**Late Time Cosmic Acceleration in Modified Gravity Models**

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**Abstract:** Some mysterious form of the energy, known as dark energy, is responsible for this cosmic acceleration. This chapter discusses thoroughly different cosmological models which can describe the observed evolution of the universe along with their detailed comparative features. These models include different modified gravity models like *f*(*R*) gravity, scalar-tensor theories, Gauss-Bonnet gravity and Braneworld models. At the local scales (galactic scale), we do not need any modification in general theory of relativity. So, we consider the mechanisms in which the effect of modification vanishes.

**1. Introduction**

There exists an uncertainty among the cosmological models that may consistently describe the observed evolution of the universe. Over the past century, the observational knowledge about the universe has been mainly acquired through electromagnetic waves (from γ-rays to radio waves), as signals carrying messages about the sources and the inter-galactic medium to our earth-based or space-based detectors of electromagnetic waves. Cosmic Microwave Background Radiation (CMBR) anisotropies gave us information about the early universe when it was about 3,80,000 years old [1]. Interestingly, another revolution came with the Gravitational Waves (GWs) [2, 3], discovered in 2015, and we got excited to think about using their characteristics to be able to fix this indeterminacy among cosmological models. Since GWs propagate freely in the early universe immediately after they are generated, it carries information about the processes that produced them. GWs provide information about the state of the universe at epochs and energy scales unreachable by any other means. At present, the Lambda Cold Dark Matter (ΛCDM) model, also treated as the standard model of the universe, is the widely accepted model largely explaining the evolution of the universe. Of course, while the cosmological constant Lambda, once put by Einstein himself into his equations of General Relativity (GR) for some historical reasons, is the simplest candidate of the *dark energy* [4] to explain the present accelerated expansion of the universe, its large value (in comparison to the energy density of the dark energy) is a serious problem, among several others. Therefore, scientists, attempted to explore the alternative theories of gravity, especially *f*(*R*) theories, in contrast to the so called standard approach to gravitation based on GR. The ongoing fervent research activity across the world in this field, There are several observational evidences which indicate that the universe is in the phase of accelerated expansion currently [5-7]. Some mysterious form of the energy, known as dark energy, is responsible for this cosmic acceleration. From the observation of the galaxy rotation curves and other observations, it is found that there exists some unknown form of the matter in the galaxies [8-12]. It is known as the *dark matter*. There are several authors, who studied the dark matter problem in alternative gravity theories in references [13-16]. We have many approaches to explain the nature of dark energy. These approaches are broadly classified into two groups (i) modified matter models (ii) modified gravity models. In both groups of dark energy models, we modify the Einstein’s General Theory of Relativity (GTR). The Lagrangian of the Einstein-Hilbert (E-H) action in GTR is modified in two different ways. In modified gravity models, the gravitational part of the E-H action is changed. A new form the matter component is added in the matter part of the action in case of modified matter models [17-23]. Quintessence, k-essence and phantom dark energy models are different modified matter models. In this chapter we will describe the modified gravity models in order to explain the cosmic acceleration.

**2. Modified gravity models**

The laws of gravity are modified in these models. The quantization of the gravity is studied in modified gravity models. A lot of research in these models has been done in order to explain the inflation and present accelerated expansion of the universe. In modified gravity models, the geometrical part of the E-H action is modified in different ways. There are many gravity models in this group like *f*(*R*) gravity, scalar-tensor theories, Braneworld models, Gauss-Bonnet gravity model and Horndeski model etc [24-35]. In this chapter, we will discuss the different modified gravity models briefly. We do not have a cosmologically viable *f*(*R*) model till now. Therefore, we want to constrain these models by using the direct detection of GWs. It will improve in our present understanding of the laws of gravity. GWs propagate freely in the early universe immediately after they are generated. This means that GWs carry unique information about the processes that produced them, and therefore about the state of the universe at epochs and energy scales unreachable by any other means. GWs can probe the the energy scales for beyond the reach of presently available observational probes of the universe based on electromagnetic emission. Since GWs are detected recently in 2015, cosmological theories are not so much probed using GWs as a tool [36-39]. Unified gravity models are used to explain the dark energy and dark matter problems by a single scalar field [40, 41].

