Stability of System of Functional Equations From a Corona Model

M. Arunkumar
Department of Mathematics
Kalaignar Karunanidhi Government Arts College,
Tiruvanamalai – 606 603, Tamil Nadu, India.
e-mail: drarun4maths@gmail.com

E. Sathya
Department of Mathematics
Kalaignar Karunanidhi Government Arts College,
Tiruvanamalai – 606 603, Tamil Nadu, India.
e-mail: sathy24mathematics@gmail.com

T. Velmurugan
Department of Mathematics
MRK College of Arts and Science,
Pazhanchanallur, Kattumannarkoil - 608 301,
TamilNadu, India.
e-mail: smmuruganvel@gmail.com

V. Alexpandiyan
Department of Mathematics
Kalaignar Karunanidhi Government Arts College,
Tiruvanamalai – 606 603, Tamil Nadu, India.
e-mail: valexpandiyan98@gmail.com

ABSTRACT

In this paper, the authors establish the generalized Ulam – Hyers stability of system of additive functional equations from a corona model in Banach space using Hyers Method. Also, we compare the results with mathematical calculations and stability analysis.

Keywords—Additive functional equatiom, Ulam – Hyers stability Banach space, Hyers method, Corona Model.

I. INTRODUCTION

In 1940 and in 1968 Ulam [34-35] proposed the general Ulam stability problem:

"When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?"

In 1941 Hyers [22] answered this problem for linear mappings. In 1951 Aoki [3] and Bourgin [14] were the authors to treat the Ulam problem for additive mappings. In 1978, according to Gruber [21], this kind of stability problems is of particular interest in probability theory and in the case of functional equations of different types. In 1978 Rassias [32] employed Hyers' ideas to new linear mappings. In 1987 Gajda and Ger [18] showed that one can get similar stability results for sub additive multi functions.

Other interesting stability results have been completed also by the following authors Aczél [1-2], Borelli and Forti [13], Cholewa [15], Czerwik [16] and Kannappan [26]. In 1982–1989 Rassias [28-29, 31] solved the above Ulam problem for different mappings. In 1999 Gavruta [18,20] answered a question of Rassias concerning the stability of the Cauchy equation. In 1983 Skof [33] was the first author to solve the Ulam problem for additive mappings on a restricted domain.

The famous Cauchy additive functional equation is

$$C\left(v_1+v_2\right)=C\left(v_1\right)+C\left(v_2\right). \tag{1.1}$$

Its stability in various settings were inspected in [3,20,22,28.31,32]. Several other types of additive functional equations in various normed spaces were discussed by Aczel, Dhombres [2], Arunkumar [4-11]. Balamurugan [12], Hyers [23-24], Jung [25], Kannappan [26]. Lee [27] and Rassias [30].

Coronaviruses are a group of related RNA viruses that cause diseases in mammals and birds. In humans and birds, they cause respiratory tract infections that can range from mild to lethal. Mild illnesses in humans include some cases of the common cold (which is also caused by other viruses, predominantly rhinoviruses), while more lethal varieties can cause SARS, MERS and COVID-19, which is causing the ongoing pandemic see [36].

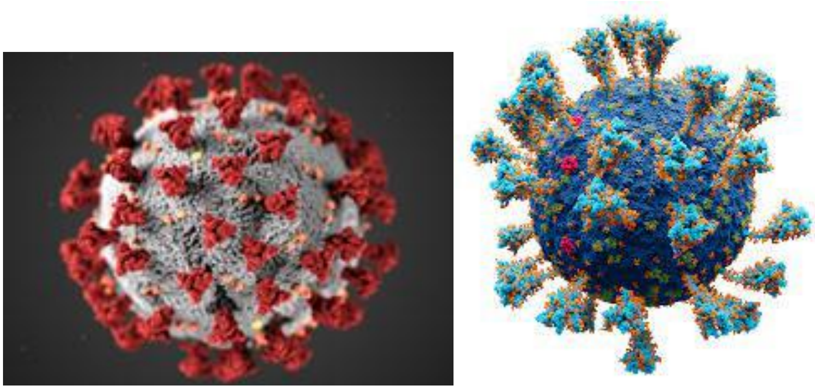

Figure : 1.1 Corona Virus

In a town or city or village, we have the following assumptions:

- v_0 denotes total number of persons;
- v_1 denotes number of susceptible persons;
- v_2 denotes number of infected persons;
- v_3 denotes number of recovered persons;
- v_4 denotes number of death persons

respectively.

Figure :1.2 v_0 ***: Total Persons***

Figure : 1.3 v_1 ***: Susceptible Persons***

Figure :1.4 v_2 **: Infected Persons**

Figure : 1.5 v_3 **: Recovered Persons**

Figure : 1.6 v_4 **: Death Persons**

With respect to this data, we have the additive functional equations of the forms

$$C_1\left(v_{10}+v_{11}+v_{12}+v_{13}+v_{14}\right)=C_1\left(v_{10}\right)+C_1\left(v_{11}\right)+C_1\left(v_{12}\right)+C_1\left(v_{13}\right)+C_1\left(v_{14}\right) \tag{1.2}$$

$$C_2\left(v_{20}+v_{21}+v_{22}+v_{23}-v_{24}\right)=C_2\left(v_{20}\right)+C_2\left(v_{21}\right)+C_2\left(v_{22}\right)+C_2\left(v_{23}\right)-C_2\left(v_{24}\right) \tag{1.3}$$

$$C_3\left(v_{30}+v_{31}+v_{32}-v_{33}+v_{34}\right)=C_3\left(v_{30}\right)+C_3\left(v_{31}\right)+C_3\left(v_{32}\right)-C_3\left(v_{33}\right)+C_3\left(v_{34}\right) \tag{1.4}$$

$$C_4\left(v_{40}-v_{41}+v_{42}+v_{43}-v_{44}\right)=C_4\left(v_{40}\right)-C_4\left(v_{41}\right)+C_4\left(v_{42}\right)+C_4\left(v_{43}\right)-C_4\left(v_{44}\right) \tag{1.5}$$

Let us assume that **+ (PLUS) denotes Yes** and **- (MINUS) denotes No** (for this case we take 0 (ZERO)).

In this paper, the authors establish the general solution in vector space and generalized Ulam – Hyers stability of system of additive functional equations (1.2), (1.3), (1.4), (1.5) in Banach space using Hyers Method. Also, we compare the results with mathematical calculations and stability analysis.

II GENERAL SOLUTION

In this subdivision, we confer about the general solution of functional equation (1.1) (1.2), (1.3), (1.4), (1.5), by considering V_1 and V_2 as real vector spaces.

