Stability of System of Functional Equations From a Corona Model

M. Arunkumar

Department of Mathematics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India.

e-mail: drarun4maths@gmail.com

T. Velmurugan

Department of Mathematics

MRK College of Arts and Science,

Pazhanchanallur, Kattumannarkoil - 608 301,

TamilNadu, India.

e-mail: smmuruganvel@gmail.com

E. Sathya

Department of Mathematics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India.

e-mail: sathy24mathematics@gmail.com

V. Alexpandiyan

Department of Mathematics

Kalaignar Karunanidhi Government Arts College, Tiruvanamalai – 606 603, Tamil Nadu, India. e-mail: valexpandiyan98@gmail.com

ABSTRACT

In this paper, the authors establish the generalized Ulam – Hyers stability of system of additive functional equations from a corona model in Banach space using Hyers Method. Also, we compare the results with mathematical calculations and stability analysis.

Keywords—Additive functional equatiom, Ulam – Hyers stability Banach space, Hyers method, Corona Model.

# INTRODUCTION

In 1940 and in 1968 Ulam [34-35] proposed the general Ulam stability problem:

“When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?”

In 1941 Hyers [22] answered this problem for linear mappings. In 1951 Aoki [3] and Bourgin [14] were the authors to treat the Ulam problem for additive mappings. In 1978, according to Gruber [21], this kind of stability problems is of particular interest in probability theory and in the case of functional equations of different types. In 1978 Rassias [32] employed Hyers’ ideas to new linear mappings. In 1987 Gajda and Ger [18] showed that one can get similar stability results for sub additive multi functions.

Other interesting stability results have been completed also by the following authors Aczél [1-2], Borelli and Forti [13], Cholewa [15], Czerwik [16] and Kannappan [26]. In 1982–1989 Rassias [28-29, 31] solved the above Ulam problem for different mappings. In 1999 Gavruta [18,20] answered a question of Rassias concerning the stability of the Cauchy equation. In 1983 Skof [33] was the first author to solve the Ulam problem for additive mappings on a restricted domain.

The famous Cauchy additive functional equation is

. (1.1)

Its stability in various settings were inspected in [3,20,22,28.31,32]. Several other types of additive functional equations in various normed spaces were discussed by Aczel, Dhombres [2], Arunkumar [4-11]. Balamurugan [12], Hyers [23-24], Jung [25], Kannappan [26]. Lee [27] and Rassias [30].

**Coronaviruses** are a group of related [RNA viruses](https://en.wikipedia.org/wiki/RNA_viruses) that cause diseases in [mammals](https://en.wikipedia.org/wiki/Mammal) and birds. In humans and birds, they cause [respiratory tract infections](https://en.wikipedia.org/wiki/Respiratory_tract_infection) that can range from mild to lethal. Mild illnesses in humans include some cases of the [common cold](https://en.wikipedia.org/wiki/Common_cold) (which is also caused by other viruses, predominantly [rhinoviruses](https://en.wikipedia.org/wiki/Rhinovirus)), while more lethal varieties can cause [SARS](https://en.wikipedia.org/wiki/SARS), [MERS](https://en.wikipedia.org/wiki/MERS) and [COVID-19](https://en.wikipedia.org/wiki/COVID-19), which is causing the [ongoing pandemic](https://en.wikipedia.org/wiki/COVID-19_pandemic) see [36].

In a town or city or village, we have the following assumptions:

•  denotes total number of persons;

•  denotes number of susceptible persons;

•  denotes number of infected persons;

•  denotes number of recovered persons;

•  denotes number of death persons

respectively.

With respect to this data, we have the additive functional equations of the forms

 (1.2)

 (1.3)

 (1.4)

 (1.5)

Let us assume that **+ (PLUS) denotes Yes** and **- (MINUS) denotes No** (for this case we take 0 (ZERO)).

In this paper, the authors establish the general solution in vector space and generalized Ulam – Hyers stability of system of additive functional equations (1.2), (1.3), (1.4), (1.5) in Banach space using Hyers Method. Also, we compare the results with mathematical calculations and stability analysis.

**II GENERAL SOLUTION**

In this subdivision, we confer about the general solution of functional equation (1.1) (1.2), (1.3), (1.4), (1.5), by considering  and  as real vector spaces.

