BANACH CONTRACTION IN ORTHOGONAL RECTANGULAR b - METRIC SPACES

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ABSTRACT

Using the notion of orthogonal sets, we introduce the idea of orthogonal contraction in rectangular b-metric space (RbMS). Furthermore, we prove the Banach contraction principle for the purposed contraction. Our results generalize and improve the results of Gordji et al.[37] and many well-known results given by some authors in RbMS.

Keywords − Fixed points (FP); orthogonal rectangular b-metric space; orthogonal set; metric space (MS).

I. INTRODUCTION

In the past few decades, fixed point theory has been effectively used to investigate a broad range of scientific subjects, bridging pure and practical approaches and even tackling highly relevant computing challenges. In particular, fixed point theory has been created for several applications, such as the study and calculation of integral equation solutions, game theory, physics, engineering, computer science, neural networks, and models in economics and related subjects. Metric FPT depends on the concept of a MS. In mathematical analysis, the most fundamental FP result is the well-known Banach contraction principle (BCP)[1]. One helpful method for demonstrating the existence and uniqueness of solutions for different numerical models is the FP hypothesis. The task of finding a point $x \in \mathscr{Y}$ such that $\phi(x) = x$ is considered a FP problem. Given a nonempty set $\mathscr{Y}$ and a map ϕ from $\mathscr{Y}$ into itself, the point $x \in \mathscr{Y}$ is referred to as a FP of ϕ.

The literature contains numerous generalizations of the idea of a MS. The concept of RMS was given by Branciari [2] and proved to be an equivalent of the BCP in such a space. Many FPT for different contractions on rectangular metric space have now been discovered (see [3]-[14]). Bakhtin [15] established b-MS as a MS generalization and demonstrated the analogue of the BCP in b-MS. Many FPTs are proven in b-MS (see [16]-[34] and the references therein).

The notion of RbMS, which was not always Hausdorff and generalized the ideas of MS, RMS, and b-MS, was first presented by George et al.[35]. He also demonstrated Kannan's and Banach's FPT for RbMS. The notion of orthogonal sets was recently given by Eshaghi Gordji et al.[36], who also provided an extension of the BCP. They also provided applications of their findings to guarantee the uniqueness and existence of solutions to differential equations of the first order. His study aims to extend the notion of an orthogonal contraction in the context of MS, as introduced by Gordji et al. [37]. We presented the notion of an ORbMS and established some FPT for banach contractions. Orthogonal rectangular b-metric space is a generalization of the metric space. In traditional metric spaces, the triangle inequality holds. In a b-metric space, this inequality is modified by a constant $b \geq 1$, allowing for greater flexibility. An orthogonal rectangular b-metric space further adapts this concept, introducing conditions related to orthogonality and possibly different types of geometric constraints that affect the nature of distance and topology in the space.

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II. PRELIMINARIES

Bakhtin [15] and Czerwikas [19] first proposed a b-MS in the following manner.

A. b-Metric Spaces

If $\mathscr{Y} \neq \emptyset$ and $s \geq 1$. Consider $\rho : \mathscr{Y} \times \mathscr{Y} \to [0,\infty)$ fulfill the given conditions $\forall\ \varpi, \varsigma, \rho \in \mathscr{Y}$

(1) $\rho(\varpi,\varsigma) = 0$ iff $\varpi = \varsigma$,
(2) $\rho(\varpi,\varsigma) = \rho(\varsigma,\varpi)$,
(3) $\rho(\varpi,\varsigma) \leq s[\rho(\varpi,\rho) + \rho(\rho,\varsigma)]$.

Then $(\mathscr{Y},\rho)$ is called b-MS with coefficient s.