Theorem 2.1:

I) If $C:V_1\to V_2$ satisfying (1.1) then $C_1:V_1\to V_2$ satisfying (1.2);

II) If $C_1:V_1\to V_2$ satisfying (1.2) then $C_2:V_1\to V_2$ satisfying (1.3);

III) If $C_2:V_1\to V_2$ satisfying (1.3) then $C_3:V_1\to V_2$ satisfying (1.4);

IV) If $C_3:V_1\to V_2$ satisfying (1.4) then $C_4:V_1\to V_2$ satisfying (1.5);

V) If $C_4:V_1\to V_2$ satisfying (1.5) then $C:V_1\to V_2$ satisfying (1.1);

for all $v_1,v_2,v_{i0},v_{i1},v_{i2},v_{i3},v_{i4}\in V_1$ where $i=1,2,3,4$ then all the functional equations are equivalent.

Proof:

Suppose $C:V_1\to V_2$ satisfies the functional equation (1.1). Changing $\left(v_1,v_2\right)$ as $\left(0,0\right),\left(v,v\right),\left(v,2v\right)$, $\left(-v,v\right)$ in (1.1) and for any $m>0$, we have

$$C\left(0\right)=0;\ C\left(2v\right)=2C\left(v\right);\ C\left(3v\right)=3C\left(v\right);\ C\left(-v\right)=-C\left(v\right);\ C\left(mv\right)=mC\left(v\right) \tag{2.1}$$

for all $v\in V_1$. Taking $\left(v_1,v_2\right)=\left(v_{10}+v_{11},\ v_{12}+v_{13}+v_{14}\right)$ in (1.1), and using (1.1) in the resulting equation with $C=C_1$, we arrive (1.2) for all $v_{10},v_{11},v_{12},v_{13},v_{14}\in V_1$.. So, I) holds.

Suppose $C_1:V_1\to V_2$ satisfies the functional equation (1.2). Changing $\left(v_{10},v_{11},v_{12},v_{13},v_{14}\right)$ as $\left(0,0,0,0,0\right),\left(v,v,0,0,0\right),\left(v,v,0,0,v\right)$, $\left(-v,v,0,0,0\right)$ in (1.2) and for any $m>0$, we have

$$C_1\left(0\right)=0;\ C_1\left(2v\right)=2C_1\left(v\right);C_1\left(3v\right)=3C_1\left(v\right);C_1\left(-v\right)=-C_1\left(v\right);C_1\left(mv\right)=mC_1\left(v\right) \tag{2.2}$$

for all $v\in V_1$. Also, interchanging v as $\frac{v}{2},\frac{v}{3},\frac{v}{m}$ in (2.2) respectively, we arrive

$$C_1\left(\frac{v}{2}\right)=\frac{1}{2}C_1\left(v\right);C_1\left(\frac{v}{3}\right)=\frac{1}{3}C_1\left(v\right);C_1\left(\frac{v}{m}\right)=\frac{1}{m}C_1\left(v\right) \tag{2.3}$$

for all $v\in V_1$. Changing $\left(v_{10},v_{11},v_{12},v_{13},v_{14}\right)=\left(v_{20},v_{21},v_{22},v_{23},-v_{24}\right)$ in (1.2) and using (2.2) in the resulting equation with $C_1=C_2$, we arrive (1.3) for all $v_{20},v_{21},v_{22},v_{23},v_{24}\in V_1$. So, II) holds.

Suppose $C_2:V_1\to V_2$ satisfies the functional equation (1.3). Changing $\left(v_{20},v_{21},v_{22},v_{23},v_{24}\right)$ as $\left(0,0,0,0,0\right),\left(v,v,0,0,0\right),\left(v,v,v,0,0\right)$, $\left(-v,v,0,0,0\right)$ in (1.3) and for any $m>0$, we have

$$C_2\left(0\right)=0;\ C_2\left(2v\right)=2C_2\left(v\right);C_2\left(3v\right)=3C_2\left(v\right);C_2\left(-v\right)=-C_2\left(v\right);C_2\left(mv\right)=mC_2\left(v\right) \tag{2.4}$$

for all $v\in V_1$. Also, interchanging v as $\frac{v}{2},\frac{v}{3},\frac{v}{m}$ in (2.4) respectively, we arrive

$$C_2\left(\frac{v}{2}\right)=\frac{1}{2}C_2\left(v\right);C_2\left(\frac{v}{3}\right)=\frac{1}{3}C_2\left(v\right);C_2\left(\frac{v}{m}\right)=\frac{1}{m}C_2\left(v\right) \tag{2.5}$$

for all $v\in V_1$. Changing $\left(v_{20},v_{21},v_{22},v_{23},v_{24}\right)=\left(v_{30},v_{31},v_{32},-v_{33},-v_{34}\right)$ in (1.3) and using (2.4) in the resulting equation with $C_2=C_3$, we arrive (1.4) for all $v_{30},v_{31},v_{32},v_{33},v_{34}\in V_1$. So, III) holds.

Suppose $C_3:V_1\to V_2$ satisfies the functional equation (1.4). Changing $\left(v_{30},v_{31},v_{32},v_{33},v_{34}\right)$ as $\left(0,0,0,0,0\right),\left(v,v,0,0,0\right),\left(v,v,v,0,0\right)$, $\left(-v,v,0,0,0\right)$ in (1.4) and for any $m>0$, we have

$$C_3(0)=0;\ C_3(2v)=2C_3(v);C_3(3v)=3C_3(v);C_3(-v)=-C_3(v);C_3(mv)=mC_3(v) \tag{2.6}$$

for all $v \in V_1$. Also, interchanging v as $\frac{v}{2},\frac{v}{3},\frac{v}{m}$ in (2.9) respectively, we arrive

$$C_3\left(\frac{v}{2}\right)=\frac{1}{2}C_3(v);C_3\left(\frac{v}{3}\right)=\frac{1}{3}C_3(v);C_3\left(\frac{v}{m}\right)=\frac{1}{m}C_3(v) \tag{2.7}$$

for all $v \in V_1$. Changing $(v_{30},v_{31},v_{32},v_{33},v_{34})=(v_{40},-v_{41},v_{42},-v_{43},-v_{44})$ in (2.9) and using (2.10) in the resulting equation with $C_3 = C_4$, we arrive (1.5) for all $v_{40},v_{41},v_{42},v_{43},v_{44} \in V_1$. So, IV) holds.