The equation of motion of the scalar field is similar to that of the damped harmonic oscillator [42-43]. The solution of the equation of motion of the scalar field is obtained in action angle formalism [44-48]. It is found that the mass of the scalar field particle known as ``scalaron” depends upon the energy density of the non-relativistic matter in the background. From this analysis we found that scalar field may behave as the non-relativistic matter, radiation and dark energy also. Further, the conformal transformation from the Jordan frame to the Einstein frame provides a scenario of two (apparently distinct) formulations of f(R) gravity theory. It is still not clear whether these two formulations are fundamentally equivalent or not [49], even though their mathematical equivalence at the level of classical action is more convincing than the physical one [50, 51]. Authors have addressed the problem of mathematical equivalence at quantum scales [52]. Anyways, as it remains today, this equivalence is a controversial topic that may show a path to deeper physics.

**2.1 *f*(*R*) gravity models**

These are the simplest modified gravity models in which the Ricci scalar (*R*) is replaced by a function of *R*. The 4-dimensional action of the *f*(*R*) gravity models is given as

 (1)

where ,  is the action of the matter fields. *f*(*R*) is a general function of the Ricci scalar *R*, g is the determinant of the metric tensor. There are two methods to derive the field equations from the action (1),

(i) metric formalism

(ii) Palatini formalism.

In metric formalism, we consider the usual relation between the spacetime metric  and Christoffel symbol. Taking the variation of the action (1) w.r.t. the metric  gives

 (2)

where *F*(*R*) is the first derivative of *f*(*R*) w.r.t. Ricci Scalar *R*,  is covariant derivative and  is the energy-momentum tensor of the matter. The trace of the field equation (2) is obtained as

 (3)

where *T* is the trace of the energy momentum tensor.

For the dynamics of the *f*(*R*) gravity models, we assume that the universe is dynamic, homogeneous and isotropic. Then the line element which describe the homogeneous and isotropic spacetime is given by the Friedmann-Lemaitre-Robertson-Walker (FLRW) spacetime. It is given by



where *a*(*t*) is the cosmic time t dependent scale factor.

is defined as



where *K* is the spatial curvature having values 1, 0, +1 for open, flat and closed geometries, respectively. describes the 3-dimensional space in spherical polar coordinates, with their usual symbols. Here, we consider the spatially flat FLRW spacetime in which the Ricci scalar *R* is



where *H* is the Hubble parameter and  is the time derivative of *H*. For equations (2) and (3), by using components of flat FLRW spacetime metric, we get



and 

Here  and  are the energy density of the matter and radiation, respectively. An overdot represents the time derivative. We have discussed about a general *f*(*R*) gravity model till now.

In the Palatini approach, we consider  and christoffel symbols as independent quantities. The field equations in this formalism is given as

 (4)

Ricci tensor in Palatini formalism is different than that in the metric formalism. The trace of the field equation (3) is given as

 (5)

In the metric formalism we have  which is not present in the trace equation of the Palatini formalism.

***2.1.1 Conformal transformation and scalar field in f(R) gravity models***

We consider that the *F*(*R*) is equivalent to a scalar field and the trace of the field equation represents the motion of the scalar field. This scalar field is defined in the conformal transformation of the space-time metric in the Jordan frame to that of the Einstein frame in *f*(*R*) gravity. The conformal transformation of the space-time metric from Jordan frame to the Einstein frame is given as

 (6)

where  is the space-time metric in the Einstein frame and  is that in the Jordan frame (physical frame).  is the conformal factor. Here we consider the quantities in Einstein frame with a tilde symbol. The Ricci scalar in both frames are related as

 (7)

where  (8)

represents the partial derivative.