B. Rectangular Metric Spaces

If $\mathscr{Y} \neq \emptyset$ and $\rho : \mathscr{Y} \times \mathscr{Y} \to [0,\infty)$ fulfills:

(1) $\rho(\varpi,\varsigma) = 0 \iff \varpi = \varsigma\ \ \forall\ \ \varpi,\varsigma \in \mathscr{Y}$;
(2) $\rho(\varpi,\varsigma) = \rho(\varsigma,\varpi)$ for all $\varpi,\varsigma \in \mathscr{Y}$;
(3) $\rho(\varpi,\varsigma) \leq \rho(\varpi,r) + \rho(r,s) + \rho(s,\varsigma)$ for all $\varpi,\varsigma \in \mathscr{Y}$ and all distinct points $r,s \in \mathscr{Y}\ \{\varpi,\varsigma\}$.

Then $(\mathscr{Y},\rho)$ is called a RMS.

C. Rectangular b-Metric Spaces

If $\mathscr{Y} \neq \emptyset$ and $\rho : \mathscr{Y} \times \mathscr{Y} \to [0,\infty)$ satisfies:

(1) $\rho(\varpi,\varsigma) = 0 \iff \varpi = \varsigma\ \forall\ \varpi,\varsigma \in \mathscr{Y}$;
(2) $\rho(\varpi,\varsigma) = \rho(\varsigma,\varpi)$ for all $\varpi,\varsigma \in \mathscr{Y}$;
(3) $\exists\ s \geq 1$ such that $\rho(\varpi,\varsigma) \leq s[\rho(\varpi,p) + \rho(p,q) + \rho(q,\varsigma)]$ for all $\varpi,\varsigma \in \mathscr{Y}$ and all distinct points $p,q \in \mathscr{Y} \setminus \{\varpi,\varsigma\}$.

Then $(\mathscr{Y},\rho)$ is called a RbMS. Gordji et al. [36] presented the notion of the orthogonal set as follows:

D. Orthogonal Set

Consider a set $\mathscr{Y} \neq \phi$ and a binary relation $\perp \subseteq \mathscr{Y} \times \mathscr{Y}$. Then $(\mathscr{Y},\perp)$ referred as an orthogonal set if following criterion is satisfied $\forall\ \varsigma \in \mathscr{Y}\ \exists\ \varpi_0$ such that $(\varsigma \perp \varpi_0)$ or $(\varpi_0 \perp \varsigma)$, where ϖ_0 is orthogonal element.

E. Orthogonal Connected Element

Consider a set $\mathscr{Y} \neq \phi$ and a binary relation $\perp \subseteq \mathscr{Y} \times \mathscr{Y}$. Any two elements from $\mathscr{Y}$ are orthogonally connected if $\varpi,\varsigma \in \mathscr{Y}$ such that $\varpi \perp \varsigma$.

F. Orthogonal Sequence

Consider $\mathscr{Y} \neq \phi$ and $(\mathscr{Y},\perp)$ is O-set then,

(i) a sequence $\{\varpi_m\}$ is known as an orthogonal sequence. if, $\varpi_m \perp \varpi_{m+1}$ or $\varpi_{m+1} \perp \varpi_m, \forall\ m \in \mathbb{N}$;
(ii) similarly, a sequence $\{\varpi_m\}$ is known as Cauchy orthogonal sequence if,

$$\varpi_m \perp \varpi_{m+1} \text{ or } \varpi_{m+1} \perp \varpi_m, \forall\ m \in \mathbb{N};$$

G. Cauchy O-Sequence

Consider that $(\mathscr{Y},\perp,\rho)$ is an O - MS. Then

(i) $(\mathscr{Y},\perp,\rho)$ is complete O-MS if every Cauchy O-sequence is converges in $\mathscr{Y}$.
(ii) And completeness of metric space imply O-completeness but inverse isn't really true.

H. O-Continuous

Consider $(\mathscr{Y},\perp,\rho)$ be an O-MS. Then

(i) a mapping $T : \mathscr{Y} \to \mathscr{Y}$ is known as O-continuous if for each O- sequence $\{\varpi_m\}_{m\in\mathbb{N}} \to \varpi \Rightarrow T(\varpi_m) \to T(\varpi)$ as $m \to \infty$.
(ii) O-continuity is relatively weak than classical continuity in classical metric spaces.