Suppose $C_4 : V_1 \to V_2$ satisfies the functional equation (1.5). Changing $(v_{40},v_{41},v_{42},v_{43},v_{44})$ as $(0,0,0,0,0)$, $(v,0,v,,0,0)$, $(v,0,v,v,0)$, $(-v,0,v,0,0)$ in (2.6) and for any $m>0$, we have

$$C_4(0)=0;\ C_4(2v)=2C_4(v);C_4(3v)=3C_4(v);C_4(-v)=-C_4(v);C_4(mv)=mC_4(v) \tag{2.8}$$

for all $v \in V_1$ Also, interchanging v as $\frac{v}{2},\frac{v}{3},\frac{v}{m}$ in (2.8) respectively, we arrive

$$C_4\left(\frac{v}{2}\right)=\frac{1}{2}C_4(v);C_4\left(\frac{v}{3}\right)=\frac{1}{3}C_4(v);C_4\left(\frac{v}{m}\right)=\frac{1}{m}C_4(v) \tag{2.9}$$

for all $v \in V_1$ Changing $(v_{40},v_{41},v_{42},v_{43},v_{44})=(v_1,0,v_2,0,0)$ in (1.5) and using (2.8) in the resulting equation with $C_4 = C$, we reach (1.1) for all $v_1, v_2 \in V_1$. So, V) holds.

Hence from the above discussions, we see all the functional equations (1.1), (1.2), (1.3), (1.4), (1.5) are equivalent to each other.

III STABILITY RESULTS: HYERS DIRECT APPROACH

In this sub division, we explore the generalized Ulam – Hyers stability of the additive functional equations (1.2), (1.3), (1.4), (1.5) in Banach space using Hyers Method. In order to prove the stability results, hereafter, assume that U be a normed space and W be a Banach space.

To provide stability theorem. We have the following assumptions:

- ✓ $v_0 = a\,v$ are total persons;
- ✓ $v_1 = \frac{av}{b}$ are susceptible persons;
- ✓ $v_2 = \frac{av}{bc}$ are infected persons;
- ✓ $v_3 = \frac{av}{bcd}$ are recovered persons;
- ✓ $v_4 = \frac{av}{bcde}$ are dead persons; respectively

For this general case, we are finding the stability results.

Theorem 3.1: If $C_1 : U \to W;\ C_2 : U \to W;\ C_3 : U \to W;\ C_4 : U \to W$ are functions fulfilling the inequalities

$$\left\| C_1(v_{10}+v_{11}+v_{12}+v_{13}+v_{14}) - \{C_1(v_{10})+C_1(v_{11})+C_1(v_{12})+C_1(v_{13})+C_1(v_{14})\} \right\| \le B_1(v_{10},v_{11},v_{12},v_{13},v_{14}) \tag{3.1}$$

$$\left\| C_2(v_{20}+v_{21}+v_{22}+v_{23}-v_{24}) - \{C_2(v_{20})+C_2(v_{21})+C_2(v_{22})+C_2(v_{23})-C_2(v_{24})\} \right\| \le B_2(v_{20},v_{21},v_{22},v_{23},v_{24}) \tag{3.2}$$

$$\left\| C_3(v_{30}+v_{31}+v_{32}-v_{33}+v_{34}) - \{C_3(v_{30})+C_3(v_{31})+C_3(v_{32})-C_3(v_{33})+C_3(v_{34})\} \right\| \le B_3(v_{30},v_{31},v_{32},v_{33},v_{34}) \tag{3.3}$$

$$\left\| C_4(v_{40}-v_{41}+v_{42}+v_{43}-v_{44}) - \{C_4(v_{40})-C_4(v_{41})+C_4(v_{42})+C_4(v_{43})-C_4(v_{44})\} \right\| \le B_4(v_{40},v_{41},v_{42},v_{43},v_{44}) \tag{3.4}$$

where $B_1, B_2, B_3, B_4 : U^5 \to [0,\infty)$ are functions with

$$\lim_{q\to\infty} \frac{B_1\left(P_1^{qr}v_{10},P_1^{qr}v_{11},P_1^{qr}v_{12},P_1^{qr}v_{13},P_1^{qr}v_{14}\right)}{P_1^{qr}}=0;\ P_1 = a\left(\frac{bcde-1}{bcde}\right) \tag{3.5}$$

$$\lim_{q\to\infty} \frac{B_2\left(P_2^{qr}v_{20},P_2^{qr}v_{21},P_2^{qr}v_{22},P_2^{qr}v_{23},P_2^{qr}v_{24}\right)}{P_2^{qr}}=0;\ P_2 = a \tag{3.6}$$

$$\lim_{q\to\infty}\frac{B_3\left(P_3^{qr}v_{30},P_3^{qr}v_{31},P_3^{qr}v_{32},P_3^{qr}v_{33},P_3^{qr}v_{34}\right)}{P_3^{qr}}=0;\ P_3=a\left(\frac{bc-1}{bc}\right) \tag{3.7}$$

$$\lim_{q\to\infty}\frac{B_4\left(P_4^{qr}v_{40},P_4^{qr}v_{41},P_4^{qr}v_{42},P_4^{qr}v_{43},P_4^{qr}v_{44}\right)}{P_4^{qr}}=0;\ P_4=a \tag{3.8}$$

for all $v_{i0},v_{i1},v_{i2},v_{i3},v_{i4}\in U$ where $i=1,2,3,4$. Then there exists a unique additive mappings $A_1:U\to W$, $A_2:U\to W$, $A_3:U\to W$, $A_4:U\to W$ are defined by

$$A_1(v_1)=\lim_{q\to\infty}\frac{C_1\left(P_1^{qr}v_1\right)}{P_1^{qr}} \tag{3.9}$$

$$A_2(v_2)=\lim_{q\to\infty}\frac{C_2\left(P_2^{qr}v_2\right)}{P_2^{qr}} \tag{3.10}$$

$$A_3(v_3)=\lim_{q\to\infty}\frac{C_3\left(P_3^{qr}v_3\right)}{P_3^{qr}} \tag{3.11}$$

$$A_4(v_4)=\lim_{q\to\infty}\frac{C_4\left(P_4^{qr}v_4\right)}{P_4^{qr}} \tag{3.12}$$

which satisfying the functional equations (1.2), (1.3), (1.4), (1.5), respectively and

$$\left\|A_1(v_1)-C_1(v_1)\right\|\le\frac{1}{P_1}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{B_1^1\left(P_1^{qr}v_1\right)}{P_1^{qr}}=\frac{1}{P_1}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{1}{P_1^{qr}}\times B_1\left(P_1^{qr}\,av_1,-P_1^{qr}\,\frac{av_1}{b},P_1^{qr}\,\frac{acv_1}{2bc},P_1^{qr}\,\frac{acdv_1}{2bcd},-P_1^{qr}\,\frac{av_1}{bcde}\right) \tag{3.13}$$

$$\left\|A_2(v_2)-C_2(v_2)\right\|\le\frac{1}{P_2}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{B_2^2\left(P_2^{qr}v_2\right)}{P_2^{qr}}=\frac{1}{P_2}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{1}{P_2^{qr}}\times B_2\left(P_2^{qr}av_2,-P_2^{qr}\,\frac{av_2}{b},P_2^{qr}\,\frac{av_2}{bc},P_2^{qr}\,\frac{a(c-1)v_2}{bc},0\right) \tag{3.14}$$

