | **-0.9270** |
| 1 | **1.0** | 0.5 | 1 | 1 | 1.5 | -0.4413 | **-0.7413** |
| 1 | **1.5** | 0.5 | 1 | 1 | 1.5 | -0.2854 | **-0.4794** |
| 1 | 1 | **0.5** | 1 | 1 | 1.5 | -0.4413 | **-0.7413** |
| 1 | 1 | **1.5** | 1 | 1 | 1.5 | -0.3083 | **-0.5179** |
| 1 | 1 | **3.5** | 1 | 1 | 1.5 | -0.0110 | **-0.0184** |
| 1 | 1 | 0.5 | **0.5** | 1 | 1.5 | -0.3506 | **-0.5890** |
| 1 | 1 | 0.5 | **1.0** | 1 | 1.5 | -0.4413 | **-0.7413** |
| 1 | 1 | 0.5 | **1.5** | 1 | 1.5 | -0.5232 | **-0.8789** |
| 1 | 1 | 0.5 | 1 | **0.5** | 1.5 | -0.4295 | **-0.8418** |
| 1 | 1 | 0.5 | 1 | **2.0** | 1.5 | -0.4614 | **-0.5167** |
| 1 | 1 | 0.5 | 1 | **4.0** | 1.5 | -0.5265 | **0.0000** |
| 1 | 1 | 0.5 | 1 | 1 | **1.2** | -0.1612 | **0.0338** |
| 1 | 1 | 0.5 | 1 | 1 | **1.5** | -0.4413 | **0.2466** |
| 1 | 1 | 0.5 | 1 | 1 | **1.8** | -0.6124 | **0.5796** |

**Table 2: Effects of various parameters on Sherwood number and Nusselt number**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$β$$ | $$λ^{\*}$$ | **K** | ***n*** | $$λ$$ | $$K\_{p}$$ | $$ε$$ | **Pr** | **B** | $$λ\_{0}$$ | $$ S\_{r}$$ | $$S\_{c}$$ | $$K\_{r}$$ | $$-\sqrt{\frac{n+1}{2}}θ^{'}(0)$$ | $$-ϕ^{'}(0)$$ |
| **0.5** | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.6928** | **1.4165** |
| **1.0** | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| **2.0** | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7570** | **1.4454** |
| 1 | **0.7** | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.9852** | **1.5479** |
| 1 | **1.0** | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | **1.5** | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.4989** | **1.3336** |
| 1 | 1 | **0.5** | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7130** | **1.4256** |
| 1 | 1 | **2.0** | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7620** | **1.4507** |
| 1 | 1 | **4.0** | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.8627** | **1.4936** |
| 1 | 1 | 1 | **1.2** | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-1.2180** | **1.7078** |
| 1 | 1 | 1 | **1.5** | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | **1.8** | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.4194** | **1.2497** |
| 1 | 1 | 1 | 1.5 | **0.5** | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | **1.5** | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.6015** | **1.3780** |
| 1 | 1 | 1 | 1.5 | **3.5** | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.3671** | **1.2817** |
| 1 | 1 | 1 | 1.5 | 0.5 | **0.5** | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.5950** | **1.3745** |
| 1 | 1 | 1 | 1.5 | 0.5 | **1.0** | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | 0.5 | **1.5** | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.8738** | **1.4971** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | **0.4** | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.3124** | **0.8943** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | **0.6** | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.2027** | **0.9468** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | **0.7** | 1 | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.1346** | **0.9820** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | **2.5** | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.8347** | **0.5865** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | **3.5** | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.8282** | **0.5634** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | **4.5** | 1 | 0.5 | 0.5 | 1 | 0.5 | **-0.8243** | **0.4684** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | **1** | 0.5 | 0.5 | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | **2** | 0.5 | 0.5 | 1 | 0.5 | **-0.7280** | **1.4325** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | **3** | 0.5 | 0.5 | 1 | 0.5 | **-0.7258** | **1.4316** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | **1.5** | 0.5 | 1 | 0.5 | **-0.7373** | **1.4316** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | **2.5** | 0.5 | 1 | 0.5 | **-0.7392** | **1.4375** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | **3.5** | 0.5 | 1 | 0.5 | **-0.7402** | **1.4379** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | **0.5** | 1 | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | **1.0** | 1 | 0.5 | **-0.7411** | **1.9373** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | **1.5** | 1 | 0.5 | **-0.7514** | **2.4511** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | **0.5** | 0.5 | **-0.7019** | **0.9331** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | **1.0** | 0.5 | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | **1.5** | 0.5 | **-0.7479** | **1.8646** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | **0.5** | **-0.7316** | **1.4339** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | **1.0** | **-0.7409** | **1.6029** |
| 1 | 1 | 1 | 1.5 | 0.5 | 1 | 1.2 | 1 | 1 | 0.5 | 0.5 | 1 | **1.5** | **-0.7474** | **1.7475** |