I. Orthogonal Preserving

Consider $\mathscr{Y} \neq \phi$ and a pair $(\mathscr{Y},\perp)$ be an O-set. Any mapping $T : \mathscr{Y} \to \mathscr{Y}$ is weakly $\perp$-preserving if $T(\varpi) \perp T(\varsigma)$ or $T(\varsigma) \perp T(\varpi)$ whenever $\varpi \perp \varsigma$ and $\perp$-preserving if $T(\varpi) \perp T(\varsigma)$ whenever $\varpi \perp \varsigma$

III. MAIN RESULTS

A. Fixed Point Theorem for Banach Contraction in RbMS

Consider $(\mathscr{Y},\rho,\perp)$ be a O-complete RbMS with coefficient $s \geq 1$ and suppose that $T:\mathscr{Y}\to\mathscr{Y}$ be $\perp$-continuous and $\perp$-preserving satisfying :

(1) $$\rho(T\varpi,T\varsigma) \leq \alpha\rho(\varpi,\varsigma)$$

for all $\varpi,\varsigma \in \mathscr{Y}$, where α are nonnegative constants with $\alpha < 1$. Then T has a unique FP.

Proof. Let $\varpi_0 \in \mathscr{Y}$ be an orthogonal element in $\mathscr{Y}$, then by definition

$$(\forall\ \varsigma \in \mathscr{Y},\varsigma \perp \varpi_0) \text{ or } (\forall\ \varsigma \in \mathscr{Y},\varpi_0 \perp \varsigma).$$

It follow that $(\varpi_0 \perp T(\varpi_0))$ or $(T(\varpi_0) \perp \varpi_0)$. Let

$$\varpi_1 = T(\varpi_0), \quad \varpi_2 = T(\varpi_1) = T^2(\varpi_0), \quad \varpi_{v+1} = T(\varpi_v) = T^{v+1}(\varpi_0), \quad \forall\ v \in \mathbb{N}.$$

Since T is $\perp$- preserving, $\{\varpi_v\}$ is an O-sequence.
Setting $\rho_v = \rho(\varpi_v,\varpi_{v+1})$. From (1), it follows that

$$\rho(\varpi_v,\varpi_{v+1}) = \rho(T\varpi_{v-1},T\varpi_v) \leq \alpha\rho(\varpi_{v-1},\varpi_v)$$

i.e.

$$\rho(\varpi_v,\varpi_{v+1}) \leq \alpha\rho(\varpi_{v-1},\varpi_v)$$

$$\rho_v \leq \alpha\rho_{v-1}.$$

By going through this process again, we get

(2) $$\rho_v \leq \alpha^v\rho_0.$$

Suppose that ϖ_0 is not a periodic point of T. If $\varpi_0 = \varpi_v$, then for any $v \geq 2$,

$$\begin{aligned}
\rho(\varpi_0,T\varpi_0) &= \rho(\varpi_v,T\varpi_v)\\
\rho(\varpi_0,\varpi_1) &= \rho(\varpi_v,\varpi_{v+1})\\
\rho_0 &= \rho_v\\
\rho_0 &\leq \alpha^v\rho_0,
\end{aligned}$$

a contradiction. Therefore, $\rho_0 = 0$ i.e., $\varpi_0 = \varpi_1$.
$\Rightarrow \varpi_0$ is a FP of T. Assume that $\varpi_v \neq \varpi_u\ \forall$ distinct $u,v \in \mathbb{N}$. Again put $\rho(\varpi_v,\varpi_{v+2}) = \rho_v^*$.
From (1) for any $v \in \mathbb{N}$, we get

$$\rho(\varpi_v,\varpi_{v+2}) = \rho(T\varpi_{v-1},T\varpi_{v+1}) \leq \alpha\rho(\varpi_{v-1},\varpi_{v+1})$$