$$\left\|A_3(v_3)-C_3(v_3)\right\|\le\frac{1}{P_3}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{B_3^3\left(P_3^{qr}v_3\right)}{P_3^{qr}}=\frac{1}{P_3}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{1}{P_3^{qr}}\times B_3\left(P_3^{qr}av_3,-P_3^{qr}\,\frac{av_3}{b},P_3^{qr}\,\frac{acv_3}{bc},0,-P_3^{qr}\,\frac{av_3}{bc}\right) \tag{3.15}$$

$$\left\|A_4(v_4)-C_4(v_4)\right\|\le\frac{1}{P_4}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{B_4^4\left(P_4^{qr}v_4\right)}{P_4^{qr}}=\frac{1}{P_4}\sum_{q=\frac{1-r}{2}}^{\infty}\frac{1}{P_4^{qr}}\times B_4\left(P_4^{qr}\,av_4,0,-P_4^{qr}\,\frac{av_4}{b},P_4^{qr}\,\frac{av_4}{b},0\right) \tag{3.16}$$

for all $v_i\in U$ where $i=1,2,3,4$ with $r=\{\pm1\}$.

Proof: Let us change

$\left(v_{10},v_{11},v_{12},v_{13},v_{14}\right)=\left(av_1,-\frac{av_1}{b},\frac{acv_1}{2bc},\frac{acdv_1}{2bcd},-\frac{av_1}{bcde}\right)$ in (3.1) and by (2.2), (2.3) of Theorem 2.1;

$\left(v_{20},v_{21},v_{22},v_{23},v_{24}\right)=\left(av_2,-\frac{av_2}{b},\frac{av_2}{bc},\frac{a(c-1)v_2}{bc},0\right)$ in (3.2) and by (2.4), (2.5) of Theorem 2.1;

$\left(v_{30},v_{31},v_{32},v_{33},v_{34}\right)=\left(av_3,-\frac{av_3}{b},\frac{acv_3}{bc},0,-\frac{av_3}{bc}\right)$ in (3.3) and by (2.6), (2.7) of Theorem 2.1;

$\left(v_{40},v_{41},v_{42},v_{43},v_{44}\right)=\left(av_4,0,-\frac{av_4}{b},\frac{av_4}{b},0\right)$ in (3.4) and by (2.8), (2.9) of Theorem 2.1;

we land

$$\begin{aligned}&\left\|C_1\left(av_1-\frac{av_1}{b}+\frac{acv_1}{2bc}+\frac{acdv_1}{2bcd}-\frac{av_1}{bcde}\right)-\left\{C_1\left(av_1\right)+C_1\left(-\frac{av_1}{b}\right)+C_1\left(\frac{acv_1}{2bc}\right)+C_1\left(\frac{acdv_1}{2bcd}\right)+C_1\left(-\frac{av_1}{bcde}\right)\right\}\right\|\\&\le B_1\left(av_1,-\frac{av_1}{b},\frac{acv_1}{2bc},\frac{acdv_1}{2bcd},-\frac{av_1}{bcde}\right)\end{aligned} \tag{3.17}$$

$$\left\| C_2\left(av_2 - \frac{av_2}{b} + \frac{av_2}{bc} + \frac{a(c-1)v_2}{bc} - 0 \right) - \left\{ C_2(av_2) + C_2\left(-\frac{av_2}{b} \right) + C_2\left(\frac{av_2}{bc} \right) + C_2\left(\frac{a(c-1)v_2}{bc} \right) - C_2(0) \right\} \right\| \le B_2\left(av_2, -\frac{av_2}{b}, \frac{av_2}{bc}, \frac{a(c-1)v_2}{bc}, 0 \right) \quad (3.18)$$

$$\left\| C_3\left(av_3 - \frac{av_3}{b} + \frac{acv_3}{bc} - 0 - \frac{av_3}{bc} \right) - \left\{ C_3(av_3) + C_3\left(-\frac{av_3}{b} \right) + C_3\left(\frac{acv_3}{bc} \right) - C_3(0) + C_3\left(-\frac{av_3}{bc} \right) \right\} \right\| \le B_3\left(av_3, -\frac{av_3}{b}, \frac{acv_3}{bc}, 0, -\frac{av_3}{bc} \right) \quad (3.19)$$

$$\left\| C_4\left(av_4 - 0 - \frac{av_4}{b} + \frac{av_4}{b} - 0 \right) - \left\{ C_4(av_4) - C_4(0) + C_4\left(-\frac{av_4}{b} \right) + C_4\left(\frac{av_4}{b} \right) - C_4(0) \right\} \right\| \le B_4\left(av_4, 0, -\frac{av_4}{b}, \frac{av_4}{b}, 0 \right) \quad (3.20)$$

respectively, which implies

$$\left\| C_1\left(a\left\{ \frac{bcde-1}{bcde} \right\} v_1 \right) - a\left\{ \frac{bcde-1}{bcde} \right\} C_1(v_1) \right\| \le B_1\left(av_1, -\frac{av_1}{b}, \frac{acv_1}{2bc}, \frac{acdv_1}{2bcd}, -\frac{av_1}{bcde} \right) = B_1^1(v_1) \quad (3.21)$$

$$\left\| C_2(av_2) - a\,C_2(v_2) \right\| \le B_2\left(av_2, -\frac{av_2}{b}, \frac{av_2}{bc}, \frac{a(c-1)v_2}{bc}, 0 \right) = B_2^2(v_2) \quad (3.22)$$

$$\left\| C_3\left(a\left\{ \frac{bc-1}{bc} \right\} v_3 \right) - a\left\{ \frac{bc-1}{bc} \right\} C_3(v_3) \right\| \le B_3\left(av_3, -\frac{av_3}{b}, \frac{acv_3}{bc}, 0, -\frac{av_3}{bc} \right) = B_3^3(v_3) \quad (3.23)$$

$$\left\| C_4(av_4) - aC_4(v_4) \right\| \le B_4\left(av_4, 0, -\frac{av_4}{b}, \frac{av_4}{b}, 0 \right) = B_4^4(v_4) \quad (3.24)$$

respectively, which yields

$$\left\| C_1(P_1 v_1) - P_1\, C_1(v_1) \right\| \le B_1^1(v_1) \quad (3.25)$$

$$\left\| C_2(P_2 v_2) - P_2\, C_2(v_2) \right\| \le B_2^2(v_2) \quad (3.26)$$

$$\left\| C_3(P_3 v_3) - P_3\, C_3(v_3) \right\| \le B_3^3(v_3) \quad (3.27)$$

$$\left\| C_4(P_4 v_4) - P_4\, C_4(v_4) \right\| \le B_4^4(v_4) \quad (3.28)$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. It follows from (3.25), (3.26), (3.27), (3.28) that

$$\left\| \frac{1}{P_1} \times C_1(P_1 v_1) - C_1(v_1) \right\| \le \frac{1}{P_1} \times B_1^1(v_1) \quad (3.29)$$

$$\left\| \frac{1}{P_2} \times C_2(P_2 v_2) - C_2(v_2) \right\| \le \frac{1}{P_2} \times B_2^2(v_2) \quad (3.30)$$

$$\left\| \frac{1}{P_3} \times C_3(P_3 v_3) - C_3(v_3) \right\| \le \frac{1}{P_3} \times B_3^3(v_3) \quad (3.31)$$

$$\left\| \frac{1}{P_4} \times C_4(P_4 v_4) - C_4(v_4) \right\| \le \frac{1}{P_4} \times B_4^4(v_4) \quad (3.32)$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Changing