**Theorem 2.1:**

I)Ifsatisfying (1.1) thensatisfying (1.2);

II) If satisfying (1.2) thensatisfying (1.3);

III)Ifsatisfying (1.3) thensatisfying (1.4);

IV)Ifsatisfying (1.4) thensatisfying (1.5);

V)If satisfying (1.5) thensatisfying (1.1);

for all  where then all the functional equations are equivalent.

**Proof:**

Suppose  satisfies the functional equation (1.1). Changing  as ,, ,  in (1.1) and for any , we have

 (2.1)

for all . Taking  in (1.1), and using (1.1) in the resulting equation with, we arrive (1.2) for all.. So, I) holds.

Suppose  satisfies the functional equation (1.2). Changing  as ,,,  in (1.2) and for any , we have

 (2.2)

for all. Also, interchanging  as  in (2.2) respectively, we arrive

 (2.3)

for all. Changing  in (1.2) and using (2.2) in the resulting equation with, we arrive (1.3) for all. So, II) holds.

Suppose  satisfies the functional equation (1.3). Changing  as ,,,  in (1.3) and for any , we have

 (2.4)

for all. Also, interchanging  as  in (2.4) respectively, we arrive

 (2.5)

for all. Changing  in (1.3) and using (2.4) in the resulting equation with, we arrive (1.4) for all. So, III) holds.

Suppose  satisfies the functional equation (1.4). Changing  as ,,,  in (1.4) and for any , we have

 (2.6)

for all. Also, interchanging  as  in (2.9) respectively, we arrive

 (2.7)

for all. Changing  in (2.9) and using (2.10) in the resulting equation with, we arrive (1.5) for all. So, IV) holds.

Suppose  satisfies the functional equation (1.5). Changing  as ,,,  in (2.6) and for any , we have

 (2.8)

for all Also, interchanging  as  in (2.8) respectively, we arrive

 (2.9)

for all Changing  in (1.5) and using (2.8) in the resulting equation with , we reach (1.1) for all. So, V) holds.

Hence from the above discussions, we see all the functional equations (1.1), (1.2), (1.3), (1.4), (1.5) are equivalent to each other.

**III STABILITY RESULTS: HYERS DIRECT APPROACH**

In this sub division, we explore the generalized Ulam – Hyers stability of the additive functional equations (1.2), (1.3), (1.4), (1.5) in Banach space using Hyers Method. In order to prove the stability results, hereafter, assume that *U* be a normed space and *W* be a Banach space.

To provide stability theorem. We have the following assumptions:

* are total persons;
*  are susceptible persons;
*  are infected persons;
*  are recovered persons;
*  are dead persons; respectively

For this general case, we are finding the stability results.

**Theorem 3.1:** If  are functions fulfilling the inequalities (3.1)

 (3.2)

 (3.3)

 (3.4)

where  are functions with

 (3.5)

 (3.6)

 (3.7)

 (3.8)

for all  where . Then there exists a unique additive mappings, , ,  are defined by

 (3.9)

 (3.10)

 (3.11)

 (3.12)

which satisfying the functional equations (1.2), (1.3), (1.4), (1.5), respectively and  (3.13)

 (3.14)

 (3.15)

 (3.16)

for all  where  with .

**Proof:** Let us change

 in (3.1) and by (2.2), (2.3) of Theorem 2.1;

 in (3.2) and by (2.4), (2.5) of Theorem 2.1;

 in (3.3) and by (2.6), (2.7) of Theorem 2.1;

 in (3.4) and by (2.8), (2.9) of Theorem 2.1;

we land

 (3.17)

 (3.18)

 (3.19)

 (3.20)

respectively, which implies

 (3.21)

 (3.22)

 (3.23)

 (3.24)

respectively, which yields

 (3.25)

 (3.26)

 (3.27)

 (3.28)

for all ; , respectively. It follows from (3.25), (3.26), (3.27), (3.28) that

 (3.29)

 (3.30)

 (3.31)