 (9)

where 

The action (8) in the Einstein frame is given as

 (10)

where  is the determinant of the space-time metric and other symbols have their usual meanings. If we consider the conformal factor  as

 (11)

and a scalar field c defined as

 (12)

Then the action (10) becomes

 (13)

where 

This scalar field is a result of the modification in the gravitational part of the Einstein’s general relativistic theory of gravity. We obtain the field equation of the scalar field on taking the variation of the action (13).

The variation of the action  w.r.t. scalar field gives

 (14)

where  and .  is the D’Alembertian operator in the Einstein frame.

Equation (14) represents the equation of motion of the scalar field. We can rewrite the equation (14) as

 (15)

where 

Further analysis can be done by specifying the form of *f*(*R*). The mass of the scalar field particle *scalaron* has been calculated for

 (16)

type models [53]. Here  is a constant and  is a model parameter having constant values. For , the effective potential of the scalar field and the mass of the scalar field particle *scalaron* was calculated. It is found that the mass of the scalaron depends upon the energy density of the non-relativistic matter in the background.

Figure 1 shows the variation of scalaron mass () with model parameter () with the energy density of matter (*ρ*) at the galactic scale = 4 10-42 (GeV)4.

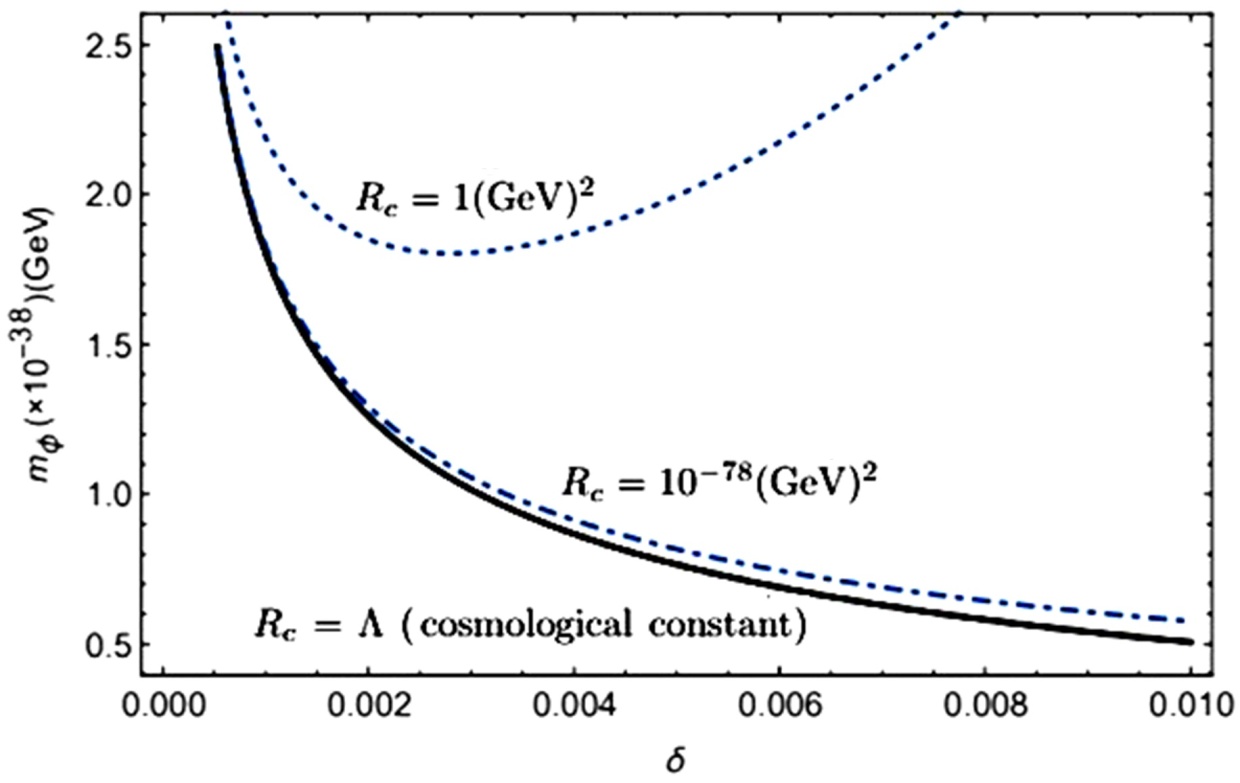


Figure 1. Variation of scalaron mass () with model parameter () with the energy density of matter (*ρ*) at the galactic scale = 4 10-42 (GeV)4 [53].