$v_1 = P_1 v_1$ and divide by P_1 in (3.29) and adding resultant inequality with (3.29) and by triangle inequality;
$v_2 = P_2 v_2$ and divide by P_2 in (3.30) and adding resultant inequality with (3.30) and by triangle inequality;
$v_3 = P_3 v_3$ and divide by P_3 in (3.31) and adding resultant inequality with (3.31) and by triangle inequality;
$v_4 = P_4 v_4$ and divide by P_4 in (3.32) and adding resultant inequality with (3.32) and by triangle inequality;
we obtain

$$\left\| \frac{1}{P_1^2} \times C_1\left(P_1^2 v_1\right) - C_1\left(v_1\right) \right\| \le \frac{1}{P_1} \times \left\{ \frac{B_1^1\left(P_1 v_1\right)}{P_1} + B_1^1\left(v_1\right) \right\} \tag{3.33}$$

$$\left\| \frac{1}{P_2^2} \times C_2\left(P_2^2 v_2\right) - C_2\left(v_2\right) \right\| \le \frac{1}{P_2} \times \left\{ \frac{B_2^2\left(P_2 v_2\right)}{P_2} + B_2^2\left(v_2\right) \right\} \tag{3.34}$$

$$\left\| \frac{1}{P_3^2} \times C_3\left(P_3^2 v_3\right) - C_3\left(v_3\right) \right\| \le \frac{1}{P_3} \times \left\{ \frac{B_3^3\left(P_3 v_3\right)}{P_3} + B_3^3\left(v_3\right) \right\} \tag{3.35}$$

$$\left\| \frac{1}{P_4^2} \times C_4\left(P_4^2 v_4\right) - C_4\left(v_4\right) \right\| \le \frac{1}{P_4} \times \left\{ \frac{B_4^4\left(P_4 v_4\right)}{P_4} + B_4^4\left(v_4\right) \right\} \tag{3.36}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Generalizing for any positive integer q, we reach

$$\left\| \frac{1}{P_1^q} \times C_1\left(P_1^q v_1\right) - C_1\left(v_1\right) \right\| \le \frac{1}{P_1} \times \sum_{t=0}^{q-1} \frac{B_1^1\left(P_1^t v_1\right)}{P_1^t} \tag{3.37}$$

$$\left\| \frac{1}{P_2^q} \times C_2\left(P_2^q v_2\right) - C_2\left(v_2\right) \right\| \le \frac{1}{P_2} \times \sum_{t=0}^{q-1} \frac{B_2^2\left(P_2^t v_2\right)}{P_2^t} \tag{3.38}$$

$$\left\| \frac{1}{P_3^q} \times C_3\left(P_3^q v_3\right) - C_3\left(v_3\right) \right\| \le \frac{1}{P_3} \times \sum_{t=0}^{q-1} \frac{B_3^3\left(P_3^t v_3\right)}{P_3^t} \tag{3.39}$$

$$\left\| \frac{1}{P_4^q} \times C_4\left(P_4^q v_4\right) - C_4\left(v_4\right) \right\| \le \frac{1}{P_4} \times \sum_{t=0}^{q-1} \frac{B_4^4\left(P_4^t v_4\right)}{P_4^t} \tag{3.40}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Thus, the sequences

$$\left\{ \frac{1}{P_1^q} \times C_1\left(P_1^q v_1\right) \right\}; \left\{ \frac{1}{P_2^q} \times C_2\left(P_2^q v_2\right) \right\}; \left\{ \frac{1}{P_3^q} \times C_3\left(P_3^q v_3\right) \right\}; \left\{ \frac{1}{P_4^q} \times C_4\left(P_4^q v_4\right) \right\}$$

are Cauchy sequences in for all $v_i \in V_1$; $i = 1,2,3,4$, respectively. Since W is complete, there exists a mappings $A_1 : U \to W$, $A_2 : U \to W$, $A_3 : U \to W$, $A_4 : U \to W$ are defined by

$$A_1(v_1) = \lim_{q\to\infty} \frac{C_1\left(P_1^q v_1\right)}{P_1^q}; \tag{3.41}$$

$$A_2(v_2) = \lim_{q\to\infty} \frac{C_2\left(P_2^q v_2\right)}{P_2^q}; \tag{3.42}$$

$$A_3(v_3) = \lim_{q\to\infty} \frac{C_3\left(P_3^q v_3\right)}{P_3^q}; \tag{3.43}$$

$$A_4(v_4) = \lim_{q\to\infty} \frac{C_4\left(P_4^q v_4\right)}{P_4^q} \tag{3.44}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Letting $q \to \infty$ in (3.37), (3.38), (3.39), (3.40) and using (3.41), (3.42), (3.43), (3.44), respectively, we arrive (3.13), (3.14), (3.15), (3.16) for all $v_{i0}, v_{i1}, v_{i2}, v_{i3}, v_{i4} \in U$ where $i = 1,2,3,4$. If, we take

$\left(v_{10}, v_{11}, v_{12}, v_{13}, v_{14}\right) = \left(P_1^q v_{10}, P_1^q v_{11}, P_1^q v_{12}, P_1^q v_{13}, P_1^q v_{14}\right)$ and divide by in (3.1) and using (3.41);

$\left(v_{20}, v_{21}, v_{22}, v_{23}, v_{24}\right) = \left(P_2^q v_{20}, P_2^q v_{21}, P_2^q v_{22}, P_2^q v_{23}, P_2^q v_{24}\right)$ and divide by in (3.2) and using (3.42);

$\left(v_{30}, v_{31}, v_{32}, v_{33}, v_{34}\right) = \left(P_3^q v_{30}, P_3^q v_{31}, P_3^q v_{32}, P_3^q v_{33}, P_3^q v_{34}\right)$ and divide by in (3.3) and using (3.43);

$\left(v_{40}, v_{41}, v_{42}, v_{43}, v_{44}\right) = \left(P_4^q v_{40}, P_4^q v_{41}, P_4^q v_{42}, P_4^q v_{43}, P_4^q v_{44}\right)$ and divide by in (3.4) and using (3.44);

we see that $A_1 : U \to W$, $A_2 : U \to W$, $A_3 : U \to W$, $A_4 : U \to W$ are additive mappings satisfying functional equations (1.2), (1.3), (1.4), (1.5) for all $v_{i0}, v_{i1}, v_{i2}, v_{i3}, v_{i4} \in U$ where $i = 1,2,3,4$.