 (3.32)

for all ; , respectively. Changing

 and divide by  in (3.29) and adding resultant inequality with (3.29) and by triangle inequality;

 and divide by  in (3.30) and adding resultant inequality with (3.30) and by triangle inequality;

 and divide by  in (3.31) and adding resultant inequality with (3.31) and by triangle inequality;

 and divide by  in (3.32) and adding resultant inequality with (3.32) and by triangle inequality;

we obtain

 (3.33)

 (3.34)

 (3.35)

 (3.36)

for all ; , respectively. Generalizing for any positive integer, we reach

 (3.37)

 (3.38)

 (3.39)

 (3.40)

for all ; , respectively. Thus, the sequences



are Cauchy sequences in for all ; , respectively. Since is complete, there exists a mappings

,,, are defined by

; (3.41)

 ; (3.42)

 ; (3.43)

 (3.44)

for all ; , respectively. Letting  in (3.37), (3.38), (3.39), (3.40) and using (3.41), (3.42), (3.43), (3.44), respectively, we arrive (3.13), (3.14), (3.15), (3.16) for all  where . If, we take

 and divide by in (3.1) and using (3.41);

 and divide by in (3.2) and using (3.42);

 and divide by in (3.3) and using (3.43);

 and divide by in (3.4) and using (3.44);

we see that ,,,are additive mappings satisfying functional equations (1.2), (1.3), (1.4), (1.5) for all  where .

To prove ,,, are unique, let us consider another mappings  fulfilling the functional equations (1.2), (1.3), (1.4), (1.5) and inequalities (3.13), (3.14), (3.15), (3.16), respectively. Now,

;

;

;



we arrive at, for all ; , respectively. Thus  are unique for all  where .

Changing

 in (3.25);

 in (3.26);

 in (3.27);

 in (3.28);

we arrive

 (3.45)

 (3.46)

 (3.47)

 (3.48)

for all ; , respectively. Again, changing

 and multiply by  in (3.45) and adding resultant inequality with (3.45) and by triangle inequality;

 and multiply by  in (3.46) and adding resultant inequality with (3.46) and by triangle inequality;

 and multiply by  in (3.47) and adding resultant inequality with (3.47) and by triangle inequality;

 and multiply by  in (3.48) and adding resultant inequality with (3.48) and by triangle inequality;

we obtain

 (3.49)

 (3.50)

 (3.51)

 (3.52)

for all ; , respectively. Generalizing for any positive integer, we reach

 (3.53)

 (3.54)

 (3.55)

 (3.56)

for all ; , respectively. The rest of the proof is analogous to that of earlier one. Thus, the proof is complete.

The following corollary is an immediate result of Theorem 3.1.

**Corollary 3.2:** If  are functions fulfilling the (3.57)

 (3.58)

 (3.59)

 (3.60)

where  be a positive constant and ;  for all  where . Then there exists a unique additive mappings ,,, which satisfying the functional equations (1.2), (1.3), (1.4), (1.5) and

 (3.61)

 (3.62)

 (3.63)

 (3.64)

for all  where .

**IV MATHEMATICAL CALCULATIONS**

In this sub division, we examine the mathematical calculation of our functional equations (1.2), (1.3), (1.4), (1.5) .

**Example: 4.1** Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if 2 persons recovered and 2 of them dead. By our data, we have



so it follows that

 (4.1)

It follows from (1.2) that



Since 2 persons are dead, the remaining persons in that village is 14. So, there is a minimum loss.

**Example: 4.2** Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if all 4 persons recovered and no dead occurs. By our data, we have



so it follows that

 (4.2)

It follows from (1.3) that



Since no persons are dead, the remaining persons in that village is 16. So, there is a no loss.

**Example: 4.3** Suppose, if 16 persons are in a village in that 10 persons are susceptible in that 4 persons are infected, in that if no persons are recovered and all the 4 persons are dead. By our data, we have



so it follows that

 (4.3)

It follows from (1.4) that



Since 4 persons are dead, the remaining persons in that village is 12. So, there is a big loss.

**Example: 4.4** Suppose, if 16 persons are in a village in that 0 persons are susceptible and if 4 persons are infected, in that if 4 persons recovered and no dead occurs. By our data, we have



so it follows that

 (4.4)

It follows from (1.2) that



Since no persons are dead, the remaining persons in that village is 16. So, there is a no loss.