$$\rho_v^* \leq \rho_{v-1}^*.$$

By going through this process again, we get

(3) $$\rho(\varpi_v,\varpi_{v+2}) \leq \alpha^v\rho_0^*.$$

For the sequence ϖ_v we consider $\rho(\varpi_v,\varpi_{v+w})$ in two cases. If w is odd say $2u+1$ then using (2) we obtain

$$\begin{aligned}
\rho(\varpi_v,\varpi_{v+2u+1}) &\leq s[\rho(\varpi_v,\varpi_{v+1}) + \rho(\varpi_{v+1},\varpi_{v+2}) + \rho(\varpi_{v+2},\varpi_{v+2u+1})]\\
&\leq s[\rho_v + \rho_{v+1}] + s^2[\rho(\varpi_{v+2},\varpi_{v+3}) + \rho(\varpi_{v+3},\varpi_{v+4}) + \rho(\varpi_{v+4},\varpi_{v+2u+1})]\\
&\leq s[\rho_v + \rho_{v+1}] + s^2[\rho_{v+2} + \rho_{v+3}] + s^3[\rho_{v+4} + \rho_{v+5}] + \ldots + s^u\rho_{v+2u}\\
&\leq s[\alpha^v\rho_0 + \alpha^{v+1}\rho_0] + s^2[\alpha^{v+2}\rho_0 + \alpha^{v+3}\rho_0] + s^3[\alpha^{v+4}\rho_0 + \alpha^{v+5}\rho_0] + \ldots + s^u\alpha^{v+2u}\rho_0\\
&\leq s\alpha^v[1 + s\alpha^2 + s^2\alpha^4 + \ldots]\rho_0 + s\alpha^{v+1}[1 + s\alpha^2 + s^2\alpha^4 + \ldots]\rho_0\\
&\leq \frac{1+\alpha}{1-s\alpha^2}s\alpha^v\rho_0 \qquad (s\alpha^2 < 1).
\end{aligned}$$

Therefore,

$$\rho(\varpi_v, \varpi_{v+2u+1}) \leq \frac{1+\alpha}{1-s\alpha^2} s\alpha^v \rho_0. \tag{4}$$

If w is even say 2u then using (2) and (3) we obtain

$$\begin{aligned}
\rho(\varpi_v, \varpi_{v+2u}) &\leq s[\rho(\varpi_v, \varpi_{v+1}) + \rho(\varpi_{v+1}, \varpi_{v+2}) + \rho(\varpi_{v+2}\varpi_{v+2u})] \\
&\leq s[\rho_v + d_{v+1}] + s^2[\rho(\varpi_{v+2}, \varpi_{v+3}) + \rho(\varpi_{v+3}, \varpi_{v+4}) + \rho(\varpi_{v+4}, \varpi_{v+2u})] \\
&\leq s[\rho_v + \rho_{v+1}] + s^2[\rho_{v+2}, +\rho_{v+3}] + s^3[\rho_{v+4}, +\rho_{v+5}] \\
&+ \ldots + s^{u-1}[\rho_{2u-4} + \rho_{2u-3}] + s^{u-1}\rho(\varpi_{v+2u-2}, \varpi_{v+2u}) \\
&\leq s[\alpha^v\rho_0 + \alpha^{v+1}\rho_0] + s^2[\alpha^{v+2}\rho_0 + \alpha^{v+3}\rho_0] + s^3[\alpha^{v+4}\rho_0 + \alpha^{v+5}\rho_0] \\
&+ \ldots + s^{u-1}[\alpha^{2u-4}\rho_0 + \alpha^{2u-3}\rho_0] + s^{u-1}\alpha^{v+2u-2}\rho_0 \\
&\leq s\alpha^v[1 + s\alpha^2 + s^2\alpha^4 + \ldots]\rho_0 + s\alpha^{v+1}[1 + s\alpha^2 + s^2\alpha^4 + \ldots]\rho_0 \\
&+ s^{u-1}\alpha^{v+2u-2}\rho_0^*,
\end{aligned}$$

i.e.