Figure 2 shows the variation of scalaron mass () with the energy density of matter (*ρ*) for *Rc* = Λ.

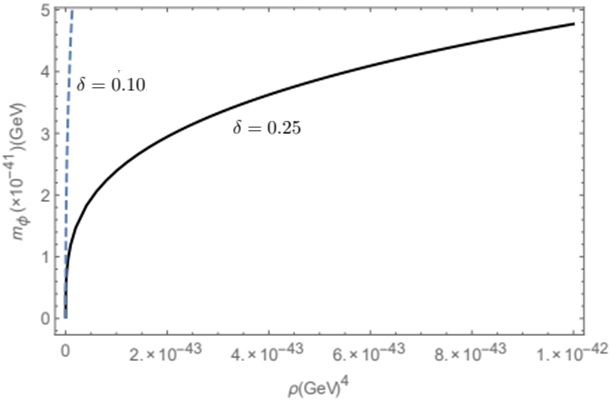


Figure 2. Variation of scalaron mass () with the energy density of matter (*ρ*) for *Rc* = Λ [53].

Figure 3 illustrates the variation of scalaron mass () with the energy density of matter (*ρ*).

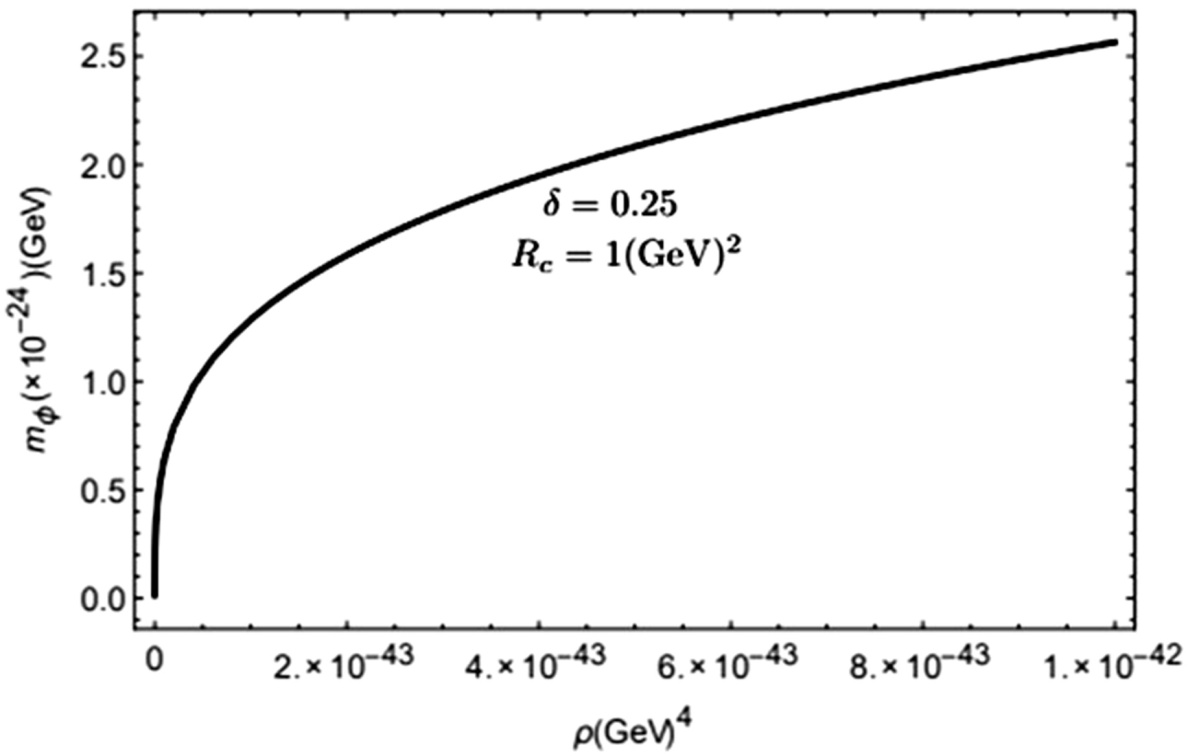


Figure 3. Variation of scalaron mass () with the energy density of matter (*ρ*) [53].