**V STABILITY ANALYSIS**

In this sub division, we inspect the stability analysis of our functional equations (1.2), (1.3), (1.4), (1.5).

**Analysis 5.1** For the functional equation (1.2):

By the definition of  in (3.5) and with the help of (4.1), we have

. (5.1)

Now, it follows from (3.61) that

 (5.2)

**Analysis 5.2** For the functional equation (1.3):

By the definition of  in (3.6) and with the help of (4.2), we have

. (5.3)

Now, it follows from (3.62) that

 (5.4)

**Analysis 5.3** For the functional equation (1.4):

By the definition of  in (3.7) and with the help of (4.3), we have

. (5.5)

Now, it follows from (3.63) that

 (5.6)

**Analysis 5.4** For the functional equation (1.5):

By the definition of  in (3.8) and with the help of (4.4), we have

. (5.7)

Now, it follows from (3.64) that

 (5.8)

**VI CONCLUSIONS**

According to Mathematical Calculations in Section IV the following are conclusions:

* In Example 4.2 and Example 4.4, there is **NO LOSS** in the village;
* In Example 4.1 there is **MINIMUM LOSS** in the village;
* In Example 4.3 there is **BIG LOSS** in the village.

Also, according to Stability Analysis in Section V the following are conclusions:

* In Analysis 5.2 and Analysis 5.4, we get **better possible upper bound**;
* In Analysis 5.1 we get **minimum upper bound**;
* In Analysis 5.3, we get **very low bound**.

So, if all the affected persons are recovered with no death, then only the village or town or city or home is stable.

This Mathematical Calculations and Stability Analysis can be done for any higher datas.