$$\begin{aligned}
\rho(\varpi_v, \varpi_{v+2u}) &\leq \frac{1+\alpha}{1-s\alpha^2} s\alpha^v\rho_0 + s^{u-1}\alpha^{v+2u-2}\rho_0^* \\
&< \frac{1+\alpha}{1-s\alpha^2} s\alpha^v\rho_0 + (s\alpha)^{2u}\alpha^{v-2}\rho_0^* \\
&\leq \frac{1+\alpha}{1-s\alpha^2} s\alpha^v\rho_0 + \alpha^{v-2}\rho_0^*.
\end{aligned}$$

Therefore,

$$\rho(\varpi_v, \varpi_{v+2u}) \leq \frac{1+\alpha}{1-s\alpha^2} s\alpha^v\rho_0 + \alpha^{v-2}\rho_0^*, \tag{5}$$

from (4) and (5) that

$$\lim_{v\to\infty} \rho(\varpi_v, \varpi_{v+w}) = 0 \; \forall \; w > 0. \tag{6}$$

Thus ϖ_v is a Cauchy sequence in $\mathscr{Y}$. By completeness of $(\mathscr{Y}, \rho)$

$$\lim_{v\to\infty} \varpi_v = \varpi_v{}^*. \tag{7}$$

We shall show that $\varpi_v{}^*$ is a fixed point of T. Again, for any $v \in \mathbb{N}$ we have

$$\begin{aligned}
\rho(\varpi_v{}^*, T\varpi_v{}^*) &\leq s[\rho(\varpi_v{}^*, \varpi_v) + \rho(\varpi_v, \varpi_{v+1}) + \rho(\varpi_{v+1}, T\varpi_v{}^*)] \\
&= s[\rho(\varpi_v{}^*, \varpi_v) + \rho_v + \rho(T\varpi_v, T\varpi_v{}^*)] \\
&\leq s[\rho(\varpi_v{}^*, \varpi_v) + \rho_v + \alpha\rho(\varpi_v, \varpi_v{}^*)].
\end{aligned} \tag{8}$$

Using (6) and (7) it follows from above inequality that $\rho(\varpi_v{}^*, T\varpi_v{}^*) = 0$ i.e., $T\varpi_v{}^* = \varpi_v{}^*$. Thus $\varpi_v{}^*$ is a fixed point of T. For uniqueness, let $\varsigma^* \in \mathscr{Y}$ be another FP of T. So we obtain $T^v\varpi^* = \varpi^*$ and $T^v\varsigma^* = \varsigma^* \; \forall \; v \in \mathbb{N}$. By the definition of orthogonality, $\exists \; \varpi_0 \in \mathscr{Y}$ so that

$$[\varpi_0 \perp \varpi^* \text{and } \varpi_0 \perp \varsigma^*]$$

or

$$[\varpi^* \perp \varpi_0 \text{ and } \varsigma^* \perp \varpi_0].$$

Since T is $\perp$ - preserving, we have

$$[T^v\varpi_0 \perp T^v\varpi^* \text{ and } T^v\varpi_0 \perp T^v\varsigma^*]$$

or

$$[T^v\varpi^* \perp T^v\varpi_0 \text{ and } T^v\varsigma^* \perp T^v\varpi_0].$$

$\forall \; v \in \mathbb{N}$. Then we obtain

$$\rho(\varpi^*, \varsigma^*) = \rho(T^v\varpi^*, T^v\varsigma^*) \leq \alpha\rho(\varpi^*, \varsigma^*)$$

$$< \rho(\varpi^*, \varsigma^*),$$

a contradiction. Therefore, $\rho(\varpi^*, \varsigma^*) = 0, i.e., \varpi^* = \varsigma^*$. Thus FP is unique.