To prove $A_1 : U \to W$, $A_2 : U \to W$, $A_3 : U \to W$, $A_4 : U \to W$ are unique, let us consider another mappings $A_1^1 : U \to W; A_2^2 : U \to W; A_3^3 : U \to W; A_4^4 : U \to W$ fulfilling the functional equations (1.2), (1.3), (1.4), (1.5) and inequalities (3.13), (3.14), (3.15), (3.16), respectively. Now,

$$\left\| A_1(v_1) - A_1^1(v_1) \right\| \le \frac{2}{P_1} \sum_{q=0}^{\infty} \frac{B_1^1\left(P_1^{q+s} v_1\right)}{P_1^{q+s}} \to 0 \quad as \quad s \to \infty ;$$

$$\left\| A_2(v_2) - A_2^2(v_2) \right\| \le \frac{2}{P_2} \sum_{q=0}^{\infty} \frac{B_2^2\left(P_2^{q+s} v_2\right)}{P_2^{q+s}} \to 0 \quad as \quad s \to \infty ;$$

$$\left\| A_3(v_3) - A_3^3(v_3) \right\| \le \frac{2}{P_3} \sum_{q=0}^{\infty} \frac{B_3^3\left(P_3^{q+s} v_3\right)}{P_3^{q+s}} \to 0 \quad as \quad s \to \infty ;$$

$$\left\| A_4(v_4) - A_4^4(v_4) \right\| \le \frac{2}{P_4} \sum_{q=0}^{\infty} \frac{B_4^4\left(P_4^{q+s} v_4\right)}{P_4^{q+s}} \to 0 \quad as \quad s \to \infty$$

we arrive at $A_1 = A_1^1;\ A_2 = A_2^2;\ A_3 = A_3^3;\ A_4 = A_4^4$, for all $v_i \in U$; $i = 1,2,3,4$, respectively. Thus $A_i\,'s$ are unique for all $v_i \in U$ where $i = 1,2,3,4$.

Changing

$$v_1 = \frac{v_1}{P_1} \text{ in (3.25);}$$

$$v_2 = \frac{v_2}{P_2} \text{ in (3.26);}$$

$$v_3 = \frac{v_3}{P_3} \text{ in (3.27);}$$

$$v_4 = \frac{v_4}{P_3} \text{ in (3.28);}$$

we arrive

$$\left\| C_1(v_1) - P_1\ C_1\left(\frac{v_1}{P_1}\right) \right\| \le B_1^1\left(\frac{v_1}{P_1}\right) \tag{3.45}$$

$$\left\| C_2(v_2) - P_2\ C_2\left(\frac{v_2}{P_2}\right) \right\| \le B_2^2\left(\frac{v_2}{P_2}\right) \tag{3.46}$$

$$\left\| C_3(v_3) - P_3\ C_3\left(\frac{v_3}{P_3}\right) \right\| \le B_3^3\left(\frac{v_3}{P_3}\right) \tag{3.47}$$

$$\left\| C_4(v_4) - P_4\ C_4\left(\frac{v_4}{P_4}\right) \right\| \le B_4^4\left(\frac{v_4}{P_4}\right) \tag{3.48}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Again, changing

$v_1 = \frac{v_1}{P_1}$ and multiply by P_1 in (3.45) and adding resultant inequality with (3.45) and by triangle inequality;

$v_2 = \frac{v_2}{P_2}$ and multiply by P_2 in (3.46) and adding resultant inequality with (3.46) and by triangle inequality;

$v_3 = \frac{v_3}{P_3}$ and multiply by P_3 in (3.47) and adding resultant inequality with (3.47) and by triangle inequality;

$v_4 = \frac{v_4}{P_4}$ and multiply by P_4 in (3.48) and adding resultant inequality with (3.48) and by triangle inequality;

we obtain

$$\left\| C_1(v_1) - P_1^2\ C_1\left(\frac{v_1}{P_1^2}\right) \right\| \le B_1^1\left(\frac{v_1}{P_1}\right) + P_1\,B_1^1\left(\frac{v_1}{P_1^2}\right) \tag{3.49}$$

$$\left\| C_2(v_2) - P_2^2 \; C_2\left(\frac{v_2}{P_2^2}\right) \right\| \le B_2^2\left(\frac{v_2}{P_2}\right) + P_2\, B_2^2\left(\frac{v_2}{P_2^2}\right) \tag{3.50}$$

$$\left\| C_3(v_3) - P_3^2 \; C_3\left(\frac{v_3}{P_3^2}\right) \right\| \le B_3^3\left(\frac{v_3}{P_3}\right) + P_3\, B_3^3\left(\frac{v_3}{P_3^2}\right) \tag{3.51}$$

$$\left\| C_4(v_4) - P_4^2 \; C_4\left(\frac{v_4}{P_4^2}\right) \right\| \le B_4^4\left(\frac{v_4}{P_4}\right) + P_4\, B_4^4\left(\frac{v_4}{P_4^2}\right) \tag{3.52}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. Generalizing for any positive integer q, we reach

$$\left\| C_1(v_1) - P_1^q \; C_1\left(\frac{v_1}{P_1^q}\right) \right\| \le \sum_{t=1}^{q} P_1^{q-1}\, B_1^1\left(\frac{v_1}{P_1^q}\right) = \frac{1}{P_1} \times \sum_{t=1}^{q} P_1^q\, B_1^1\left(\frac{v_1}{P_1^q}\right) \tag{3.53}$$

$$\left\| C_2(v_2) - P_2^q \; C_2\left(\frac{v_2}{P_2^q}\right) \right\| \le \sum_{t=1}^{q} P_2^{q-1}\, B_2^2\left(\frac{v_2}{P_2^q}\right) = \frac{1}{P_2} \times \sum_{t=1}^{q} P_2^q\, B_2^2\left(\frac{v_2}{P_2^q}\right) \tag{3.54}$$

$$\left\| C_3(v_3) - P_3^q \, C_3\left(\frac{v_3}{P_2^q}\right) \right\| \le \sum_{t=1}^{q} P_3^{q-1}\, B_3^3\left(\frac{v_3}{P_3^q}\right) = \frac{1}{P_3} \times \sum_{t=1}^{q} P_3^q\, B_3^3\left(\frac{v_3}{P_3^q}\right) \tag{3.55}$$

$$\left\| C_4(v_4) - P_4^q \; C_4\left(\frac{v_4}{P_4^q}\right) \right\| \le \sum_{t=1}^{q} P_4^{q-1}\, B_4^4\left(\frac{v_4}{P_4^q}\right) = \frac{1}{P_4} \times \sum_{t=1}^{q} P_4^q\, B_4^4\left(\frac{v_4}{P_4^q}\right) \tag{3.56}$$

for all $v_i \in U$; $i = 1,2,3,4$, respectively. The rest of the proof is analogous to that of earlier one. Thus, the proof is complete.