##### REFERENCES

1. J. Aczél, Lectures on Functional Equations and Their Applications, Academic Press, New York/London, 1966.
2. J. Aczel and J. Dhombres, Functional Equations in Several Variables, Cambridge Univ, Press, 1989.
3. T. Aoki, “On the stability of the linear transformation in Banach spaces”, J. Math. Soc. Japan, 2 (1950), 64-66.
4. M. Arunkumar, “Solution and Stability of Arun-Additive functional equations”, International Journal Mathematical Sciences and Engineering Applications, Vol 4, No. 3, (2010), 33-46.
5. M. Arunkumar, C. Leela Sabari, “Solution and stability of a functional equation originating from a chemical equation”, International Journal Mathematical Sciences and Engineering Applications, 5 (2) (2011), 1-8.
6. M. Arunkumar, P. Agilan, “Additive functional equation and inequality are Stable in Banach space and its applications”, Malaya Journal of Matematik (MJM), Vol 1, Issue 1, (2013), 10-17.
7. M. Arunkumar, S. Karthikeyan, “Functional Equation Originating From n-Consecutive Terms Of An Arithmetic Progression Are Stable In Banach Space: Direct and Fixed Point Methods”, Jamal Academic Research Journal an Interdisciplinary, (2015), 111-118 .
8. M.Arunkumar, E.Sathya, S.Karthikeyan, G. Ganapathy, T. Namachivayam, “Stability of System of Additive Functional Equations in Various Banach Spaces: Classical Hyers Methods”, Malaya Journal of Matematik, Volume 6, Issue 1, 2018, 91-112.
9. M. Arunkumar, E.Sathya, C.Pooja, “Stability of a functional equation originates from a corona Model with additive solution in Banach spaceusing direct and fixed point methods”, Recent Advancement ofMathematics in Science and Technology, JPS Scientific Publication India,First Edition, Book Chapter, 103-113, 2021, (ISBN): 978-81-950475-0-5.219
10. M. Arunkumar, S. Tamilarasan, R. Kondandan, E. Sathya, “Stability analysisof system of additive functional equations from a hotel modelVia FixedPoint Method”, Recent Advancement of Mathematics in Science andTechnology, JPS Scientific Publication India, First Edition, Book Chapter,25-34, 2021, (ISBN): 978-81-950475-0-5.
11. M. Arunkumar, E. Sathya, “Stability Of A Ramanujan Type AdditiveFunctional Equation”, Journal of Computational Mathematica, Vol. 5, Issue1, (2021), 11 -27.
12. K. Balamurugan, A.Manjula, M.Arunkumar, E.Sathya, “Stability of anadditive functional equation originating from a telescoping series”, Rabindra Bharati Journal Of Philosophy (ISSN : 0973-0087), Vol. : XXIV,No:02, 2023, 56-65.
13. C. Borelli, G.L. Forti, “On a general Hyers–Ulam stability result”, Internat. J. Math. Sci. 18 (1995) 229–236.
14. D.G. Bourgin, “Classes of transformations and bordering transformations”, Bull. Amer. Math. Soc. 57 (1951) 223–237.
15. P.W. Cholewa, “Remarks on the stability of functional equations”, Aequationes Math. 27 (1984)76–86.
16. S. Czerwik, “On the stability of the quadratic mapping in normed spaces”, Abh. Math. Sem. Univ.Hamburg 62 (1992) 59–64.
17. G.L. Forti, “Hyers–Ulam stability of functional equations in several variables”, AequationesMath. 50 (1995) 143–190.
18. Z. Gajda, R. Ger, Subadditive multifunctions and Hyers–Ulam stability, in: General Inequalities,Vol. 5, in: Internat. Schriftenreiche Numer. Math., Vol. 80, Birkhäuser, Basel/Boston, 1987.
19. P. Gavruta, “A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings”, J. Math. Anal. Appl., 184 (1994), 431-436.
20. P. Gavruta, An answer to a question of John M. Rassias concerning the stability of Cauchyequation, in: Advances in Equations and Inequalities, in: Hadronic Math. Series, 1999, pp. 67–71.
21. P.M. Gruber, “Stability of isometries”, Trans. Amer. Math. Soc. 245 (1978) 263–277.
22. D.H. Hyers, “On the stability of the linear functional equation”, Proc.Nat. Acad.Sci.,U.S.A.,27 (1941) 222-224.
23. D.H. Hyers, “The stability of homomorphisms and related topics, in: Global Analysis-Analysison Manifolds”, Teubner—Texte zur Mathematik, Vol. 57, 1983, pp. 140–153.
24. D.H. Hyers, G. Isac,Th.M. Rassias, Stability of functional equations in several variables, Birkhauser, Basel, 1998.
25. S.M. Jung, On the Hyers–Ulam stability of the functional equations that have the quadraticproperty, J. Math. Anal. Appl. 222 (1998) 126–137.
26. Pl. Kannappan, Functional Equations and Inequalities with Applications, Springer Monographs in Mathematics, 2009.
27. D.O. Lee, “Hyers-Ulam stability of an addtiive type functional equation”, J. Appl. Math. and Computing, 13 (2003) no.1-2, 471-477.
28. J.M. Rassias, “On approximately of approximately linear mappings by linear mappings”, J. Funct. Anal. USA, 46, (1982) 126-130.
29. J.M. Rassias, “Solution of a problem of Ulam”, J. Approx. Theory 57 (1989) 268–273.
30. John. M Rassias, M. Arunkumar, E.Sathya, “Stability of Difference a –Additive and General Alternative a - Additive Functional Equations”, International Conference on Recent Advances in Technology, Engineering and Science ’July 2016 (ICRATES’16) ISBN:978-93-5258-740-7.
31. K. Ravi, M. Arunkumar and J.M. Rassias, “On the Ulam stability for the orthogonally general Euler-Lagrange type functional equation”, International Journal of Mathematical Sciences, Autumn 2008 Vol.3, No. 08, 36-47.
32. Th.M. Rassias, “On the stability of the linear mapping in Banach spaces”, Proc.Amer.Math. Soc., 72 (1978), 297-300.
33. F. Skof, Sull’, “approssimazione delle applicazioni localmente δ-additive”, Atti Accad. Sci. TorinoCl. Sci. Fis. Mat. Natur. 117 (1983) 377–389.
34. S.M. Ulam, A Collection of Mathematical Problems, Interscience Publishers, New York, 1968,p. 63.
35. S.M. Ulam, Problems in Modern Mathematics, Science Editions,Wiley, NewYork, 1964 (Chapter VI, Some Questions in Analysis: 1, Stability).
36. <https://en.wikipedia.org/wiki/Coronavirus>