B. Example

Consider $\mathscr{Y} = [0, \infty)$ and orthogonal relation $\perp$ defined on $\mathscr{Y}$ by $\varpi \perp \varsigma \iff \varpi\varsigma \leq \varpi$, i.e., $\varpi = 0$ or $\varsigma \leq 1$. Let $\rho : \mathscr{Y} \times \mathscr{Y} \to [0, \infty)$ be defined by $\rho(\varpi, \varsigma) = |\varpi - \varsigma|^2$, then ρ is a RbMS with $s = 2$. It is easy to see that $(\mathscr{Y}, \perp, \rho)$ is O-complete ORbMS.

Define a mapping $T : \mathscr{Y} \to \mathscr{Y}$ by

$$T\varpi = \begin{cases} \frac{\varpi}{3}, \ 0 \leq \varpi \leq 3 \\ 0, \ 3 < \varpi \leq 12. \end{cases}$$

It is easy to check T is an OP and OC selfmap on $\mathscr{Y}$ and $|T\varpi - T\varsigma|^2 \leq \frac{1}{9}|\varpi - \varsigma|^2 \ \forall \ \varpi, \varsigma \in \mathscr{Y}$.
So, T satisfy all the condition of above theorem, then T has a unique FP.

C. Example

Let $\mathscr{Y} = A \cup B$, where $A = \left\{ \frac{1}{n} : n \in \{2,3,4,5\} \right\}$ and $B = [1,2]$. and orthogonal relation $\perp$ defined on $\mathscr{Y}$ by $\varpi \perp \varsigma \iff \varpi\varsigma \leq \varpi$, i.e., $\varpi = 0$ or $\varsigma \leq 1$. Let $\rho : \mathscr{Y} \times \mathscr{Y} \to [0, \infty)$ such that $\rho(\varpi, \varsigma) = \rho(\varsigma, \varpi)$ for all $\varpi, \varsigma \in \mathscr{Y}$ and

$$\rho(\frac{1}{2}, \frac{1}{3}) = \rho(\frac{1}{4}, \frac{1}{5}) = 0.03; \rho(\frac{1}{2}, \frac{1}{5}) = \rho(\frac{1}{3}, \frac{1}{4}) = 0.02;$$
$$\rho(\frac{1}{2}, \frac{1}{4}) = \rho(\frac{1}{5}, \frac{1}{3}) = 0.06; \rho(\varpi, \varsigma) = |\varpi - \varsigma|^2 \text{ otherwise.}$$

Then ρ is a RbMS with $s = 3$. It is easy to see that $(\mathscr{Y}, \perp, \rho)$ is O-complete ORbMS.
Define a mapping $T : \mathscr{Y} \to \mathscr{Y}$ by

$$T\varpi = \begin{cases} \frac{1}{4}, \ \varpi \in A \\ \frac{1}{5}, \ \varsigma \in B. \end{cases}$$

It is easy to check T is an OP and OC selfmap on $\mathscr{Y}$. So T satisfy all the condition of above theorem, then T has a unique FP ($\varpi = \frac{1}{4}$).

IV. CONCLUSION

The development of the field of fixed point theory depends on the generalization of the Banach contraction principle on complete metric spaces. This generalization comes up by either introducing new types of contractions or by working on a more general space, such as rectangular b-metric spaces. Research on the fixed points of mappings that satisfy orthogonal sets has been intensively pursued in various studies over the last decade, yielding numerous findings. This paper introduces a novel generalized orthogonal contractive condition in rectangular b-metric spaces. Additionally, we provide examples that illustrate the effectiveness and practical applicability of the results presented. From our primary results, we may also obtain certain fixed-point results for mappings meeting an orthogonal contractive condition in rectangular b-metric spaces. These results improve and generalize the main results of Gordji et al.[37]. The ability to use the Banach contraction principle in these spaces could enhance modeling capabilities for systems where distances or interactions are not accurately described by classical metric space assumptions. Extending fixed point theorems like Banach's to orthogonal rectangular b-metric spaces broadens the applicability of mathematical and computational techniques to more complex and possibly more realistic models, especially in fields like computer science, engineering, and physics.

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