The following corollary is an immediate result of Theorem 3.1.

Corollary 3.2: If $C_1 : U \to W;\ C_2 : U \to W;\ C_3 : U \to W;\ C_4 : U \to W$ are functions fulfilling the

$$\left\| C_1(v_{10} + v_{11} + v_{12} + v_{13} + v_{14}) - \left\{ C_1(v_{10}) + C_1(v_{11}) + C_1(v_{12}) + C_1(v_{13}) + C_1(v_{14}) \right\} \right\| \le \begin{cases} \psi \\ \psi \times \sum_{j=0}^{4} \| v_{1j} \|^{\varphi} \\ \psi \times \prod_{j=0}^{4} \| v_{1j} \|^{\varphi} \\ \psi \times \left\{ \prod_{j=0}^{4} \| v_{1j} \|^{\varphi} + \sum_{j=0}^{4} \| v_{1j} \|^{5\varphi} \right\} \end{cases} \tag{3.57}$$

$$\left\| C_2(v_{20} + v_{21} + v_{22} + v_{23} - v_{24}) - \left\{ C_2(v_{20}) + C_2(v_{21}) + C_2(v_{22}) + C_2(v_{23}) - C_2(v_{24}) \right\} \right\| \le \begin{cases} \psi \\ \psi \times \sum_{j=0}^{4} \| v_{2j} \|^{\varphi} \end{cases} \tag{3.58}$$

$$\left\| C_3(v_{30} + v_{31} + v_{32} - v_{33} + v_{34}) - \left\{ C_3(v_{30}) + C_3(v_{31}) + C_3(v_{32}) - C_3(v_{33}) + C_3(v_{34}) \right\} \right\| \le \begin{cases} \psi \\ \psi \times \sum_{j=0}^{4} \| v_{3j} \|^{\varphi} \end{cases} \tag{3.59}$$

$$\left\| C_4(v_{40} - v_{41} + v_{42} + v_{43} - v_{44}) - \left\{ C_4(v_{40}) - C_4(v_{41}) + C_4(v_{42}) + C_4(v_{43}) - C_4(v_{44}) \right\} \right\| \le \begin{cases} \psi \\ \psi \times \sum_{j=0}^{4} \| v_{4j} \|^{\varphi} \end{cases} \tag{3.60}$$

where ψ be a positive constant and $\varphi \ne 1$; $5\varphi \ne 1$ for all $v_{i0}, v_{i1}, v_{i2}, v_{i3}, v_{i4} \in U$ where $i = 1,2,3,4$. Then there exists a unique additive mappings $A_1 : U \to W$, $A_2 : U \to W$, $A_3 : U \to W$, $A_4 : U \to W$ which satisfying the functional equations (1.2), (1.3), (1.4), (1.5) and

$$\left\|A_1(v_1)-C_1(v_1)\right\| \le \begin{cases} \left(\dfrac{\psi}{|P_1-1|}\right); \\ \left(\dfrac{\psi\left\{a^{\varphi}+\left(\dfrac{a}{b}\right)^{\varphi}+\left(\dfrac{ac}{2bc}\right)^{\varphi}+\left(\dfrac{acd}{2bcd}\right)^{\varphi}+\left(\dfrac{a}{bcde}\right)^{\varphi}\right\}\|v_1\|^{\varphi}}{|P_1-P_1^{\varphi}|}\right); \\ \left(\dfrac{\psi\left\{a^{\varphi}\left(\dfrac{a}{b}\right)^{\varphi}\left(\dfrac{ac}{2bc}\right)^{\varphi}\left(\dfrac{acd}{2bcd}\right)^{\varphi}\left(\dfrac{a}{bcde}\right)^{\varphi}\right\}\|v_1\|^{5\varphi}}{|P_1-P_1^{5\varphi}|}\right); \\ \left(\dfrac{\psi\left\{a^{5\varphi}+\left(\dfrac{a}{b}\right)^{5\varphi}+\left(\dfrac{ac}{2bc}\right)^{5\varphi}+\left(\dfrac{acd}{2bcd}\right)^{5\varphi}+\left(\dfrac{a}{bcde}\right)^{5\varphi}+a^{\varphi}\left(\dfrac{a}{b}\right)^{\varphi}\left(\dfrac{ac}{2bc}\right)^{\varphi}\left(\dfrac{acd}{2bcd}\right)^{\varphi}\left(\dfrac{a}{bcde}\right)^{\varphi}\right\}}{|P_1-P_1^{5\varphi}|}\right); \end{cases} \tag{3.61}$$

$$\left\|A_2(v_2)-C_2(v_2)\right\| \le \begin{cases} \left(\dfrac{\psi}{|P_2-1|}\right); \\ \left(\dfrac{\psi\left\{a^{\varphi}+\left(\dfrac{a}{b}\right)^{\varphi}+\left(\dfrac{ac}{2bc}\right)^{\varphi}+\left(\dfrac{a(c-1)}{2bc}\right)^{\varphi}\right\}\|v_2\|^{\varphi}}{|P_2-P_2^{\varphi}|}\right); \end{cases} \tag{3.62}$$

$$\left\|A_3(v_3)-C_3(v_3)\right\| \le \begin{cases} \left(\dfrac{\psi}{|P_3-1|}\right); \\ \left(\dfrac{\psi\left\{a^{\varphi}+\left(\dfrac{a}{b}\right)^{\varphi}+2\left(\dfrac{a}{bc}\right)^{\varphi}\right\}\|v_3\|^{\varphi}}{|P_3-P_3^{\varphi}|}\right); \end{cases} \tag{3.63}$$

$$\left\|A_4(v_4)-C_4(v_4)\right\| \le \begin{cases} \left(\dfrac{\psi}{|P_4-1|}\right); \\ \left(\dfrac{\psi\left\{a^{\varphi}+2\left(\dfrac{a}{b}\right)^{\varphi}\right\}\|v_4\|^{\varphi}}{|P_4-P_4^{\varphi}|}\right); \end{cases} \tag{3.64}$$

for all $v_i \in U$ where $i = 1,2,3,4$.

IV MATHEMATICAL CALCULATIONS

In this sub division, we examine the mathematical calculation of our functional equations (1.2), (1.3), (1.4), (1.5) .