Figure 4 gives the variation of the equation of state (*w*) with the energy density ()of scalar field. It is found that *w* is zero at and becomes positive at higher densities.

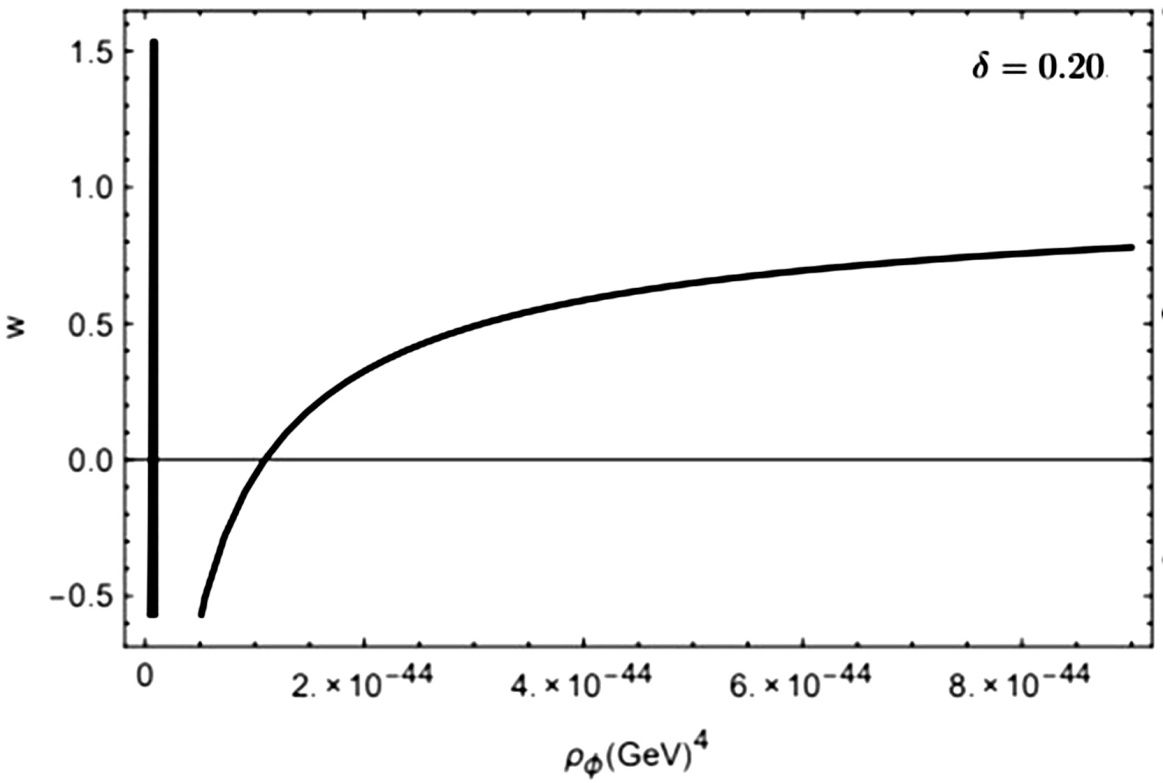


Figure 4. Variation of the equation of state (*w*) with the energy density ()of scalar field [53].

The equation of state (*w*) of the scalar field for a power-law model of *f*(*R*) is also derived and established that it varies with the energy density of non-relativistic matter. From this analysis we found that scalar field may behave as the non-relativistic matter, radiation and dark energy also.