Table 1 show that the varying parameters' effects including the $β,$ **K,**$ Kp$**,** $λ$**,** $, n$and$ λ^{\*}$on the **skin friction coefficient**. Specifically, Table 1 demonstrates that a raise in the$ β,$ **K,**$ λ $**,** $ n$and$ λ^{\*}$leads to a corresponding increase **skin friction coefficient**. Conversely, rise in value of the $Kp $results in a decrease in the **skin friction coefficient**.

Table 2 illustrates the varying parameters' effects including the $β,$ **K,**$ Kp$**, Sc, Sr,**$ Kr,$$ε, λ^{\*}$**,**$ λ$**, Pr,**$ B, n$and$ λ\_{0}$on the **Nusselt number**. Specifically, Table 2 demonstrates that a raise in the$ λ^{\*}$**,**$ λ$**, Pr,**$ B and n$ leads to a corresponding increase in the **Nusselt number**. Conversely, a raise in values of the $β,$ **K,**$ Kp$**, Sc, Sr,**$ Kr,$$ε$ and $λ\_{0}$ results in a decrease in the **Nusselt number**

We observe that table 2 effects of different parameters likes $β,$ **K,**$ Kp,$ **Sc, Sr,**$ Kr,ε, λ^{\*}, λ$ **Pr,**$ B, n$and$ λ\_{0}$on **Sherwood number**. We noticed that increases the value $β,$ **K,**$ Kp$**, Sc, Sr,**$ Kr,$$ε$and$ λ\_{0}$respectably, also increases the value of **Sherwood number**. We investigate increases the value of$ λ^{\*}$**,**$ λ$**, Pr,**$ B and n$ the **Sherwood number** decrease.

**Conclusions**

* The velocity of the sheet decreases when increase the values of Casson parameter, Porosity parameter, Micropolar parameter and Power index parameter respectively.
* On raising the value of Diffusion parameter and Buoyancy parameter, value of fluid velocity increase.
* On increasing the value of Micro inertia density parameter, Micropolar parameter and Spin gradient viscosity parameter, Micro-rotation of the fluid is increase.
* It is seen that Micro-rotation improves with increase in Power index near the surface, whereas reverse effects on throughout the flow field.
* As the value of heat generation and Prandtl number parameter rises, temperature of fluid increases with heat generation and inverse effect of Prandtl number.
* On Schmidt number and chemical reaction parameter are increasing, concentration parameter is decreasing.
* Concentration parameter of the fluid is oscillating when increasing the value of heat generation and Prandtl number.
* Shows that concentration profile decreases near the boundary and after$ η=0.1961$ sometime we observe the opposite action it means concentration increases with increases in the Soret number.
* When we increase the values of power index parameter there is initial raise and then fall in the concentration profile.
* **Skin friction** shows same effect to$ β,$ **K,**$ λ $**,** $ n$$ λ^{\*}$andopposite effect to$ Kp$.
* **Nusselt number** shows same effect to$ λ^{\*}$**,**$ λ$**, Pr,**$ B n$ andinverse effect to$ β, K, Kp, Sc, Sr, Kr, ε λ\_{0} $.
* **Sherwood number** shows same effect to$ β,$ **K,**$ Kp$**, Sc, Sr,**$ Kr,$$ε$$ λ\_{0}$ andinverse effect to$ λ^{\*}$**,**$ λ$**, Pr,**$ B n $.

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