Example: 4.1 Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if 2 persons recovered and 2 of them dead. By our data, we have

$$\left(v_{10}, v_{11}, v_{12}, v_{13}, v_{14}\right) = \left(av_1, -\frac{av_1}{b}, \frac{acv_1}{2bc}, \frac{acdv_1}{2bcd}, -\frac{av_1}{bcde}\right)$$

so it follows that

$$\begin{aligned} &a = 16; \\ &\frac{a}{b} = 10 \Rightarrow b = \frac{8}{5}; \\ &\frac{a}{bc} = 4 \Rightarrow c = \frac{16}{4b} = \frac{5}{2}; \\ &\frac{av}{bcd} = 2; \Rightarrow d = \frac{16}{2bc} = 2; \\ &\frac{av}{bcde} = 2 \Rightarrow e = \frac{16}{2bcd} = 1. \end{aligned} \tag{4.1}$$

It follows from (1.2) that

$$\begin{aligned} &C_1\left(16v_1 - 10v_1 + 5v_1 + 5v_1 - 2v_1\right) = 16C_1\left(v_1\right) - 10C_1\left(v_1\right) + 5C_1\left(v_1\right) + 5\,C_1\left(v_1\right) - 2C_1\left(v_1\right) \\ &C_1\left(14v_1\right) = 14C_1\left(v_1\right). \end{aligned}$$

Since 2 persons are dead, the remaining persons in that village is 14. So, there is a minimum loss.

Example: 4.2 Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if all 4 persons recovered and no dead occurs. By our data, we have

$$\left(v_{20}, v_{21}, v_{22}, v_{23}, v_{24}\right) = \left(av_2, -\frac{av_2}{b}, \frac{av_2}{bc}, \frac{a(c-1)v_2}{bc}, 0\right)$$

so it follows that

$$\begin{aligned} &a = 16; \\ &\frac{a}{b} = 10 \Rightarrow b = \frac{16}{10} = \frac{8}{5}; \\ &\frac{a}{bc} = 4 \Rightarrow c = \frac{16}{4b} = \frac{16}{4*\frac{8}{5}} = \frac{5}{2}. \end{aligned} \tag{4.2}$$

It follows from (1.3) that

$$\begin{aligned} &C_2\left(16v_2 - 10v_2 + 4v_2 + 6v_2 - 0\right) = 16C_2\left(v_2\right) - 10C_2\left(v_2\right) + 4C_2\left(v_2\right) + 6C_2\left(v_2\right) - C_2\left(0\right) \\ &C_2\left(16v_2\right) = 16C_2\left(v_2\right). \end{aligned}$$

Since no persons are dead, the remaining persons in that village is 16. So, there is a no loss.

Example: 4.3 Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if no persons are recovered and all the 4 persons are dead. By our data, we have

$$\left(v_{30}, v_{31}, v_{32}, v_{33}, v_{34}\right) = \left(av_3, -\frac{av_3}{b}, \frac{acv_3}{bc}, 0, -\frac{av_3}{bc}\right)$$

so it follows that

$$\begin{aligned} &a = 16; \\ &\frac{a}{b} = 10 \Rightarrow b = \frac{8}{5}; \\ &\frac{a}{bc} = 4 \Rightarrow c = \frac{16}{4b} = \frac{5}{2}. \end{aligned} \tag{4.3}$$

It follows from (1.4) that

$$\begin{aligned} &C_3\left(16v_3 - 10v_3 + 10v_3 + 0 - 4v_3\right) = 16C_3\left(v_3\right) - 10C_3\left(v_3\right) + 10C_3\left(v_3\right) - 4C_3\left(v_3\right) \\ &C_3\left(12v_3\right) = 12C_3\left(v_3\right). \end{aligned}$$

Since 4 persons are dead, the remaining persons in that village is 12. So, there is a big loss.

Example: 4.4 Suppose, if 16 persons are in a village in that 0 persons are susceptible and if 4 persons are infected, in that if 4 persons recovered and no dead occurs. By our data, we have

$$\left(v_{40}, v_{41}, v_{42}, v_{43}, v_{44}\right) = \left(av_4, 0, -\frac{av_4}{b}, \frac{av_4}{b}, 0\right)$$

so it follows that

$$a = 16; \qquad \frac{a}{b} = 10 \Rightarrow b = \frac{8}{5}. \tag{4.4}$$

It follows from (1.2) that

$$C_4\left(16v_4 - 10v_4 + 10v_4\right) = 16C_4\left(v_4\right) - 10C_4\left(v_4\right) + 10C_4\left(v_4\right)$$
$$C_4\left(16v_1\right) = 16C_4\left(v_4\right).$$

Since no persons are dead, the remaining persons in that village is 16. So, there is a no loss.

V STABILITY ANALYSIS

In this sub division, we inspect the stability analysis of our functional equations (1.2), (1.3), (1.4), (1.5).

Analysis 5.1 For the functional equation (1.2):

By the definition of P_1 in (3.5) and with the help of (4.1), we have

$$P_1 = a\left(\frac{bcde - 1}{bcde}\right) = 14. \tag{5.1}$$

Now, it follows from (3.61) that

$$\left\|A_1(v_1) - C_1(v_1)\right\| \le \frac{\psi}{|14 - 1|} = \frac{\psi}{13}. \tag{5.2}$$

Analysis 5.2 For the functional equation (1.3):

By the definition of P_2 in (3.6) and with the help of (4.2), we have

$$P_2 = a = 16. \tag{5.3}$$

Now, it follows from (3.62) that

$$\left\|A_2(v_2) - C_2(v_2)\right\| \le \frac{\psi}{|16 - 1|} = \frac{\psi}{15}. \tag{5.4}$$

Analysis 5.3 For the functional equation (1.4):

By the definition of P_3 in (3.7) and with the help of (4.3), we have

$$P_3 = a\left(\frac{bc - 1}{bc}\right) = 12. \tag{5.5}$$

Now, it follows from (3.63) that

$$\left\|A_3(v_3) - C_3(v_3)\right\| \le \frac{\psi}{|12 - 1|} = \frac{\psi}{11}. \tag{5.6}$$

Analysis 5.4 For the functional equation (1.5):

By the definition of P_4 in (3.8) and with the help of (4.4), we have

$$P_4 = a = 16. \tag{5.7}$$

Now, it follows from (3.64) that

$$\left\|A_4(v_4) - C_4(v_4)\right\| \le \frac{\psi}{|16 - 1|} = \frac{\psi}{15}. \tag{5.8}$$

VI CONCLUSIONS

According to Mathematical Calculations in Section IV the following are conclusions:

- In Example 4.2 and Example 4.4, there is **NO LOSS** in the village;
- In Example 4.1 there is **MINIMUM LOSS** in the village;
- In Example 4.3 there is **BIG LOSS** in the village.

Also, according to Stability Analysis in Section V the following are conclusions:

- In Analysis 5.2 and Analysis 5.4, we get **better possible upper bound**;
- In Analysis 5.1 we get **minimum upper bound**;
- In Analysis 5.3, we get **very low bound**.

So, if all the affected persons are recovered with no death, then only the village or town or city or home is stable.

This Mathematical Calculations and Stability Analysis can be done for any higher datas.

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