**2.2 Scalar-Tensor theories**

The action of the scalar-tensor theories is

 (17)

where  is the scalar field, is a function of the scalar field  and  is the action of the matter fields. *f* is a function of the scalar field  and the Ricci scalar *R*. Here, we have chosen units for which. Now we consider the conformal transformation of the space-time metric from Jordan frame to the Einstein frame as in the case of *f*(*R*) gravity. The conformal factor is given as

 (18)

Let us consider the model

 (19)

Under the conformal transformation, action (17) of scalar tensor theories becomes

 (20)

in the Einstein frame. Here we have defined a new scalar field  given as

 (21)

The potential of the new scalar field is given by

 (22)

**2.3 Gauss Bonnet Dark energy models**

We have taken a general function of the Ricci scalar *R* in the Lagrangian density of the *f*(*R*) gravity models. In scalar-tensor theories we consider a function of the Ricci scalar and a scalar field. It is also possible to choose a combination of Ricci tensors and Riemann tensors in the Lagrangian. In these models a Gauss-Bonnet term is coupled with the scalar field. The action of Gauss Bonnet models is given by

 (23)

where

 (24)

where  is the Ricci tensor and is the Riemann tensor.

**2.4 Braneworld models**

In braneworld models of dark energy, we consider that the particles are in three dimensional brane, which is embedded in a five dimensional bulk space-time.

 (25)

where  is the spacetime metric in the 5D bulk, is the metric in brane. Other symbols

 (26)

where  and  are Planck masses in five and four dimensions respectively.

**3. Comparison of the different modified gravity models**

Table 1 gives a comparative study among the different modified gravity models by focusing on their main characteristic features..

**Table 1.** Comparison of the different modified gravity models

|  |  |  |  |
| --- | --- | --- | --- |
| **Sl.**  **No.** | **Modified gravity models** | **Features of the different models** | **References** |
| 1. | *f*(*R*) gravity | 4-dimensional gravity theory, Lagrangian is a function of the Ricci scalar *R* only | *f*(*R*) gravity models are studied by authors of references [24-27]. |
| 2. | Scalar-tensor theories | Lagrangian is a function of scalar field and Ricci scalar | These theories are given in references [28, 29] |
| 3. | Gauss-Bonnet gravity models | Lagrangian is a function of Ricci scalar, Ricci tensor, Riemann tensor and scalar field | Different Gauss-Bonnet models are discussed in references [30, 31]. |
| 4. | Braneworld models | Lagrangian is a function of the Ricci Scalar and extra dimensions are considered. | These models are studied in the references [32, 33]. |

**4. Conclusions**

We have studied the modified gravity models like *f*(*R*) gravity, scalar-tensor theories, Gauss-Bonnet gravity and Braneworld models. In modified gravity theories dark energy and dark matter problems are explained by modifying the E-H action of gravity. To study the gravity at quantum scales, we modify the E-H action and replace the Ricci scalar by a function of Ricci scalar *R*. We do not have a suitable *f*(*R*) gravity model to explain the whole cosmic evolution. Similarly other modified gravity models faces different problems. Scalar-tensor theories are more general theories of gravity. In these models gravity is mediated by scalar field and metric tensor. Brans-Dicke theory is an example of these theories. *f*(*R*) gravity is a case of scalar-tensor theories in which  and . At the local scales (galactic scale), we do not need any modification in GTR. So, we consider the mechanisms in which the effect of modification vanishes. In chameleon mechanism, the mass of the scalar field depends upon the energy density of the normal matter. In high density regions, the mass of the scalar field increases and the motion of the scalar field seize and effect of the modification is weared off. Similarly, there are other screening mechanisms, which are used to vanish the modification of gravity. In Braneworld models, large extra dimensions are responsible for the late time cosmic acceleration. A scalar field, Ricci tensor, Ricci scalar and Riemann tensor is considered in the Gauss-Bonnet model.

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